Output Growth and Inflation across Space and Time

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Abstract

It is common for comparisons to be made of output growth and inflation across groups of countries, yet such comparisons can result in inconsistencies. We address two problems: (i) how to measure aggregate real output and inflation for groups of countries and (ii) how to construct measures of real GDP for a group of countries where the country measures of real GDP are consistent across time and space. A method is proposed for harmonizing conflicting estimates of OECD member-country real GDP, ensuring consistency over space and overall group consistency over time. A new measure of OECD inflation is also proposed.

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“Econometricians have an ambivalent attitude towards economic data. At one level, the “data” are the world that we want to explain, the basic facts that economists purport to elucidate. At the other level, they are the source of all our troubles.”

Zvi Griliches (1985; 196)

1. Introduction

Providing consistent estimates of real GDP across countries and time is important for many policy-relevant purposes, such as assessing convergence of living standards; see Eurostat (2012) and the World Bank (2013). The OECD publishes estimates of Purchasing Power Parities (PPPs) on an annual basis and these PPPs can be used to generate estimates of real GDP for member countries that are comparable across countries for the given year. However, the resulting estimates of relative real GDP are inconsistent with national estimates of real GDP growth for the member countries.

We use OECD data for the years 2000-2012 in order to study two problems. First, how can estimates be constructed of OECD aggregate real GDP and associated measures of aggregate OECD inflation? Index number theory is used to decompose national nominal GDP into price and quantity (or volume) components, but constructing estimates of inflation and real GDP growth for a group of countries that use different currencies is a more complicated operation.[1]

Second, how can PPP information be used in conjunction with country data on real GDP growth to construct estimates of OECD member country real GDP that are in principle comparable across space and time? Using our proposed solution as a benchmark, we show that if PPP data are available only infrequently, as is the case for the World Bank provided PPP data used in the Penn World Tables, then estimates can differ considerably as new PPP information becomes available. This is rather inconvenient: studies of competitiveness and living standards convergence across countries will want to use real GDP series that are not subject to violent revision.

In section 2, we study the first measurement problem using just national data and exchange rate information. Sections 3-6 use the OECD PPP data and study the second measurement problem (and revisit the first problem). In section 3, we propose a harmonized method for constructing estimates of member country GDP volumes that are comparable across time and space. In sections 4 and 5, we compare our harmonized estimates with estimates of member country real GDP that are generated by using PPP

[1] “At the national level, current price (value) data can typically be decomposed into a volume (or quantity) series and price series. At the international level, a second ‘price’ component enters the picture in the form of a conversion rate from the domestic to a common currency. The implication is that values can be expressed at current market exchange rates (or current international prices, if purchasing power parities - *PPPs* - are used); and at constant exchange rates (or constant international prices).” OECD (2001; 6).

data for only one year. These base year estimates are then projected to all other years using national growth rates of GDP. Section 4 uses the PPPs for 2000 and section 5 uses the PPPs for 2012. We find that the resulting two panel sets of real GDP estimates are very different from each other and they also are very different from our harmonized estimates developed in section 3. Section 6 considers the context where PPPs are only available infrequently, as is the case with World Bank provided PPPs. [2] We use PPP information for 2000 and 2012 to provide interpolated estimates of country volumes, finding that the current interpolation method implemented in the widely used Penn World Tables did not work well with our OECD data base. In contrast, we find that our proposed method produces estimates that are much closer to our preferred harmonized estimates of section 3. Section 7 concludes.

2. OECD Growth and Inflation Using Market Exchange Rates

Our first measure of aggregate GDP growth over the member countries in the OECD during the years 2000-2012 uses national growth rates of GDP and domestic prices converted into US dollars at market exchange rates. The aggregation principle used to form OECD aggregate GDP volumes and prices in this section is the same that is used to aggregate prices and quantities across different regions in a country: each commodity in each region is regarded as a separate commodity in the index number formula. In what follows, we use the OECD ordering of countries, which is as in Table 1.

Table 1: OECD Country Codes

1= Australia	10= France	19= Korea	28= Slovenia
2= Austria	11= Germany	20= Luxembourg	29= Spain
3= Belgium	12= Greece	21= Mexico	30= Sweden
4= Canada	13= Hungary	22= Netherlands	31= Switzerland
5= Chile	14= Iceland	23= New Zealand	32= Turkey
6= Czech Republic	15= Ireland	24= Norway	33=U.K.
7= Denmark	16= Israel	25= Poland	34= United States
8= Estonia	17= Italy	26= Portugal	
9= Finland	18= Japan	27=Slovak Republic	

The country values for nominal GDP in the national currencies for the years 2000-2012 can be obtained from the OECD electronic data base, OECD.Stat. [3] We convert these

[2] For example, the World Bank provided PPPs for 155 countries for the year 2005 and has just provided a new set of PPPs for 2011. How can these two benchmark sets of PPPs be used in conjunction with national data in order to provide estimates of country real GDP that are comparable across all years from 2005 to 2012? The interpolation method explained in section 6 could be used in this context.
[3] OECD.Stat Table B1-GE: Gross domestic product (expenditure approach); National currency, current prices, millions, annual data.

estimates into billions and denote the estimate for country n in year t by V_n^t. The corresponding volume estimates can be obtained from the same source, [4] and we similarly convert these estimates into billions and denote these volumes (or quantities) by Q_n^t for n = 1,...,34 and t = 2000, ...,2012. The corresponding country price level for country n in year t is defined as $P_n^t \equiv V_n^t/Q_n^t$ for n = 1,...,34 and t = 2000,...,2012. These national price levels and volumes are listed in the Appendix; see Tables A1 and A2.

Since the country volumes Q_n^t are measured in domestic currency units (which are not comparable across countries), we need to convert the domestic nominal values of GDP into common currency units using the average exchange rates for each year. In principle, the numeraire country could be any of the 34 OECD countries but it seems reasonable to choose the largest country as the numeraire country. The OECD has conveniently done this for us, converting each country's nominal GDP into US dollars at the average market exchange rates for the given year.[5] We convert these estimates into billions and denote the US dollar estimate for nominal GDP for country n in year t by v_n^t.

The year t, country n US dollar price level for GDP, p_n^t, is initially defined as v_n^t/Q_n^t where the country volumes or real outputs Q_n^t have already been defined using national data. The resulting p_n^t were normalized so that $p_n^{2000} = 1$ for n = 1,...,34. The Q_n^t were then normalized in the opposite direction so that US dollar values were preserved. Denote the resulting normalized Q_n^t as q_n^t for n = 1,...,34 and t = 2000,...,2012.[6] These US dollar price levels p_n^t and the corresponding volumes q_n^t are listed in Tables A3 and A4 in the Appendix.

We are now in a position to calculate aggregate OECD real output and the corresponding price level for the years 2000-2012 using the price and volume data, p_n^t and q_n^t, as inputs into an index number formula. It will be useful to discuss the choice of index number formula in the context of providing index levels for two periods, say periods 0 and 1.[7] Suppose there are N commodities to be aggregated. Denote the price and quantity vectors for period t by $p^t \equiv [p_1^t,...,p_N^t]$ and $q^t \equiv [q_1^t,...,q_N^t]$ for t = 0,1. The value of transactions in the N commodities during period t is defined as $v^t \equiv \sum_{n=1}^N p_n^t q_n^t \equiv p^t \cdot q^t$.[8] The problem of choosing functional forms for the price and quantity indexes is usually phrased as follows: find two suitable functions of 4N variables, a price index function $P(p^0,p^1,q^0,q^1)$ and a quantity index function $Q(p^0,p^1,q^0,q^1)$, such that the product

[4] OECD.Stat TableB1-GE: Gross domestic product (GDP); National currency, constant prices, national base year, millions, annual data.

[5] OECD.Stat Table B1-GE: Gross domestic product (expenditure approach); US dollars, current prices, current exchange rates, millions, annual data.

[6] Note that $v_n^t = p_n^t q_n^t$ for n = 1,...,34 and t = 2000,...,2012. For each n, the US dollar volumes q_n^t are proportional to the national volumes Q_n^t; i.e., we have $q_n^t = \lambda_n Q_n^t$ for t = 2000,...,2012 for each country n where λ_n is the factor of proportionality for country n.

[7] For materials on the historical development of index number theory, see Diewert (1993) and Balk (2008).

[8] The inner product of two vectors $x \equiv [x_1,...,x_N]$ and $y \equiv [y_1,...,y_N]$ of the same dimension N is defined as $x \cdot y \equiv \sum_{n=1}^N x_n y_n$.

of these two functions is equal to the value ratio, v^1/v^0. Thus the functions P and Q are to satisfy the following equation:

$$(1)\ p^1\cdot q^1/p^0\cdot q^0 = P(p^0,p^1,q^0,q^1)Q(p^0,p^1,q^0,q^1).$$

It can be seen that if the functional form for either the price or quantity index is determined then the functional form of the corresponding quantity or price index is also determined using equation (1).[9]

Two natural choices for the functional form for the price index are the well-known Laspeyres and Paasche price indexes, P_L and P_P, defined as follows:[10]

$$(2)\ P_L(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^0/p^0\cdot q^0\ ;$$
$$(3)\ P_P(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^1/p^0\cdot q^1\ .$$

Using (1), it can be seen that quantity indexes that match up with P_L and P_P are Q_P and Q_L defined as follows:

$$(4)\ Q_P(p^0,p^1,q^0,q^1) \equiv p^1\cdot q^1/p^1\cdot q^0\ ;$$
$$(5)\ Q_L(p^0,p^1,q^0,q^1) \equiv p^0\cdot q^1/p^0\cdot q^0\ .$$

The Paasche and Laspeyres price and quantity indexes are equally plausible. The problem is that they can generate quite different estimates of growth and inflation. A natural solution to this problem is to take a symmetric average of these two equally plausible estimates; taking the geometric mean of these two price indexes (and of the two corresponding quantity indexes) leads to indexes that have very good axiomatic properties.[11] This leads to the Fisher (1922) ideal price and quantity indexes, P_F and Q_F, defined as follows:[12]

$$(6)\ P_F(p^0,p^1,q^0,q^1) \equiv [P_L(p^0,p^1,q^0,q^1)P_P(p^0,p^1,q^0,q^1)]^{1/2};$$
$$(7)\ Q_F(p^0,p^1,q^0,q^1) \equiv [Q_L(p^0,p^1,q^0,q^1)Q_P(p^0,p^1,q^0,q^1)]^{1/2}.$$

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[9] Once P and Q satisfying (1) have been chosen, the corresponding price levels for periods 0 and 1, say P^0 and P^1, and the corresponding quantity (or volume) levels for periods 0 and 1, say Q^0 and Q^1, are generally determined as follows: $P^0 \equiv 1$; $P^1 \equiv P(p^0,p^1,q^0,q^1)$; $Q^0 \equiv v^0 = p^0\cdot q^0$ and $Q^1 \equiv v^0Q(p^0,p^1,q^0,q^1) = v^1/P(p^0,p^1,q^0,q^1)$. Note that the price and quantity indexes can be interpreted as *ratios* of aggregate price and quantity *levels*; i.e., we have $P(p^0,p^1,q^0,q^1) = P^1/P^0$ and $Q(p^0,p^1,q^0,q^1) = Q^1/Q^0$.

[10] It can be seen that the Laspeyres price index uses the "basket" of period 0 quantities, q^0, and prices out this basket at the prices of period 0 (in the denominator) and prices out the same basket at the prices of period 1 (in the numerator) and takes the ratio of these costs as the price index. The Paasche index is similar but uses the "basket" of period 1 quantities, q^1, as the common quantity vector in the numerator and denominator.

[11] See Fisher (1922) and Diewert (1992) (1997).

[12] It can be verified that $P_FQ_F = v^1/v^0$; i.e., the Fisher price and quantity indexes satisfy equation (1).

There is one more choice that needs some discussion: namely, should fixed base or chained Fisher indexes be used when aggregating over many periods? The chain system measures the change in prices going from one period to a subsequent period using a bilateral index number formula involving the prices and quantities pertaining to the two adjacent periods. [13] These one period rates of change (the links in the chain) are then cumulated to yield the relative levels of prices over the entire period under consideration. If the bilateral price index is P, the chain system generates the following sequence of price levels for the first three periods:

(8) $1, P(p^0,p^1,q^0,q^1), P(p^0,p^1,q^0,q^1)\, P(p^1,p^2,q^1,q^2)$.

The fixed base system of price levels using the same bilateral index number formula P simply computes the level of prices in period t relative to the base period 0 as $P(p^0,p^t,q^0,q^t)$. The fixed base sequence of price levels for periods 0, 1 and 2 is:

(9) $1, P(p^0,p^1,q^0,q^1), P(p^0,p^2,q^0,q^2)$.

There are two major problems associated with the use of fixed base indexes in the context of annual time series data: (i) over longer periods of time, it becomes more difficult to match up products in the current period with the corresponding products in a distant base period, leading to less accurate index numbers; and (ii) fixed base indexes are subject to revisions (that can be substantial) when the base period is finally changed.

When using fixed base Paasche or Laspeyres indexes, the revision problem can become massive. [14] Thus a major advantage of the chain system in the context of aggregating annual data is that chaining will reduce the spread between the Paasche and Laspeyres indexes. [15] These two indexes each provide an asymmetric perspective on the amount of price change that has occurred between the two periods under consideration and it could be expected that a single point estimate of the aggregate price change should lie between these two estimates. Chaining will usually lead to a smaller difference between the two and hence to estimates that are closer to the "truth". [16]

[13] The chain principle was introduced independently into the economics literature by Lehr (1885; 45-46) and Marshall (1887; 373). Both authors observed that the chain system would mitigate the difficulties due to the introduction of new commodities into the economy, a point also mentioned by T.P. Hill (1993; 388). Fisher (1911; 203) introduced the term "chain system".

[14] The US Bureau of Economic Analysis used to provide long term estimates of US GDP back to 1926 using fixed base Laspeyres volume indexes. When the base year was changed, the resulting Laspeyres estimates of real GDP growth changed massively and this fact led the BEA to switch to chained Fisher indexes in the early 1990s.

[15] See Diewert (1978; 895) and T.P. Hill (1988) (1993; 387-388).

[16] There is a more elaborate justification for chaining annual data that is based on aggregating over observations that have the most "similar" price structures; see R.J. Hill (2001), (2004) (2009) and Diewert (2009). Typically, adjacent annual observations will have more similar price structures than a pair of

For year t = 2001,...,2012, denote the chained Fisher aggregate OECD volume level for by Q^t and the corresponding US dollar price level by P^t, and define the OECD *volume growth rate* γ^t and the corresponding OECD *US dollar inflation rate* ρ^t in percentage points as follows:

(10) $\gamma^t \equiv 100[(Q^t/Q^{t-1}) - 1]$;
(11) $\rho^t \equiv 100[(P^t/P^{t-1}) - 1]$.

The chained Fisher OECD aggregate price and volume levels, P^t and Q^t, for the years 2000-2012 are listed in Table 2 along with the corresponding percentage point annual growth rates, ρ^t and γ^t, for the years 2001-2012. For comparison purposes, we also provide the aggregate OECD chained Laspeyres and Paasche indexes, $P_L{}^t$ and $P_P{}^t$, over the same period. It can be seen that the chained Fisher, Laspeyres and Paasche price levels are all very close to each other so that for this particular application, the choice of index number formula does not matter very much.

Table 2: OECD Annual Aggregate Volumes Q^t and Price Levels in US Dollars P^t, $P_L{}^t$ and $P_P{}^t$, Price Levels in Euros $P_{EU}{}^t$, PPP Price Levels $P_{PPP}{}^t$ and Percentage Point Changes, 2000-2012

Year t	Q^t	P^t	$P_L{}^t$	$P_P{}^t$	γ^t	ρ^t	$\rho_{EU}{}^t$	$P_{EU}{}^t$	$P_{PPP}{}^t$
2000	26694.3	1.000	1.000	1.000				1.000	1.000
2001	27022.9	0.979	0.979	0.980	1.23	-2.06	0.84	1.008	1.030
2002	27432.9	1.008	1.008	1.008	1.52	2.92	-2.14	0.987	1.055
2003	28007.3	1.108	1.108	1.108	2.09	9.90	-8.35	0.904	1.080
2004	28896.6	1.193	1.193	1.193	3.18	7.71	-2.10	0.885	1.107
2005	29670.9	1.227	1.227	1.227	2.68	2.84	2.68	0.909	1.133
2006	30566.7	1.256	1.256	1.256	3.02	2.35	1.46	0.922	1.161
2007	31374.2	1.339	1.339	1.340	2.64	6.63	-2.27	0.902	1.189
2008	31410.0	1.411	1.410	1.411	0.11	5.36	-1.56	0.887	1.217
2009	30267.1	1.373	1.373	1.373	-3.64	-2.69	2.60	0.911	1.231
2010	31138.6	1.401	1.401	1.402	2.88	2.07	7.06	0.975	1.248
2011	31688.5	1.478	1.477	1.478	1.77	5.44	0.46	0.979	1.270
2012	32162.6	1.453	1.453	1.454	1.50	-1.63	6.42	1.042	1.289

observations chosen from different decades. However, it is not always best to use chained indexes. T.P. Hill (1993; 388), drawing on the earlier research of Szulc (1983) and T.P. Hill (1988; 136-137), noted that it is not appropriate to use the chain system when prices oscillate or "bounce" to use Szulc's (1983; 548) term. This bouncing phenomenon can occur when aggregating subannual data when there are seasonal fluctuations or periodic sales (deeply discounted prices). However, in the context of more or less smoothly trending prices and quantities as is the usual case using annual data, T.P. Hill (1993; 389) recommended the use of chained symmetrically weighted indexes such as the Fisher ideal index. Thus in this paper, we will use chained Fisher indexes when aggregating over countries.

The sample average of the year to year growth rates for OECD real GDP using US dollar weights, the γ^t, was 1.58% per year. It can be seen that there was only one year where OECD real growth was negative: 2009 (−3.64%). The sample average of the OECD inflation rates ρ^t (measured in US dollars at market exchange rates) was 3.24% per year. However, what is striking is the variability in these US dollar inflation rates.

The principles used to construct our OECD aggregate measures of real GDP, Q^t, are the same principles used to construct country wide estimates of real GDP within a country. Estimates of real GDP aggregate output growth over regions within the country use regional price levels as weights for the regional volumes. In constructing national estimates of real output, the national statistician does not assume that the quantities or volumes in each region are comparable across regions; all that is assumed is that whatever is being measured at the regional level *is comparable over time*. This is the same principle that is being used to construct the above OECD real output measures Q^t; there is no assumption that the country units of measurement are comparable across countries.

The one difference in our suggested method for constructing OECD real GDP as opposed to methods used to construct national estimates of real GDP is that we needed to convert national values of GDP into a common currency using annual average market exchange rates. We chose to make this conversion using US dollars as the numeraire currency. If we chose another currency to be the numeraire currency, the unit of measurement would change, but the overall OECD *growth rates* for real GDP would remain the same; i.e., the γ^t listed in Table 2 *do not change* if we converted all country nominal GDP estimates into a different common currency at annual average market exchange rates and then applied the same methodology to construct the overall OECD volume estimates.[17] On the other hand, switching to a different numeraire currency dramatically affects the inflation rates ρ^t; the OECD aggregate price level estimates P^t and the resulting inflation rates ρ^t defined by (11) change with each choice of a numeraire currency.

In order to illustrate the dependence of the above OECD GDP inflation rates on the choice of the numeraire country, we computed the aggregate OECD price and volume levels, $P_{EU}{}^t$ and $Q_{EU}{}^t$, using Germany as the numeraire country. Thus instead of using the US dollar estimates for nominal GDP for country n in year t defined earlier by $v_n{}^t$, for Euro zone countries we use the reported national value estimates of GDP. For non-Euro zone countries, we converted the $v_n{}^t$ into Euros using the implied OECD exchange rate that can be obtained by dividing the national value estimate of GDP for Germany (or any other Euro zone country) by the corresponding US dollar measure. The same Fisher index

[17] In order for this statement to be true, we need our chosen bilateral index number formula to satisfy the following two tests: $Q(\lambda p^0,p^1,q^0,q^1) = Q(p^0,p^1,q^0,q^1)$ for all scalar $\lambda > 0$ and $Q(p^0,\lambda p^1,q^0,q^1) = Q(p^0,p^1,q^0,q^1)$ for all scalar $\lambda > 0$. The Fisher, Laspeyres and Paasche bilateral quantity indexes all satisfy these homogeneity-in-prices properties.

number methodology was then used to construct $P_{EU}{}^t$ and $Q_{EU}{}^t$. The resulting Euro based price index $P_{EU}{}^t$ and inflation growth rates $\rho_{EU}{}^t \equiv 100[(P_{EU}{}^t/P_{EU}{}^{t-1}) - 1]$ are listed in Table 2. Comparing the inflation measures using the US and then Germany as the numeraire countries shows that the resulting price levels, P^t and $P_{EU}{}^t$, and inflation rates, ρ^t and $\rho_{EU}{}^t$, are *completely different*. P^t trends upward from 1.00 in 2000 to end up at 1.45 in 2012 whereas the Euro based OECD price level trends downward to 0.89 in 2008 and then trend upward to end up at 1.04 in 2012. The explanation for these diverging results is simple: they are driven by large exchange rate movements over the sample period.[18]

Our conclusion at this point is that our *first approach* to measuring OECD real output and inflation using national GDP data and market exchange rates between countries is (perhaps) satisfactory for measuring real output but that it is *not* satisfactory for measuring inflation. A satisfactory inflation measure will be introduced in the following section when we introduce our *second approach* to measuring aggregate OECD inflation.

The analysis presented in this section made no assumption that the goods and services produced in any country were comparable to the goods and services produced in any other country. In the following section, it will be assumed that the goods and services produced in each country *are comparable across countries* and different measures of OECD growth and inflation will be derived.

3. OECD Growth and Inflation Measurement Using Annual PPP Information

The OECD (in close cooperation with Eurostat) produces an annual series of Purchasing Power Parities (PPPs) that enable the comparison of real GDP of member countries with each other.[19] For each OECD country n and each year t, $PPP_n{}^t$ is an estimate of the number of units of the national currency of country n that is required to purchase one dollar of US GDP in year t.[20] We divide the country n nominal value of GDP in year t in domestic currency, $V_n{}^t$, by the corresponding $PPP_n{}^t$ in order to obtain an estimate, $r_n{}^t$, of country n's *real GDP* in year t in units that are comparable across countries for year t:[21]

$$(12)\ r_n{}^t \equiv V_n{}^t/PPP_n{}^t\ ; \qquad\qquad n = 1,\ldots,34;\ t = 2000,\ldots,2012.$$

[18] US prices in terms of Euros declined markedly from 2000 to 2008 and this explains the large number of negative $\rho_{EU}{}^t$ over this period; the number of Euros it took to buy one US dollar in 2000, 2008 and 2012 was 1.085, 0.683 and 0.778, respectively.

[19] The construction of these PPPs is explained in the Eurostat and OECD PPP Manual; see Eurostat (2012). The International Comparison Program (ICP) of the World Bank constructed PPPs for over 150 countries for 2005 and 2011. The ICP methodology is explained in World Bank (2013).

[20] OECD.Stat, Table 4: PPPs and Exchange Rates; PPPGDP; Purchasing Power Parities for GDP; National currency per US dollar; Annual; 2000-2012. This table is reproduced in the Appendix as Table A5.

[21] These relative GDP volume measures for year t are not comparable across years.

Once the r_n^t have been calculated, they can be summed so that $r^t \equiv \sum_{n=1}^{34} r_n^t$ and then the *year t country n share of OECD real output* can be defined as follows:[22]

$$(13)\ s_n^t \equiv r_n^t/r^t \ ; \qquad\qquad n = 1,...,34;\ t = 2000,...,2012.$$

These country shares of OECD real GDP are listed in Table 3, which enables the comparison of GDP volumes across all OECD countries within each year.[23] Note that country 34, the US, has the largest share (around 35-37%), followed by country 18, Japan, (10-11%) and country 11, Germany (7%).

We can use this information to construct estimates of overall real GDP growth and inflation across OECD countries. A natural method is to use the country shares in Table 3 as weights for national rates of growth of real GDP. The year t growth factor for country n can be defined as Q_n^t/Q_n^{t-1} where Q_n^t is country n's GDP volume in year t, and the OECD Laspeyres type growth factor (or chain link) for year t, Γ_L^t, as the following weighted average of the national growth factors:

$$(14)\ \Gamma_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(Q_n^t/Q_n^{t-1}) \ ; \qquad\qquad t = 2001,...,2012.$$

The measure of OECD GDP volume growth defined by (14) is the method used by the OECD to calculate their official measure of OECD volume growth. It certainly is a sensible measure, using country (one plus) growth rates going from year t−1 to year t, Q_n^t/Q_n^{t-1}, weighted by the country real volume shares s_n^{t-1} for year t−1, which were derived using PPPs. However, the above formula suffers from the same problem that the standard Laspeyres formula has: namely, it does not treat the periods in a symmetric fashion.

The counterpart to the Laspeyres-type formula defined by (14) is the following *Paasche-type formula*:[24]

$$(15)\ \Gamma_P^t \equiv \left[\sum_{n=1}^{34} s_n^t(Q_n^t/Q_n^{t-1})^{-1}\right]^{-1} ; \qquad\qquad t = 2001,...,2012.$$

[22] Note that the country shares s_n^t can be constructed without using country exchange rates (in principle). Using definitions (12) and (13), it can be seen that the s_n^t can be written in the following form: $s_n^t = [V_n^t/PPP_n^t]/[\sum_{i=1}^N (V_i^t/PPP_i^t)]$ for all n and t. Compare these "real" shares s_n^t with the corresponding country US dollar shares $S_n^t = [V_n^t/e_n^t]/[\sum_{i=1}^N (V_i^t/e_i^t)]$ defined in the Appendix. All of the measures derived in this section are independent of country exchange rates.

[23] Row n+1 in the Table gives the shares for country n where we use the standard ordering of OECD countries listed in the previous section. Since the PPPs used by the OECD are invariant to the choice of the numeraire country (up to a scalar factor), it can be verified that the country shares listed in Table 3 are also invariant to the choice of numeraire country.

[24] Suppose that there is only one homogeneous commodity in each country's GDP. Then the volume for country n in year t, Q_n^t, should be equal to the number of units of this homogeneous commodity. Under these conditions, it can be seen that both Γ_L^t and Γ_P^t equal $\sum_{n=1}^{34} Q_n^t / \sum_{n=1}^{34} Q_n^{t-1}$.

Table 3: Country Shares of OECD Real GDP 2000-2012

n	s_n^{2000}	s_n^{2001}	s_n^{2002}	s_n^{2003}	s_n^{2004}	s_n^{2005}	s_n^{2006}	s_n^{2007}	s_n^{2008}	s_n^{2009}	s_n^{2010}	s_n^{2011}	s_n^{2012}
1	0.019	0.019	0.020	0.020	0.020	0.020	0.020	0.020	0.020	0.022	0.022	0.022	0.023
2	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008
3	0.010	0.010	0.010	0.010	0.010	0.009	0.009	0.009	0.009	0.010	0.010	0.010	0.010
4	0.031	0.031	0.031	0.031	0.031	0.032	0.031	0.031	0.031	0.031	0.031	0.032	0.031
5	0.005	0.005	0.005	0.005	0.006	0.006	0.007	0.007	0.007	0.007	0.008	0.008	0.008
6	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.007	0.006	0.006	0.006
7	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
8	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
9	0.005	0.005	0.005	0.005	0.005	0.004	0.005	0.005	0.005	0.005	0.005	0.005	0.005
10	0.054	0.055	0.055	0.053	0.052	0.052	0.052	0.052	0.052	0.053	0.052	0.052	0.052
11	0.074	0.074	0.074	0.073	0.072	0.072	0.072	0.072	0.073	0.071	0.072	0.073	0.073
12	0.007	0.007	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.007	0.007	0.006
13	0.004	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005	0.005
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.004	0.004	0.004	0.004
16	0.005	0.005	0.005	0.005	0.005	0.005	0.004	0.005	0.004	0.005	0.005	0.005	0.005
17	0.051	0.052	0.050	0.049	0.047	0.046	0.047	0.047	0.048	0.047	0.046	0.045	0.044
18	0.115	0.114	0.113	0.112	0.111	0.109	0.106	0.105	0.103	0.099	0.101	0.098	0.097
19	0.028	0.029	0.030	0.030	0.031	0.031	0.031	0.031	0.031	0.032	0.033	0.034	0.033
20	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
21	0.035	0.034	0.034	0.035	0.035	0.036	0.037	0.038	0.039	0.039	0.040	0.043	0.044
22	0.016	0.017	0.017	0.016	0.016	0.016	0.016	0.016	0.017	0.017	0.016	0.016	0.016
23	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
24	0.006	0.006	0.005	0.005	0.006	0.006	0.007	0.006	0.007	0.006	0.007	0.007	0.007
25	0.014	0.014	0.014	0.014	0.015	0.015	0.015	0.016	0.016	0.018	0.018	0.018	0.019
26	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006	0.006
27	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.003	0.003	0.003	0.003	0.003	0.003
28	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
29	0.030	0.031	0.032	0.033	0.033	0.033	0.035	0.036	0.036	0.036	0.034	0.033	0.032
30	0.009	0.008	0.009	0.009	0.009	0.008	0.008	0.009	0.009	0.008	0.009	0.009	0.009
31	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.008	0.009	0.009	0.009	0.009	0.009
32	0.021	0.019	0.019	0.018	0.020	0.022	0.023	0.024	0.026	0.025	0.027	0.028	0.029
33	0.054	0.056	0.056	0.056	0.057	0.056	0.056	0.054	0.054	0.053	0.052	0.051	0.051
34	0.361	0.359	0.357	0.361	0.363	0.365	0.361	0.357	0.352	0.353	0.352	0.351	0.353

Note: n denotes the country code, given in Table 1.

The corresponding Fisher-type formula for OECD volume growth for year t is defined as follows:[25]

(16) $\Gamma_F{}^t \equiv [\Gamma_L{}^t\,\Gamma_P{}^t]^{1/2}$; $\qquad$ t = 2001,...,2012.

The growth factors (or chain link indexes) defined by (14)-(16) can be multiplied together to generate OECD aggregate volume levels. The growth factors can also be transformed into growth rates, $\gamma_L{}^t$, $\gamma_P{}^t$ and $\gamma_F{}^t$ (in percentage points), by using the following definitions for t = 2001,...,2012:

(17) $\gamma_L{}^t \equiv 100[\Gamma_L{}^t - 1]$; $\gamma_P{}^t \equiv 100[\Gamma_P{}^t - 1]$; $\gamma_F{}^t \equiv 100[\Gamma_F{}^t - 1]$.

The annual OECD volume growth measures defined by (17) as well as our earlier US dollar weighted measures γ^t are listed in Table 4. It can be seen that the Laspeyres, Paasche and Fisher measures of OECD growth explained in this section are virtually identical so that moving from the OECD Laspeyres-type measure of overall volume growth to the Fisher measure did not make much difference for this data set.[26] It can also be seen that our preferred Fisher measure of OECD growth in comparable units across countries, $\gamma_F{}^t$, grew on average about 1/10 of a percentage point more rapidly per year than our preferred measure of OECD GDP growth using US dollar weights, γ^t. Although this is not a large difference in growth rates, it is significant and so users need to decide which measure, $\gamma_F{}^t$ or γ^t, best suits their needs.

The measure γ^t can be defined using just national information on domestic price and quantity (or volume) indexes and exchange rates while the measure $\gamma_F{}^t$ requires information on domestic values, domestic volume indexes and PPPs. PPPs are not likely to be nearly as accurate as national measures of price and volume change due to the difficulties in matching products across countries. There are additional difficulties with the treatment of international trade in the construction of PPPs.

The γ^t measure has the problem that large fluctuations in exchange rates can lead to fluctuations in the γ^t while the PPP based $\gamma_F{}^t$ measures are theoretically independent from exchange rate movements.[27] Thus one has to weigh the disadvantage of possibly less

[25] If the PPPs are independent of the choice of the numeraire country (up to a scalar factor), then the growth factors, $\Gamma_L{}^t\,\Gamma_P{}^t$ and $\Gamma_F{}^t$ will not depend on the choice of the numeraire country.

[26] Recall that the official OECD measure of real GDP growth is the Laspeyres measure, $\gamma_L{}^t$. Our estimates differ slightly from the official measures due to rounding. The exchange-rate-weighted growth rates γ^t should be somewhat lower than the PPP-weighted growth rates $\gamma_F{}^t$ due to the Balassa-Samuelson effect and this expectation is realized for the OECD data. We would expect the divergence to grow as less rich countries are added to the list of countries.

[27] Exchange rate movements do not directly affect the domestic rates of growth (the $Q_n{}^t/Q_n{}^{t-1}$) but as we have seen, they do affect the *weights* used to aggregate the country real growth rates into the overall OECD

reliable PPPs against the advantage of having aggregate growth measures that are independent from exchange rate movements.[28]

Table 4: Annual Percentage Point Changes in OECD PPP Based Laspeyres, Paasche and Fisher Volume Measures γ_L^t, γ_P^t and γ_F^t, US Dollar Weighted Volume Measures γ^t and Laspeyres, Paasche and Fisher PPP Based Inflation Measures, ρ_L^t, ρ_P^t and ρ_F^t: 2001-2012

Year t	γ_L^t	γ_P^t	γ_F^t	γ^t	ρ_L^t	ρ_P^t	ρ_F^t
2001	1.291	1.296	1.294	1.231	3.216	2.798	3.007
2002	1.683	1.677	1.680	1.517	2.514	2.312	2.413
2003	2.167	2.161	2.164	2.094	2.386	2.290	2.338
2004	3.327	3.333	3.330	3.175	2.526	2.501	2.513
2005	2.832	2.831	2.831	2.680	2.341	2.336	2.338
2006	3.153	3.159	3.156	3.019	2.512	2.500	2.506
2007	2.707	2.707	2.707	2.642	2.452	2.435	2.443
2008	0.191	0.190	0.191	0.114	2.359	2.343	2.351
2009	-3.571	-3.574	-3.573	-3.639	1.092	1.081	1.086
2010	2.995	3.001	2.998	2.879	1.408	1.389	1.399
2011	1.956	1.963	1.960	1.766	1.763	1.762	1.762
2012	1.543	1.530	1.537	1.496	1.507	1.498	1.502
Average	1.689	1.690	1.690	1.581	2.173	2.104	2.138

The OECD real output shares, s_n^t defined by (13), can also be used as weights for national GDP inflation rates. We define the *OECD Laspeyres, Paasche and Fisher PPP based chain link price indexes*, Π_L^t, Π_P^t and Π_F^t for t = 2001,...,2012, as follows:[29]

(18) $\Pi_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(P_n^t/P_n^{t-1})$; $\Pi_P^t \equiv [\sum_{n=1}^{34} s_n^t(P_n^t/P_n^{t-1})^{-1}]^{-1}$; $\Pi_F^t \equiv [\Pi_L^t\,\Pi_P^t]^{1/2}$.

These chain link indexes can be multiplied together to generate the corresponding OECD aggregate price levels. The resulting Fisher OECD price level index for year t is denoted by P_{PPP}^t and it is listed in the last column of Table 2.[30] The inflation growth factors can

Laspeyres, Paasche and Fisher growth rates. Exchange rate fluctuations are large enough to materially affect the weights, which in turn lead to material fluctuations in the overall OECD volume growth rates.

[28] It will generally be the case that the s_n^t will be greater than the corresponding S_n^t for countries n that are relatively poor and thus the index of OECD aggregate real GDP growth defined in section 2 will tend to be a more *plutocratic index* (since rich countries get larger share weights in this index) compared to the more *democratic index* of OECD aggregate real GDP growth defined in section 3. Thus one could choose between the two indexes based on one's preferences over weights. We owe this point to Marshall Reinsdorf.

[29] The official OECD measure of household inflation over member countries is the Laspeyres measure defined in (18) where household consumption replaces GDP; see the OECD (2014).

[30] This price index satisfies the time reversal test whereas its Laspeyres and Paasche counterparts do not satisfy this important test. Hence the Fisher PPP based inflation index P_{PPP}^t is our preferred measure of OECD aggregate inflation.

also be transformed into *growth rates*, $\rho_L{}^t$, $\rho_P{}^t$ and $\rho_F{}^t$ in percentage points, by using the following definitions for t = 2001,...,2012:

(19) $\rho_L{}^t \equiv 100[\Pi_L{}^t - 1]$; $\rho_P{}^t \equiv 100[\Pi_P{}^t - 1]$; $\rho_F{}^t \equiv 100[\Pi_F{}^t - 1]$.

These PPP based inflation rates (in percentage points) are listed in the last 3 columns of Table 4. The sample averages of the $\rho_L{}^t$, $\rho_P{}^t$ and $\rho_F{}^t$ are 2.17, 2.10 and 2.14 percentage points. Viewing Table 4, it can be seen that there are some significant differences between the three measures of OECD inflation that are PPP based.[31] Comparing the numbers in tables 2 and 4, it can be seen that the PPP based estimates of OECD inflation are much more reasonable than the estimates that were based on country exchange rates that were listed in Table 2, the ρ^t and $\rho_{EU}{}^t$. Our conclusion is that the OECD Fisher price index $P_{PPP}{}^t$ is a much better measure of OECD inflation than the indexes that used exchange rates instead of PPPs.

Now we come to the most difficult problem: how can we use PPP information and national growth rates to obtain estimates of member country GDP volumes that are comparable across time and space? The Eurostat (2012) *Manual* offers the following advice:

> "To trace the evolution of relative GDP volume levels between countries over time, it is necessary to select one of the reference years as a base year and to extrapolate its relative GDP volume levels over the other years. Extrapolation is done by applying the relative rates of GDP volume growth observed in the different countries. This provides a time series of volume indices at a constant uniform price level that replicates exactly the relative movements of GDP volume growth of each country." Eurostat (2012; 18).

We implement this strategy in sections 4 and 5 below, where we choose the relative country GDP volumes given by the country shares of OECD aggregate GDP for 2000 (section 4) and for 2012 (section 5) and we use national growth rates for country GDP volumes to extrapolate these base shares to all time periods. However, it will be seen that the resulting comparable country volumes over time and space differ considerably, depending upon which base year is chosen. This is rather inconvenient: studies of competitiveness of OECD countries and living standards convergence across countries will want to use country volume series that are not subject to violent revision.[32]

[31] In view of these differences in the three indexes of OECD GDP inflation, it may be preferable for the OECD to replace their Laspeyres type indexes of OECD household inflation by their Fisher counterparts.

[32] McCarthy (2013; 484-486) explains in some detail why estimates of real GDP based on national growth information do not match up exactly with relative GDP estimates based on PPP benchmark information. The PPP information is generally not as accurate as national price index information due to the difficulty of matching representative products across countries. However, country methodology for constructing

Our suggested solution to the problem of harmonizing national growth rates of GDP with the country shares of OECD aggregate real GDP rests on two principles. First, the resulting harmonized estimates of country volumes must be consistent with the real annual cross country volume shares s_n^t listed in Table 3 above. Second, OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes Γ_F^t defined by (16).

Using these principles, the country GDP volumes are uniquely determined (up to a scalar units-of-measurement factor). To see this, first define the OECD volume index that chains the Γ_F^t defined by (16) into a time series index, Q_H^t. Define Q_H^t as follows:

(20) $Q_H^{2000} \equiv 1$; $Q_H^t \equiv Q_H^{t-1}\, \Gamma_F^t$; t = 2001,...,2012.

Now use the country shares of OECD real GDP s_n^t listed in Table 3 and the aggregate index Q_H^t to define the following preliminary *harmonized country volumes* for country n in year t, q_{Hn}^t, as follows:

(21) $q_{Hn}^t \equiv Q_H^t\, s_n^t$; n = 1,...,34; t = 2000,...,2012.

Note that for each year t, $\sum_{n=1}^{34} q_{Hn}^t = \sum_{n=1}^{34} Q_H^t\, s_n^t = Q_H^t\,(\sum_{n=1}^{34} s_n^t) = Q_H^t$ and so the harmonized volumes satisfy the two principles listed above. In principle, the country volumes defined by (21) are independent of country prices and exchange rates.[33]

It is of interest to define US dollar prices for real GDP for each country. Recall that the value of country n's nominal GDP converted into US dollars at market exchange rates for year t was defined as v_n^t. The corresponding *harmonized US dollar price* of a unit of (comparable across countries) real GDP for country n in year t is defined as follows:

(22) $p_{Hn}^t \equiv v_n^t/q_{Hn}^t$; n = 1,...,34; t = 2000,...,2012.

In order to make the harmonized volumes and prices defined by (21) and (22) comparable to the country prices and volumes expressed in US dollars that are listed in the Appendix

national price indexes differs considerably across countries; e.g., some countries may use out of date reference expenditure baskets, some countries use Carli indexes at the elementary level while others use the Jevons or Dutot indexes which generate lower estimates of inflation at the elementary level and some countries may use quality adjustment methods more extensively than others. All of these methodological differences lead to inconsistencies between the time series and cross sectional estimates. Finally, the index number formulae used at the national levels and in the construction of the benchmark PPPs are in general not transitive and so it is impossible to achieve perfect consistency.

[33] However, in practice, the PPPs do not do a perfect job in eliminating exchange rate effects (since adjusted exchange rates are used in place of true PPPs to deflate international trade flows). If the relative PPPs are independent of the choice of the numeraire country, then the relative volumes defined by (20) will also be independent of this choice.

in tables A3 and A4, we impose a normalization on the prices defined by (22) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (22) by a constant that sets the resulting p_{H34}^{2000} equal to 1 and the quantities or volumes defined by (21) are all multiplied by this constant. The resulting normalized q_{Hn}^{t} and p_{Hn}^{t} are listed in tables 5 and 6.

Note that $q_{H34}^{2000} = q_{34}^{2000}$ and $p_{H34}^{2000} = p_{34}^{2000} = 1$ so that country GDP volumes are measured as multiples of a bundle of US GDP in the year 2000. Thus the price levels in Table 6 measure the US dollar value of constant bundle of GDP that is (in theory) comparable across countries. The price levels in Table 6 are comparable across space and time, whereas the price levels p_n^t listed in Table A3 of the Appendix are only comparable across time for each country.

From Table 6, it can be seen that the countries with the lowest price levels (in US dollars) in 2012 are countries 13, 21, 25 and 32 (Hungary, Mexico, Poland and Turkey) with price levels in the 0.76 to 0.77 range. Countries with relatively high price levels in 2012 are countries 1 (Australia, $p_{H1}^{2012} = 2.00$), 4 (Canada, $p_{H4}^{2012} = 1.62$), 7 (Denmark, 1.77), 9 (Finland, 1.58), 18 (Japan, 1.76), 20 (Luxembourg, 1.56), 23 (New Zealand, 1.55), 24 (Norway, 2.01), 30 (Sweden, 1.69) and 31 (Switzerland, 1.96). These price level estimates are (imperfect) [34] indicators of the competiveness of the country on international markets, with lower price levels indicating greater competiveness.

A problem with the volume estimates listed in Table 5 is that they do not respect national growth rates of GDP by country; only the aggregate OECD growth rate is respected. In the following two sections, we will derive alternative country volume estimates that are comparable over time and space. These alternative estimates will respect country growth rates but they will not reproduce the real OECD country expenditure shares listed in Table 3 for all time periods.

[34] The price levels p_{Hn}^{t} are imperfect indicators of competiveness because not all components of GDP are internationally traded. Moreover, these price levels are not independent of the choice of the numeraire currency (US dollars in this case). They are also imperfect because they depend heavily on the accuracy of the underlying PPPs and these PPPs are subject to considerable error variances due to the difficulties involved in matching product prices (and quantities) across countries.

Table 5: Harmonized OECD Country GDP Volumes in Comparable US Dollar Units of Measurement $q_{Hn}{}^{t}$

n	$q_{Hn}{}^{2000}$	$q_{Hn}{}^{2001}$	$q_{Hn}{}^{2002}$	$q_{Hn}{}^{2003}$	$q_{Hn}{}^{2004}$	$q_{Hn}{}^{2005}$	$q_{Hn}{}^{2006}$	$q_{Hn}{}^{2007}$	$q_{Hn}{}^{2008}$	$q_{Hn}{}^{2009}$	$q_{Hn}{}^{2010}$	$q_{Hn}{}^{2011}$	$q_{Hn}{}^{2012}$
1	537.3	553.2	572.5	598.6	618.7	637.2	658.2	686.6	685.8	718.5	740.3	764.3	786.4
2	231.6	227.6	235.2	239.3	245.8	246.0	258.4	262.7	268.2	259.9	266.6	273.8	279.1
3	283.4	285.9	296.6	295.8	297.6	299.9	308.5	314.9	320.7	315.3	325.3	330.1	334.5
4	874.1	886.8	895.8	930.9	960.5	1006.5	1027.2	1050.1	1050.5	1018.4	1053.4	1080.3	1094.9
5	147.6	152.5	155.5	160.9	172.3	183.5	219.1	232.4	223.7	219.7	252.1	281.6	295.0
6	159.7	167.8	171.3	180.2	187.7	193.5	204.1	218.4	218.2	214.8	210.3	213.1	212.0
7	153.9	153.7	157.9	154.4	159.8	159.9	167.4	171.2	177.0	168.8	177.8	176.6	178.2
8	13.5	14.2	15.6	17.1	18.3	19.8	22.0	24.1	23.9	21.2	21.3	23.2	23.9
9	132.9	134.2	136.8	135.5	143.0	143.2	149.1	159.0	163.6	151.6	152.6	155.8	156.5
10	1533.0	1587.0	1628.6	1593.3	1613.0	1654.4	1701.9	1756.9	1772.1	1740.1	1760.9	1790.8	1791.5
11	2117.5	2144.2	2162.4	2202.5	2242.8	2281.5	2360.8	2431.0	2464.8	2330.8	2431.2	2516.6	2551.1
12	199.2	210.8	224.6	233.5	241.7	240.4	254.9	258.2	269.0	263.2	246.0	226.3	215.8
13	121.3	133.0	142.3	146.3	149.8	152.2	157.4	158.2	165.8	162.1	162.4	164.2	164.9
14	8.1	8.5	8.5	8.4	9.0	9.2	9.3	9.6	10.2	9.5	8.9	9.0	9.1
15	109.8	115.4	124.4	130.5	136.6	143.4	154.2	164.5	153.2	144.5	146.5	150.6	151.4
16	147.0	146.3	147.5	139.9	146.8	143.5	144.2	152.8	151.2	155.0	163.1	173.8	178.1
17	1466.5	1515.3	1471.0	1478.8	1466.6	1473.6	1530.6	1581.8	1614.8	1549.1	1533.1	1537.5	1524.6
18	3287.0	3288.1	3316.3	3357.2	3438.6	3458.3	3471.1	3548.1	3468.9	3234.7	3389.8	3350.2	3390.1
19	808.4	837.8	894.1	908.8	951.4	975.1	1003.2	1054.8	1056.5	1048.2	1116.9	1148.5	1163.3
20	23.4	23.2	24.5	25.8	27.2	28.2	31.7	33.7	33.3	31.2	33.4	34.9	35.4
21	987.1	983.6	1000.8	1043.2	1086.4	1150.3	1232.1	1273.9	1315.8	1282.3	1353.6	1475.9	1519.7
22	468.3	481.5	492.7	484.2	494.8	509.4	531.7	554.5	570.8	541.2	545.2	550.5	545.9
23	82.1	84.2	87.1	90.0	92.9	93.6	97.7	101.6	100.7	104.2	105.4	108	110.4
24	162.3	163.2	160.6	164.4	178.6	195.8	214.4	218.5	236.5	211.3	221.9	234.2	248.8
25	404.3	408.3	422.3	430.9	454.9	467.7	491.1	530.9	555.6	572.8	604.4	630.8	645.2
26	182.0	185.7	189.6	191.2	190.9	200.4	207.7	213.4	214.2	211.4	214.7	209.3	201.9
27	59.3	63.3	66.6	68.9	72.2	77.5	84.7	93.6	101.5	97.6	99.5	100.6	102.9
28	34.9	35.8	37.7	38.6	40.7	41.8	43.6	45.7	47.5	43.7	43.0	43.5	42.7
29	858.1	896.4	949.8	978.8	1014.8	1056.9	1144.6	1202.1	1221.6	1172.3	1145.6	1129.2	1118.6
30	248.0	244.8	249.6	256.5	267.6	262.5	277.0	292.5	295.4	276.7	290.8	301.7	309.2
31	233.6	234.8	241	238.7	243.0	244.4	261.7	280.9	296.5	290.0	300.3	313.1	321.8
32	589.3	546.9	546.5	553.4	630.4	694.6	764.7	811.3	863.7	829.6	909.8	975.7	1024.8
33	1552.1	1604.6	1646.0	1686.4	1756.3	1784.4	1839.8	1839.7	1816.7	1716.6	1758.8	1753.8	1788.8
34	10290	10356	10489	10833	11243	11643	11854	12040	11904	11517	11817	12032	12270

Table 6: Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement $p_{Hn}{}^{t}$

n	$p_{Hn}{}^{2000}$	$p_{Hn}{}^{2001}$	$p_{Hn}{}^{2002}$	$p_{Hn}{}^{2003}$	$p_{Hn}{}^{2004}$	$p_{Hn}{}^{2005}$	$p_{Hn}{}^{2006}$	$p_{Hn}{}^{2007}$	$p_{Hn}{}^{2008}$	$p_{Hn}{}^{2009}$	$p_{Hn}{}^{2010}$	$p_{Hn}{}^{2011}$	$p_{Hn}{}^{2012}$
1	0.763	0.706	0.760	0.931	1.095	1.193	1.239	1.432	1.534	1.403	1.740	1.988	1.998
2	0.829	0.842	0.883	1.061	1.186	1.240	1.258	1.428	1.544	1.476	1.417	1.519	1.414
3	0.821	0.813	0.853	1.054	1.216	1.258	1.297	1.460	1.582	1.501	1.448	1.555	1.444
4	0.829	0.807	0.820	0.930	1.033	1.127	1.245	1.356	1.431	1.313	1.497	1.609	1.626
5	0.528	0.468	0.451	0.473	0.576	0.671	0.706	0.745	0.803	0.783	0.863	0.892	0.910
6	0.368	0.384	0.458	0.529	0.607	0.672	0.727	0.826	1.033	0.918	0.944	1.014	0.927
7	1.040	1.044	1.101	1.377	1.532	1.611	1.639	1.819	1.943	1.839	1.760	1.889	1.768
8	0.420	0.438	0.471	0.577	0.659	0.702	0.764	0.913	0.993	0.918	0.892	0.971	0.936
9	0.917	0.929	0.989	1.212	1.322	1.367	1.395	1.548	1.662	1.579	1.551	1.684	1.581
10	0.865	0.843	0.892	1.125	1.274	1.292	1.325	1.470	1.598	1.506	1.457	1.554	1.458
11	0.891	0.877	0.928	1.100	1.216	1.213	1.230	1.367	1.470	1.415	1.359	1.442	1.343
12	0.632	0.616	0.650	0.826	0.943	0.999	1.027	1.183	1.270	1.220	1.196	1.281	1.153
13	0.382	0.396	0.466	0.571	0.680	0.725	0.715	0.860	0.930	0.781	0.785	0.837	0.756
14	1.072	0.937	1.043	1.309	1.466	1.769	1.788	2.124	1.651	1.273	1.409	1.556	1.492
15	0.886	0.912	0.989	1.216	1.364	1.413	1.445	1.578	1.724	1.560	1.430	1.501	1.391
16	0.844	0.836	0.765	0.847	0.861	0.932	1.006	1.089	1.333	1.257	1.335	1.402	1.353
17	0.753	0.742	0.833	1.024	1.183	1.212	1.224	1.345	1.429	1.363	1.341	1.429	1.321
18	1.439	1.265	1.200	1.282	1.354	1.322	1.255	1.228	1.398	1.557	1.621	1.760	1.758
19	0.660	0.602	0.644	0.708	0.759	0.866	0.949	0.995	0.882	0.796	0.909	0.970	0.971
20	0.866	0.871	0.921	1.130	1.251	1.333	1.343	1.522	1.642	1.586	1.557	1.662	1.556
21	0.645	0.693	0.711	0.671	0.698	0.736	0.770	0.811	0.830	0.686	0.762	0.783	0.771
22	0.822	0.832	0.889	1.112	1.233	1.253	1.275	1.411	1.526	1.471	1.426	1.513	1.411
23	0.655	0.635	0.711	0.924	1.093	1.216	1.127	1.331	1.296	1.138	1.361	1.511	1.551
24	1.037	1.047	1.195	1.368	1.456	1.553	1.586	1.801	1.919	1.793	1.897	2.095	2.008
25	0.424	0.466	0.469	0.503	0.556	0.650	0.696	0.801	0.953	0.752	0.777	0.818	0.759
26	0.645	0.648	0.698	0.847	0.971	0.957	0.972	1.086	1.176	1.107	1.066	1.137	1.051
27	0.344	0.334	0.367	0.483	0.584	0.618	0.659	0.801	0.929	0.894	0.877	0.953	0.888
28	0.572	0.572	0.614	0.756	0.831	0.855	0.892	1.036	1.149	1.127	1.093	1.156	1.063
29	0.676	0.679	0.723	0.903	1.029	1.070	1.080	1.199	1.304	1.241	1.209	1.288	1.182
30	0.997	0.929	1.005	1.227	1.353	1.412	1.441	1.581	1.646	1.466	1.592	1.776	1.694
31	1.096	1.118	1.190	1.402	1.540	1.574	1.548	1.604	1.768	1.757	1.829	2.105	1.961
32	0.452	0.358	0.426	0.548	0.622	0.695	0.694	0.798	0.846	0.741	0.804	0.794	0.769
33	0.962	0.926	0.985	1.112	1.265	1.301	1.350	1.553	1.480	1.286	1.305	1.404	1.382
34	1.000	1.026	1.047	1.063	1.092	1.125	1.169	1.203	1.237	1.252	1.266	1.291	1.324

4. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Base Period Shares of OECD Real GDP

We generate comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2000, the s_n^{2000} listed in Table 3 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix. This is a typical strategy in forming estimates of real GDP that rely on PPPs that are only produced infrequently. Our purpose in listing these estimates is to evaluate how different the resulting estimates are from our preferred harmonized volume estimates, q_{Hn}^t, listed in Table 5 above.

Define preliminary *base period estimates of country GDP volumes for year t and country* n, q_{Bn}^t, as follows:

$$(23)\ q_{Bn}^t \equiv s_n^{2000}\,(Q_n^t/Q_n^{2000})\,; \qquad n = 1,...,34\ ;\ t = 2000,...,2012.$$

The above estimates are obviously based on the country shares of real OECD GDP that prevailed in 2000 (the s_n^{2000}) and the long term country growth rates of real GDP (the Q_n^t/Q_n^{2000}). The companion *country US dollar price levels* for country n and year t, p_{Bn}^t, are defined as follows:

$$(24)\ p_{Bn}^t \equiv v_n^t/q_{Bn}^t\,; \qquad n = 1,...,34\ ;\ t = 2000,...,2012$$

where v_n^t is the nominal value of GDP for country n in year t converted into US dollars at market exchange rates for that year.

In order to make the volumes and prices defined by (23) and (24) comparable to the country prices and volumes expressed in US dollars that are listed tables 5 and 6 in the previous section, we impose a normalization on the prices defined by (24) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (24) by a constant that sets the resulting p_{B34}^{2000} equal to 1 and the volumes defined by (23) are all multiplied by this constant. The resulting normalized p_{Bn}^t are listed in Table 7.[35]

[35] The entries in tables 5 and 6 enable one to recover the US dollar values of GDP, equal to $v_n^t = p_{Hn}^t q_{Hn}^t$ for n = 1,...,34 and t = 2000,...,2012. Then the q_{Bn}^t can be recovered as v_n^t/p_{Bn}^t.

Table 7: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Bn}^{t} Based on Country Growth Rates of GDP Volumes and Year 2000 Country Shares of OECD Output

n	p_{Bn}^{2000}	p_{Bn}^{2001}	p_{Bn}^{2002}	p_{Bn}^{2003}	p_{Bn}^{2004}	p_{Bn}^{2005}	p_{Bn}^{2006}	p_{Bn}^{2007}	p_{Bn}^{2008}	p_{Bn}^{2009}	p_{Bn}^{2010}	p_{Bn}^{2011}	p_{Bn}^{2012}
1	0.763	0.699	0.756	0.930	1.094	1.191	1.233	1.432	1.507	1.414	1.764	2.014	2.008
2	0.829	0.821	0.874	1.060	1.186	1.212	1.246	1.386	1.509	1.454	1.406	1.506	1.416
3	0.821	0.814	0.873	1.068	1.200	1.230	1.270	1.418	1.550	1.488	1.448	1.550	1.460
4	0.829	0.804	0.802	0.928	1.031	1.144	1.255	1.367	1.433	1.312	1.499	1.611	1.622
5	0.528	0.468	0.448	0.469	0.571	0.667	0.793	0.844	0.848	0.820	0.981	1.070	1.083
6	0.368	0.391	0.466	0.546	0.624	0.667	0.711	0.817	0.990	0.907	0.891	0.953	0.875
7	1.040	1.036	1.117	1.361	1.531	1.573	1.620	1.810	2.015	1.929	1.917	2.023	1.917
8	0.420	0.434	0.478	0.596	0.685	0.727	0.799	0.972	1.095	1.042	0.997	1.077	1.028
9	0.917	0.917	0.977	1.163	1.286	1.294	1.316	1.479	1.630	1.568	1.500	1.618	1.539
10	0.865	0.857	0.922	1.127	1.261	1.287	1.326	1.484	1.629	1.556	1.498	1.592	1.494
11	0.891	0.875	0.933	1.132	1.258	1.268	1.283	1.423	1.535	1.473	1.418	1.507	1.413
12	0.632	0.626	0.680	0.848	0.960	0.989	1.022	1.152	1.291	1.252	1.207	1.281	1.175
13	0.382	0.419	0.505	0.612	0.712	0.742	0.728	0.879	0.988	0.870	0.867	0.920	0.848
14	1.072	0.940	1.055	1.269	1.422	1.629	1.591	1.841	1.500	1.155	1.249	1.360	1.297
15	0.886	0.912	1.012	1.259	1.418	1.454	1.515	1.682	1.748	1.595	1.497	1.582	1.472
16	0.844	0.833	0.770	0.796	0.810	0.816	0.838	0.907	1.055	1.008	1.074	1.149	1.101
17	0.753	0.752	0.817	1.010	1.138	1.160	1.190	1.329	1.459	1.412	1.352	1.438	1.352
18	1.439	1.261	1.203	1.279	1.352	1.311	1.228	1.202	1.352	1.486	1.549	1.672	1.658
19	0.660	0.600	0.640	0.695	0.745	0.839	0.899	0.943	0.818	0.730	0.836	0.885	0.879
20	0.866	0.842	0.904	1.148	1.286	1.349	1.453	1.645	1.768	1.690	1.726	1.888	1.798
21	0.645	0.691	0.715	0.695	0.723	0.781	0.834	0.878	0.917	0.786	0.874	0.943	0.921
22	0.822	0.839	0.917	1.123	1.245	1.277	1.311	1.457	1.592	1.512	1.453	1.542	1.444
23	0.655	0.628	0.693	0.893	1.053	1.141	1.087	1.289	1.265	1.133	1.369	1.524	1.550
24	1.037	1.033	1.143	1.326	1.475	1.681	1.837	2.071	2.388	2.026	2.240	2.580	2.549
25	0.424	0.465	0.477	0.503	0.557	0.646	0.683	0.796	0.943	0.756	0.793	0.833	0.776
26	0.645	0.649	0.708	0.874	0.985	1.012	1.049	1.177	1.280	1.225	1.175	1.236	1.139
27	0.344	0.344	0.381	0.495	0.597	0.636	0.683	0.831	0.988	0.962	0.922	0.983	0.920
28	0.572	0.570	0.620	0.759	0.844	0.856	0.882	1.003	1.118	1.095	1.033	1.097	1.016
29	0.676	0.684	0.751	0.938	1.074	1.122	1.179	1.329	1.456	1.382	1.318	1.384	1.279
30	0.997	0.905	0.975	1.195	1.319	1.309	1.351	1.516	1.603	1.409	1.509	1.697	1.642
31	1.096	1.111	1.210	1.412	1.542	1.544	1.567	1.678	1.911	1.894	1.983	2.337	2.216
32	0.452	0.353	0.394	0.488	0.577	0.656	0.675	0.786	0.881	0.779	0.849	0.827	0.823
33	0.962	0.936	0.999	1.112	1.276	1.292	1.345	1.497	1.419	1.229	1.257	1.334	1.337
34	1.000	1.023	1.039	1.059	1.088	1.123	1.158	1.189	1.212	1.221	1.236	1.260	1.282

The differences between the entries in tables 6 and 7 are very large. If we take each column in Table 6, subtract the corresponding entries in the same column of Table 7 and then take the absolute value of the differences, we find that the *average* absolute difference grows from 0 in 2000 to 9.4 percentage points in 2012.[36] The maximum absolute difference grows from 0 in 2000 to 54.0 percentage points in 2012. These are massive differences in price levels, which translate into massive differences in GDP levels. This problem of the inconsistency with national growth rates is well known but most users of PPP adjusted country real volume estimates are not aware of how large these inconsistencies are.[37]

In the following section, we undertake a computation that is similar to the computations in the present section except that we use the real volume shares of 2012 as the benchmark shares instead of the shares of 2000.

5. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Current Period Shares of OECD Real GDP

We generate comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2012, the s_n^{20012} listed in Table 3 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix. This method for forming comparable country GDP volumes is used by the World Bank when the International Comparisons Project produces a new set of PPPs.[38] The methodology is straightforward and follows the approach used in the previous section except that the 2012 country volume shares are used in place of the 2000 shares.

The preliminary *end of sample period estimates of country GDP volumes for year t and country* n, q_{En}^t, is defined as follows:

$$(25)\quad q_{En}^t \equiv s_n^{2012}\,(Q_n^t/Q_n^{2012})\;; \qquad\qquad n = 1,...,34\;;\; t = 2000,...,2012.$$

The above estimates are obviously based on the country shares of real OECD GDP that prevailed in 2012 (the s_n^{2012}) and the levels of real GDP in year t relative to the corresponding country n level in 2012 (the Q_n^t/Q_n^{2012}). The companion *country US dollar price levels* for country n and year t, p_{En}^t, are defined as follows:

[36] The sequence of *average* absolute differences in percentage points over the 13 years is as follows: 0, 0.8, 1.6, 2.0, 2.2, 3.8, 5.2, 6.4, 7.6, 6.8, 8.0, 9.3, 9.4. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 0, 2.9, 5.2, 6.0, 5.4, 14.0, 25.1, 28.2, 46.9, 24.8, 34.3, 48.5, 54.0.
[37] See the Eurostat-OECD Manual on this point; Eurostat (2012; 18).
[38] See Chapter 18 in the World Bank (2013). The Penn World Tables use the extrapolation methodology described in this section and the previous section to construct estimates of comparable real GDP for periods subsequent to the last available ICP round and prior to the first available ICP round; see Summers and Heston (1991) and Feenstra, Inklaar and Timmer (2013).

(26) $p_{En}{}^{t} \equiv v_n{}^t/q_{En}{}^t$; $n = 1,...,34$; $t = 2000,...,2012$

where $v_n{}^t$ is the nominal value of GDP for country n in year t converted into US dollars at market exchange rates for that year.

In order to make the volumes and prices defined by (25) and (26) comparable to the harmonized country prices and volumes expressed in US dollars that are listed tables 5 and 6 in section 3, we impose a normalization on the prices defined by (26) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (26) by a constant that sets the resulting $p_{E34}{}^{2000}$ equal to 1 and the quantities or volumes defined by (25) are all multiplied by this constant. The resulting normalized $p_{En}{}^t$ are listed in Table 8.[39]

It can be seen that there are substantial differences between the price levels listed in Table 8 as compared to the price levels listed in Table 7 and the harmonized price levels listed in Table 6. If we take each column in Table 6, subtract the corresponding entries in the same column of Table 8 and then take the absolute value of the differences, we find that the average absolute difference for 2000 over the 34 countries is 6.0 percentage points, which increases to 7.9 percentage points for 2005 and then gradually decreases to 4.2 percentage points in 2012. Over all observations, the maximum absolute deviation is 35.6 percentage points.[40] Again these are large differences in price levels, which translate into large differences in GDP levels.

For comparing real GDP levels across time and space, the results presented in this section indicate that the strategy of using national growth rates and a single cross country comparison of real GDP levels will not lead to stable comparisons. The harmonization strategy suggested in section 3 will lead to stable comparisons and if the accuracy of the annual sequence of PPPs is roughly constant, the resulting harmonized estimates seem to be preferable to the consistent national growth rate estimates that are based on a single cross country comparison.

[39] As in the previous section, the $q_{En}{}^t$ can be recovered as $v_n{}^t/p_{En}{}^t$.

[40] The sequence of *average* absolute differences over the 34 countries in percentage points over the 13 years is as follows: 6.0, 6.1, 6.2, 7.0, 7.8, 7.9, 6.2, 6.8, 5.9, 5.4, 5.1, 5.2, 4.2. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 24.6, 25.9, 32.3, 35.6, 33.1, 27.1, 20.5, 22.0, 16.0, 24.7, 18.7, 12.6, 6.3. Recall that we normalized the price level of the US to be 1 in 2000 for the $p_{Hn}{}^t$ and the $p_{En}{}^t$. If instead of using the normalizations $p_{H34}{}^{2000} = p_{E34}{}^{2000} = 1$ when constructing tables 6 and 9, we used the normalizations $p_{H34}{}^{2012} = p_{E34}{}^{2012} = 1$, we would find that the absolute differences between the resulting $p_{Hn}{}^t$ and $p_{En}{}^t$ would equal 0 for all countries n for t = 2012. Thus the choice of normalization (and hence of the units of measurement) can affect the results.

Table 8: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement $p_{En}{}^{t}$ Based on Country Growth Rates of GDP Volumes and Year 2012 Country Shares of OECD Output

n	$p_{En}{}^{2000}$	$p_{En}{}^{2001}$	$p_{En}{}^{2002}$	$p_{En}{}^{2003}$	$p_{En}{}^{2004}$	$p_{En}{}^{2005}$	$p_{En}{}^{2006}$	$p_{En}{}^{2007}$	$p_{En}{}^{2008}$	$p_{En}{}^{2009}$	$p_{En}{}^{2010}$	$p_{En}{}^{2011}$	$p_{En}{}^{2012}$
1	0.735	0.674	0.728	0.896	1.055	1.148	1.188	1.380	1.452	1.363	1.700	1.940	1.935
2	0.802	0.794	0.845	1.025	1.147	1.172	1.205	1.340	1.459	1.406	1.360	1.456	1.369
3	0.786	0.779	0.836	1.022	1.149	1.178	1.216	1.358	1.485	1.425	1.386	1.484	1.398
4	0.805	0.781	0.779	0.901	1.001	1.111	1.218	1.327	1.391	1.274	1.455	1.564	1.574
5	0.430	0.381	0.365	0.381	0.465	0.542	0.645	0.686	0.690	0.667	0.798	0.871	0.881
6	0.378	0.401	0.478	0.560	0.639	0.684	0.729	0.838	1.015	0.930	0.914	0.977	0.897
7	0.929	0.925	0.998	1.215	1.367	1.405	1.447	1.617	1.800	1.723	1.713	1.807	1.713
8	0.370	0.383	0.421	0.525	0.604	0.641	0.704	0.857	0.965	0.919	0.879	0.949	0.907
9	0.912	0.912	0.972	1.157	1.279	1.287	1.309	1.471	1.621	1.560	1.492	1.610	1.531
10	0.817	0.810	0.871	1.065	1.191	1.216	1.253	1.402	1.539	1.470	1.415	1.504	1.412
11	0.820	0.805	0.859	1.041	1.158	1.167	1.181	1.309	1.412	1.355	1.305	1.387	1.301
12	0.601	0.595	0.647	0.806	0.913	0.940	0.972	1.095	1.228	1.191	1.148	1.218	1.117
13	0.330	0.362	0.436	0.528	0.615	0.640	0.628	0.759	0.852	0.751	0.748	0.794	0.732
14	1.195	1.048	1.176	1.414	1.584	1.816	1.773	2.052	1.671	1.287	1.392	1.516	1.445
15	0.811	0.835	0.927	1.153	1.298	1.331	1.387	1.540	1.601	1.460	1.371	1.448	1.347
16	1.005	0.992	0.916	0.947	0.964	0.971	0.997	1.079	1.255	1.200	1.278	1.367	1.311
17	0.712	0.712	0.773	0.955	1.076	1.097	1.126	1.258	1.380	1.336	1.279	1.360	1.279
18	1.479	1.295	1.236	1.314	1.389	1.346	1.262	1.234	1.389	1.526	1.592	1.718	1.703
19	0.706	0.642	0.684	0.744	0.797	0.898	0.961	1.008	0.875	0.781	0.894	0.947	0.940
20	0.726	0.706	0.758	0.962	1.078	1.131	1.219	1.379	1.482	1.417	1.447	1.583	1.507
21	0.523	0.560	0.580	0.563	0.586	0.633	0.676	0.712	0.743	0.637	0.709	0.764	0.747
22	0.778	0.794	0.867	1.063	1.178	1.208	1.240	1.378	1.506	1.430	1.374	1.459	1.366
23	0.635	0.609	0.671	0.866	1.020	1.106	1.053	1.249	1.226	1.098	1.328	1.477	1.502
24	0.791	0.788	0.872	1.012	1.125	1.283	1.402	1.581	1.822	1.546	1.710	1.969	1.945
25	0.401	0.441	0.452	0.477	0.527	0.612	0.648	0.755	0.894	0.716	0.751	0.789	0.735
26	0.576	0.579	0.632	0.781	0.880	0.904	0.937	1.052	1.143	1.094	1.050	1.105	1.018
27	0.322	0.322	0.356	0.462	0.558	0.594	0.639	0.777	0.924	0.899	0.861	0.919	0.860
28	0.580	0.578	0.628	0.769	0.855	0.867	0.893	1.016	1.132	1.109	1.046	1.111	1.029
29	0.605	0.613	0.672	0.840	0.961	1.005	1.056	1.189	1.303	1.237	1.180	1.239	1.145
30	0.996	0.904	0.974	1.194	1.318	1.307	1.350	1.514	1.601	1.407	1.507	1.695	1.641
31	0.940	0.952	1.037	1.211	1.322	1.324	1.343	1.438	1.638	1.624	1.700	2.004	1.899
32	0.409	0.319	0.357	0.441	0.522	0.594	0.610	0.711	0.797	0.705	0.768	0.748	0.745
33	0.963	0.938	1.000	1.113	1.278	1.294	1.347	1.498	1.420	1.230	1.258	1.335	1.338
34	1.000	1.023	1.039	1.059	1.088	1.123	1.158	1.189	1.212	1.221	1.236	1.260	1.282

6. OECD Growth and Inflation Using Adjusted Country Annual GDP Volume Growth Rates and OECD Shares of Real GDP for Two Benchmark Years

The OECD provides annual PPPs so that estimates of relative GDP volumes can be constructed for all member countries for each year. However, the World Bank's ICP PPPs are only available at infrequent intervals.[41] We now consider using the benchmark GDP shares for the years 2000 and 2012 along with information on national GDP growth rates in order to interpolate between the benchmark years. We propose an interpolation method that leads to country shares of real GDP that are exactly consistent with the shares s_n^{2000} for the year 2000 and the shares s_n^{2012} for the year 2012.

We begin by using the methodology of section 3 to construct country measures of real GDP that jump from the year 2000 to the year 2012. The *long term growth factor for country n* can be defined as Q_n^{2012}/Q_n^{2000} where Q_n^t is country n's GDP volume in year t.[42] We use these long term growth factors along with the year 2000 country shares of OECD real GDP, s_n^{2000}, in order to define the *OECD Laspeyres type long term growth factor*, Γ_L, as the following weighted average of the national long term growth factors:

$$(27)\ \Gamma_L \equiv \textstyle\sum_{n=1}^{34} s_n^{2000}(Q_n^{2012}/Q_n^{2000})\ ; \qquad t = 2001,...,2012.$$

The counterpart to the Laspeyres type formula defined by (27) is the following *Paasche type formula* that uses the shares of 2012 and reciprocal long term growth rates:

$$(28)\ \Gamma_P \equiv [\textstyle\sum_{n=1}^{34} s_n^{2012}(Q_n^{2012}/Q_n^{2000})^{-1}]^{-1}\ ; \qquad t = 2001,...,2012.$$

A symmetric average of the two indexes leads to the following Fisher type formula for *OECD long term volume growth* going from the year 2000 to the year 2012:

$$(29)\ \Gamma_F \equiv [\Gamma_L\,\Gamma_P]^{1/2}\ ; \qquad t = 2001,...,2012.$$

The long term indexes defined by (27)-(29) turn out to be 1.2207, 1.2209 and 1.2208 respectively, so that there is practically no difference in the three indexes for this data set.[43] We will use the Fisher measure as our preferred measure of OECD volume growth between 2000 and 2012. We use this measure in order to define country volumes for 2012.

[41] The World Bank has produced benchmark PPPs for over 150 countries for 2005 and 2011.

[42] These long term country growth factors are conveniently listed in the last column of Table A2 in the Appendix.

[43] Note that (29) defines a direct comparison of the data of 2000 with the data of 2012 whereas in section 3 above, we used chained Fisher type indexes to go from 2000 to 2012. The chained Fisher index for 2012 relative to 2000 is equal to 1.2203, which is very close to its direct counterpart, 1.2208.

Preliminary estimates of country GDP volumes in comparable units for the years 2000 and 2012, q_{In}^{2000} and q_{In}^{2012} (the index I indicates that these are interpolated estimates), are defined as follows:

$$(30)\ q_{In}^{2000} \equiv s_n^{2000}\ ;\ q_{In}^{2012} \equiv \Gamma_F\, s_n^{2012}\ ; \qquad n = 1,...,34.$$

The volumes defined by (30) will be imposed as constraints on our interpolation scheme. Define the *implied long term growth factor* over the years 2000-2012 for country n, g_n, that is implied by the estimates of country levels given by equations (30):

$$(31)\ g_n \equiv q_{In}^{2012}/q_{In}^{2000}\ ; \qquad n = 1,...,34.$$

These growth factors are not necessarily equal to the national growth factors G_n that are implied by the national growth rates listed in Table A2 of the Appendix:

$$(32)\ G_n \equiv Q_n^{2012}/Q_n^{2000}\ ; \qquad n = 1,...,34.$$

Thus for each country n, there is an "error" factor or discrepancy, $E_n \equiv g_n/G_n$ between the implied growth rates g_n defined by (31) and the national growth rates between 2000 and 2012, G_n defined by (32). We will distribute these errors in a proportional manner and use the resulting adjusted national growth rates to interpolate between the two benchmark observations. Thus define the *country n proportional annualized discrepancy factor*, α_n, as follows:[44]

$$(33)\ \alpha_n \equiv [g_n/G_n]^{1/12}\ ; \qquad n = 1,...,34.$$

The q_{In}^t for non-benchmark years t can now be defined as follows:[45]

$$(34)\ q_{In}^t \equiv q_{In}^{t-1}(Q_n^t/Q_n^{t-1})\alpha_n\ ; \qquad n = 1,...,34\ ;\ t = 2001,....,2011.$$

Once the q_{In}^t have been defined, the corresponding US dollar price levels p_{In}^t are defined in the usual way:

$$(35)\ p_{In}^t \equiv v_n^t/q_{In}^t\ ; \qquad n = 1,...,34\ ;\ t = 2001,....,2011.$$

[44] The average of the g_n/G_n was 1.03. The maximum ratio was 1.27 (Norway) and the minimum ratio was 0.81 (Israel). The PPP based growth rates treat changes in the terms of trade differently than the nationally based growth rates and so fluctuations in the price of oil probably explain the Norwegian divergence. For the three largest countries, the ratio was 1.05 (Germany), 0.94 (Japan) and 0.97 (US).
[45] It can be verified that if we apply definitions (34) for t = 2012, we obtain the q_{In}^{2012} defined by (30).

In order to make the volumes and prices defined by (34) and (35) comparable to the harmonized country prices and volumes expressed in US dollars that are listed tables 5 and 6 in section 3, we impose a normalization on the prices defined by (35) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (35) by a constant that sets the resulting p_{I34}^{2000} equal to 1 and the quantities or volumes defined by (34) are all multiplied by this constant. The resulting normalized p_{In}^{t} are listed in Table 9.[46]

Again, it can be seen that there are some substantial differences between the price levels listed in Table 9 as compared to the price levels listed in Table 6 but the discrepancies are much reduced as compared to the discrepancies when only one benchmark set of PPPs is used. The overall sample average absolute discrepancy is now only 1.9 percentage points. The average absolute difference for 2000 over the 34 countries is 0, which increases to 3.2 percentage points for 2005 and 2007 and then gradually decreases to 0.3 percentage points in 2012. Over all observations, the maximum absolute deviation is 12.5 percentage points.[47]

Some tentative conclusions can be drawn from the tables in this section and the previous sections. First, the interpolation method which is consistent with benchmark expenditure shares for two widely separated years seems to work quite well and if the benchmark PPPs are of equal quality, the interpolation method is much better than simply projecting the country shares from a single benchmark using national growth rates.[48] Second, if it is too expensive to prepare annual PPPs for a group of countries, then the interpolation method will probably generate comparable country real GDP volumes that are close to our preferred harmonized volumes described in section 3, provided that benchmark PPPs are calculated every three to five years.

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[46] As usual, the q_{In}^{t} can be recovered as v_n^{t}/p_{In}^{t}.

[47] The sequence of *average* absolute differences over the 34 countries in percentage points over the 13 years is as follows: 0, 1.0, 1.8, 2.0, 2.3, 3.2, 3.0, 3.2, 2.8, 2.6, 1.7, 1.2, 0.06. The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 0, 3.9, 9.7, 11.9, 9.4, 10.7, 8.3, 12.5, 12.3, 9.9, 6.1, 3.2, 0.08. The reason why the differences are not all equal to 0 for 2012 is that the direct aggregate Fisher index going from 2000 to 2012 differs slightly from its chained counterpart defined in section 3.

[48] "Better" means "more consistent" with our preferred harmonized volumes that could be calculated if annual PPPs were available.

Table 9: OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{In}^{t} Based on Adjusted Country Growth Rates of GDP Volumes and Year 2000 and 2012 Country Shares of OECD Output

n	p_{In}^{2000}	p_{In}^{2001}	p_{In}^{2002}	p_{In}^{2003}	p_{In}^{2004}	p_{In}^{2005}	p_{In}^{2006}	p_{In}^{2007}	p_{In}^{2008}	p_{In}^{2009}	p_{n}^{2010}	p_{In}^{2011}	p_{In}^{2012}
1	0.763	0.699	0.755	0.928	1.092	1.189	1.229	1.427	1.502	1.408	1.756	2.004	1.997
2	0.829	0.821	0.874	1.060	1.185	1.211	1.244	1.385	1.507	1.452	1.404	1.503	1.413
3	0.821	0.813	0.871	1.064	1.195	1.224	1.262	1.409	1.538	1.475	1.434	1.533	1.443
4	0.829	0.804	0.803	0.929	1.032	1.145	1.256	1.369	1.435	1.314	1.501	1.613	1.625
5	0.528	0.461	0.435	0.449	0.539	0.620	0.726	0.762	0.754	0.719	0.848	0.912	0.909
6	0.368	0.393	0.471	0.554	0.635	0.683	0.731	0.845	1.028	0.947	0.934	1.003	0.926
7	1.040	1.029	1.102	1.333	1.490	1.521	1.556	1.727	1.909	1.815	1.792	1.878	1.768
8	0.420	0.431	0.470	0.582	0.664	0.699	0.762	0.920	1.028	0.971	0.921	0.988	0.936
9	0.917	0.919	0.981	1.171	1.297	1.308	1.334	1.502	1.658	1.599	1.533	1.658	1.580
10	0.865	0.856	0.918	1.120	1.250	1.274	1.310	1.463	1.602	1.527	1.467	1.556	1.457
11	0.891	0.871	0.926	1.117	1.237	1.241	1.251	1.381	1.483	1.417	1.359	1.438	1.342
12	0.632	0.625	0.678	0.844	0.954	0.981	1.012	1.139	1.275	1.235	1.189	1.259	1.153
13	0.382	0.415	0.495	0.594	0.685	0.707	0.687	0.822	0.914	0.798	0.787	0.827	0.755
14	1.072	0.951	1.080	1.314	1.489	1.727	1.706	1.998	1.646	1.283	1.404	1.546	1.492
15	0.886	0.908	1.003	1.241	1.391	1.420	1.473	1.627	1.684	1.528	1.428	1.502	1.391
16	0.844	0.848	0.796	0.838	0.867	0.889	0.928	1.022	1.210	1.177	1.274	1.387	1.353
17	0.753	0.751	0.813	1.004	1.129	1.149	1.176	1.311	1.436	1.387	1.325	1.407	1.320
18	1.439	1.267	1.215	1.298	1.379	1.343	1.265	1.243	1.405	1.552	1.627	1.764	1.758
19	0.660	0.605	0.650	0.713	0.770	0.874	0.944	0.999	0.874	0.786	0.908	0.969	0.971
20	0.866	0.832	0.882	1.107	1.225	1.271	1.352	1.512	1.605	1.516	1.530	1.653	1.555
21	0.645	0.681	0.694	0.664	0.681	0.725	0.763	0.791	0.814	0.688	0.754	0.800	0.771
22	0.822	0.838	0.913	1.117	1.235	1.264	1.295	1.437	1.567	1.485	1.424	1.509	1.410
23	0.655	0.628	0.693	0.893	1.053	1.142	1.087	1.289	1.266	1.134	1.370	1.524	1.550
24	1.037	1.013	1.098	1.249	1.362	1.522	1.631	1.802	2.036	1.694	1.836	2.073	2.007
25	0.424	0.465	0.476	0.500	0.552	0.640	0.676	0.786	0.930	0.743	0.778	0.816	0.759
26	0.645	0.644	0.698	0.857	0.959	0.978	1.007	1.123	1.212	1.152	1.098	1.148	1.050
27	0.344	0.343	0.379	0.490	0.590	0.626	0.671	0.814	0.965	0.936	0.894	0.951	0.888
28	0.572	0.572	0.625	0.767	0.856	0.872	0.902	1.029	1.151	1.132	1.072	1.142	1.062
29	0.676	0.680	0.741	0.920	1.046	1.086	1.133	1.269	1.381	1.302	1.234	1.287	1.182
30	0.997	0.908	0.980	1.204	1.332	1.325	1.372	1.543	1.636	1.441	1.548	1.745	1.693
31	1.096	1.100	1.186	1.370	1.480	1.467	1.474	1.562	1.761	1.728	1.790	2.089	1.960
32	0.452	0.351	0.390	0.480	0.564	0.638	0.652	0.755	0.842	0.740	0.802	0.777	0.769
33	0.962	0.939	1.005	1.121	1.290	1.310	1.367	1.525	1.450	1.260	1.292	1.374	1.381
34	1.000	1.026	1.044	1.068	1.100	1.138	1.176	1.211	1.238	1.250	1.269	1.297	1.323

The interpolation method that we described in this section is not the only possible method that could be used to calculate comparable real GDP series over time and space when benchmark PPPs are only available infrequently. In particular, econometricians may prefer to use an interpolation method that is based on the Kalman filter; see Rao, Rambaldi and Doran (2010) (2011) for the description of such a method.[49] However, statistical agencies are generally reluctant to adopt methods that rely heavily on econometrics so the simple method of interpolation described here is proposed as an attractive alternative.

Feenstra, Inklaar and Timmer (2013; 17-19) use a simple interpolation method to harmonize their new Penn World Table estimates of real GDP growth (in comparable units of measurement) using both ICP information between two benchmarks and national information on GDP growth. Their interpolation method is similar to our suggested method, in that their interpolated estimates are consistent with the relative GDP levels for the PPP benchmark years. The key to their interpolation method is the construction of interpolated PPPs between ICP benchmarks. We explain their method using our notation and adapting their analysis to the problem of constructing PPPs for the years 2001-2011, given that we have PPPs for the benchmark years 2000 and 2012. Recall that the domestic price level for country n in year t was defined as $P_n^t \equiv V_n^t/Q_n^t$ for n = 1,...,34 and t = 2000,...,2012 (and these price levels are listed in Appendix Table A1). Recall also that PPP_n^t was defined as the number of units of the national currency of country n that is required to purchase one dollar of US (real) GDP in year t (and these OECD PPPs are listed in Appendix Table A5). Using our notation, their interpolated PPP for country n in year t, PPP_{FITn}^t, is defined as follows:

$$(36)\ PPP_{FITn}^t \equiv (1 - w^t)(PPP_n^{2000})(P_n^t/P_n^{2000}) + w^t(PPP_n^{2012})(P_n^t/P_n^{2012})\ ; \quad n = 1,...,34;\ t = 2000, 2001, ...,2012$$

where the weight function w^t is defined as follows:

$$(37)\ w^t \equiv (t - 2000)/12\ ; \qquad t = 2000, 2001, ...,2012.$$

Thus w^t grows linearly in t with $w^{2000} = 0$ and $w^{2012} = 1$. Note that $PPP_{FITn}^{2000} = PPP_n^{2000}$ and $PPP_{FITn}^{2012} = PPP_n^{2012}$ so that the interpolated PPPs coincide with the actual PPPs for the two benchmark years, 2000 and 2012. Thus the interpolated PPP for country n in year t, PPP_{FITn}^t, is a simple weighted average of two extrapolated PPPs for country n. The first index in the weighted average uses the PPPs for 2000, PPP_n^{2000}, and pushes these PPPs forward using the normalized domestic inflation rates P_n^t/P_n^{2000} and the second index uses

[49] For additional methods for harmonizing cross sectional and time series estimates of real GDP, see Rao, Rambaldi and Balk (2013). Summers and Heston (1991; 340-341) also used an econometric method to reconcile the differences between national growth rates and ICP generated estimates of relative GDP levels.

the PPPs for 2012, PPP_n^{2012}, and pushes these PPPs backwards using the normalized domestic inflation rates P_n^t/P_n^{2012}.

We do not construct OECD real volumes for our sample period using the complete Feenstra, Inklaar and Timmer (FIT) methodology but we experiment with their method of weighting. Recall that tables 7 and 8 listed the OECD country US dollar prices (in comparable units across time and space), p_{Bn}^t and p_{En}^t, where the prices p_{Bn}^t (p_{En}^t) were based on country growth rates of GDP volumes and year 2000 (2012) country shares of OECD output. Using the weights w^t defined by (37), define the FIT type US dollar price levels, p_{FITn}^t, as the following weighted averages of the 2000 US dollar prices p_{Bn}^{2000} and 2012 US dollar prices p_{En}^{2012}:

$$(38)\ p_{FITn}^t \equiv (1-w^t)p_{Bn}^{2000} + w^t p_{En}^{2012}\ ; \qquad n = 1,...,34;\ t = 2000, 2001, ...,2012.$$

It can be seen that $p_{FITn}^{2000} = p_{Bn}^{2000}$ and $p_{FITn}^{2012} = p_{En}^{2012}$ for n = 1,...,34. Now compare the prices p_{FITn}^t to our preferred Harmonized US dollar price levels p_{Hn}^t listed in Table 6. Take the absolute value of the differences, $p_{Hn}^t - p_{FITn}^t$. The sample average absolute difference (over time periods t and countries n) is 12.2 percentage points. The within year absolute difference grows from 0 in 2000 to 25.3 percentage points in 2008.[50] The maximum absolute difference grows from 0 in 2000 to 83.4 percentage points in 2007. These are large differences in price levels, which translate into large differences in real GDP levels.

The relative volumes generated by dividing the US dollar GDP values by the corresponding US dollar prices defined by (38) are no longer independent of the choice of the numeraire country. Thus instead of taking the weighted arithmetic means of the prices p_{Bn}^{2000} and p_{En}^{2012}, take the corresponding weighted geometric means and denote the resulting prices by p_{FITGn}^t.[51] Compare the prices p_{FITGn}^t to our preferred Harmonized US dollar price levels p_{Hn}^t listed in Table 6 and take the absolute value of the differences, $p_{Hn}^t - p_{FITGn}^t$. The sample average absolute difference is now 13.6 percentage points, which is larger than the average differences using the weighted arithmetic means. The within year absolute difference grows from 0 in 2000 to 28.6 percentage points in 2008.[52] The maximum absolute difference grows from 0 in 2000 to 84.7 percentage points in 2007.

[50] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 6.2, 6.2, 9.3, 14.5, 15.8, 14.7, 23.0, 25.3, 14.9, 9.4, 14.8, 4.2. The differences are nonzero in 2013 even though the corresponding PPPs for 2012 are exactly consistent with the OECD PPPs for 2012. Thus while the US dollar prices p_{Hn}^{2012} equal λp_{FITn}^{2012} for n = 1,...,34, the factor of proportionality λ is not equal to one and thus the differences are nonzero in 2013.

[51] Usually, taking geometric means (rather than arithmetic means) of two indexes leads to indexes that have better invariance and homogeneity properties. For examples of this phenomenon, see Diewert (1997) and Hill and Fox (1997).

[52] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 5.2, 5.0, 10.8, 16.7, 18.6, 17.5, 26.3, 28.6, 16.2, 11.6, 16.0, 4.2.

Why do the above variants of the interpolation method suggested by Feenstra, Timmer and Inklaar generate US dollar price levels (and the corresponding country real GDP levels) that are so different from the Harmonized country US dollar price levels that are listed in Table 6? The reason is that the interpolated PPPs defined by (36) (and their geometric counterparts) depend on country inflation rates, which are quite variable.[53] In order to eliminate the effects of country inflation rates, we tried the following variant of the FIT interpolation method: instead of using equations (36) to interpolate the PPPs between the years 2000 and 2012, use the following equations to define the interpolated PPP_{In}^t for country n and year t:

$$(39)\ PPP_{In}^t \equiv (1 - w^t)PPP_n^{2000} + w^t\, PPP_n^{2012}\ ; \qquad n = 1,\ldots,34;\ t = 2000, 2001, \ldots,2012$$

where the weight w^t is still defined by (37). Now return to our description for the construction of the harmonized country estimates for GDP and US dollar price levels that is in section 3 but replace PPP_n^t in equations (12) by their interpolated counterparts PPP_{In}^t defined by (39). Denote the resulting US dollar price levels by p_{In}^t. We compare the prices p_{In}^t to our preferred US dollar price levels p_{Hn}^t listed in Table 6 and as usual, take the absolute value of the differences, $p_{Hn}^t - p_{In}^t$. The sample average absolute difference (over time periods t and countries n) is only 2.48 percentage points. The within year average absolute difference grows from 0 in 2000 to 4.3 percentage points in 2005.[54] The maximum absolute difference is 20.6 percentage points in 2003. The performance of this interpolation method is much better than the previous interpolation method but still not quite as good as our suggested interpolation method that was described at the beginning of this section (which generated an average absolute difference of only 1.92 percentage points).

The above numerical experiments with interpolation methods that are similar in spirit to the method used by Feenstra, Timmer and Inklaar are not conclusive since it assumes that the "truth" is best defined by the harmonized parities p_{Hn}^t defined earlier in section 3. However, at a minimum, the numerical experiments do show that the method of

[53] More precisely, the FIT method interacts country inflation rates with the linear in time weights in equations (36) and *these weights are independent of the magnitude of economic price and quantity data that pertain to the countries* whereas our interpolation method depends only on country volume indexes over the sample period and the relative volumes generated by the PPPs at the beginning and end of the sample period. If the α_n defined by (33) were all equal to unity, then the matrix of country real volumes generated by extrapolating the base period relative GDP volumes forward by national growth rates would be equal to the matrix of country real volumes generated by extrapolating the final period relative GDP volumes backwards (after normalization to a common base) and our interpolation method would generate this common matrix of comparable over time and space real GDP volumes. Under the same conditions, the FIT method would not generate the same matrix (except by chance).

[54] The sequence of within year average absolute differences in percentage points over the 13 years is as follows: 0, 1.1, 1.9, 2.9, 4.0, 4.3, 3.7, 3.7, 3.6 2.3, 2.4, 1.9, 0.3. The sequence of within year maximum absolute differences in percentage points over the 13 years is as follows: 0, 6.7, 13.8, 20.6, 20.3, 18.6, 15.0, 13.3, 15.8, 8.2, 6.8, 5.8, 0.5.

interpolation between benchmark Purchasing Power Parity rounds does matter. Additional research into alternative methods of interpolation is required.

7. Conclusion

A number of interesting points emerged from our investigations. If our focus is on measuring overall OECD GDP growth and PPP information is unavailable, then the method that is explained in section 2 may be used. The overall OECD growth measures, γ^t, do not depend on PPPs or the choice of the numeraire currency but exchange rate fluctuations can cause perhaps unwarranted fluctuations. We computed γ^t using the US and then Germany as the numeraire country and found that while the Fisher index of OECD real GDP growth remained invariant, the accompanying Fisher price indexes, P^t and P_{EU}^t, exhibited wildly different rates of growth. Thus these price indexes are useless as indicators of OECD inflation.

Three alternative measures of overall OECD GDP growth were defined in section 3: the Laspeyres, Paasche and Fisher measures, γ_L^t, γ_P^t and γ_F^t. These measures depended on the annual OECD PPP information. The Laspeyres measure is the official OECD measure for overall OECD growth but the Fisher measure seems preferable on conceptual grounds. However, for our data set, all three measures were very close to each other.

The Fisher measure of OECD growth, γ_F^t, (which used real GDP share weights constructed using PPPs) grew on average about 1/10 of a percentage point more rapidly per year over the period 2000-2012 than our section 2 measure of OECD GDP growth γ^t, (which used exchange rate based share weights). This result was expected since the section 3 PPP based share weights s_n^t for rich countries (which generally have lower rates of GDP growth) are generally smaller than the corresponding section 2 exchange rate based share weights S_n^t.

Section 3 also introduced three measures of OECD aggregate GDP price inflation, the Laspeyres, Paasche and Fisher measures ρ_L^t, ρ_P^t and ρ_F^t defined by (19). These inflation measures used PPP based country share weights to weight the country inflation rates and were much more satisfactory than the section 2 measures of OECD aggregate inflation. The three measures differed somewhat so the choice of index matters. Our preference is for the Fisher measure ρ_F^t since it satisfies a time reversal test whereas the other two indexes do not.

We used two principles in section 3 to generate our *harmonized estimates of real GDP for OECD countries*: (i) The resulting harmonized estimates of country volumes q_{Hn}^t must be consistent with the real volume shares s_n^t listed in Table 3 and (ii) OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes γ_F^t.

Once the harmonized estimates of real GDP q_{Hn}^t have been generated, companion *US dollar country price levels* p_{Hn}^t can be generated as $p_{Hn}^t \equiv v_n^t/q_{Hn}^t$ where v_n^t is the exchange rate converted US dollar nominal value of GDP for country n in year t. These country price levels are useful (but imperfect) indicators of a country's competitiveness in year t.

In sections 4 and 5, alternative measures of comparable levels of real GDP and the accompanying US dollar price levels were constructed. The measures constructed in section 4 used the PPP information for 2000 and national growth rates for real GDP by country whereas the estimates constructed in section 5 substituted the PPP information for 2012. We found tremendous discrepancies in these estimates as compared to the harmonized estimates constructed in section 3.

The results listed in sections 3 to 5 show that it is very hazardous for analysts interested in comparative levels of GDP across countries to use national growth rates and a *single* cross country comparison of real GDP levels. Eventually, the single cross country comparison is replaced by another single cross country comparison but the new set of comparable GDP levels across time and space can be vastly different from the earlier set of GDP levels, particularly for small countries. These results reinforce the case for using the harmonized series that were defined in section 3. Using the section 3 methodology, the previously constructed harmonized estimates of relative GDP levels remain unchanged as another year of data is added.

If PPP computations for a group of countries are only done on an infrequent basis (rather than on an annual basis as is the case for the OECD), then the interpolation method explained in section 6 may prove to be a useful method for obtaining comparable GDP levels that are consistent with the GDP relative levels for the two benchmark years. The results in section 6 also indicate that different interpolation methods can generate very different results. In particular, the present interpolation method used in the Penn World Tables did not work well with our OECD data base.

Of course, the harmonization methods that have been suggested in this paper can be applied to any other value aggregate, such as consumption, investment or domestic absorption. The results in this paper show that if countries want to compare the size of their economies or measure expenditure growth or price inflation for a group of countries, it is absolutely essential that those countries undertake regular cross country comparisons of prices.

Appendix

This Appendix lists the underlying OECD data, some supplementary tables and notes. The source for all of the data listed in this Appendix is OECD.Stat. The country price levels P_n^t using domestic currencies (normalized to equal unity in 2000) are listed in Table A1 and the corresponding volumes are listed in Table A2.

It can be seen that Country 18, Japan, had the lowest rate of domestic inflation, which was actually a deflation. Country 32, Turkey, had the highest rate of domestic inflation, which was 533% over the sample period. The countries that exhibited the fastest rates of real GDP growth over the sample period were countries 5 (Chile), 27 (Slovak Republic), 32 (Turkey), 19 (Korea) and 25 (Poland) with growth rates equal to 168%, 167%, 162%, 159% and 156% respectively.

In order to obtain the country volume levels Q_n^t that match up with the price levels P_n^t in Table A1, the entries in the rows labeled 1-34 need to be multiplied by the country volume levels for 2000, the Q_n^{2000} for n = 1,...,34. These year 2000 levels are as follows: 706.89, 208.47, 252.54, 1076.58, 42094.99, 2269.70, 1293.96, 6.16, 132.19, 1439.60, 2047.50, 135.04, 13089.05, 683.75, 105.64, 506.17, 1198.29, 509860.00, 603236.00, 22.00, 6020.65, 417.96, 118.38, 1481.24, 744.38, 127.32, 31.18, 18.57, 629.91, 2265.45, 432.41, 166.66, 987.14, 10289.70. The units are in billions of year 2000 domestic currency units.

Table A3 lists the country n, year t US dollar price levels for GDP, p_n^t, and Table A4 lists the corresponding volume levels, q_n^t. Note that $p_n^t q_n^t$ equals v_n^t, the year t value of country n's GDP in current US dollars.

It is useful to spell out in a bit more detail what the US dollar period t share weights for country n, $S_n^t \equiv v_n^t/\sum_{i=1}^N v_i^t$ look like in terms of domestic values and exchange rates. Let e_n^t be the number of units of domestic currency it takes for country n to purchase one dollar of US currency in year t. Then country n's domestic currency value for its GDP in year t, V_n^t, is related to the corresponding US dollar value for year t, v_n^t by the equations $V_n^t = v_n^t e_n^t$ for all t and n. Similarly, the domestic currency price level for country n in year t, P_n^t, is related to the corresponding US dollar price level for year t, p_n^t by the equations $P_n^t = p_n^t e_n^t$ for all t and n. Substituting these equations into the definition for the country US dollar shares of OECD GDP, we find that $S_n^t = [V_n^t/e_n^t]/[\sum_{i=1}^N (V_i^t/e_i^t)]$ for all n and t. Note that these shares do not depend on which country is chosen as the numeraire currency. The country GDP values in domestic currencies, V_n^t, will move smoothly over time but exchange rate fluctuations will introduce erratic movements in the shares S_n^t over time t.

The exchange rate based Laspeyres and Paasche price indexes P_L^t and P_L^t that appear in Table 2 are built up using the following chain link indexes: $P_{L,link}^t \equiv \sum_{n=1}^N S_n^{t-1} (p_n^t/p_n^{t-1}) = \sum_{n=1}^N [V_n^{t-1}/e_n^{t-1}] [(P_n^t/e_n^t)/(P_n^{t-1}/e_n^{t-1})]/[\sum_{i=1}^N (V_i^{t-1}/e_i^{t-1})]$ and $P_{P,link}^t \equiv [\sum_{n=1}^N S_n^t (p_n^t/p_n^{t-1})^{-1}]^{-1} = \{\sum_{n=1}^N [V_n^t/e_n^t] [(P_n^t/e_n^t)/(P_n^{t-1}/e_n^{t-1})]^{-1}]/[\sum_{i=1}^N (V_i^t/e_i^t)]\}^{-1}$. It can be seen that exchange rate fluctuations affect not only the domestic share weights in the above expressions but they also interact directly with the country inflation rates, P_n^t/P_n^{t-1}. Thus intertemporal exchange rate variation "noise" will tend to drown out the country inflation trends.

Define the exchange rate based Laspeyres, Paasche and Fisher chain link volume indexes as $Q_{L,link}^t \equiv \sum_{n=1}^N S_n^{t-1} (q_n^t/q_n^{t-1}) = \sum_{n=1}^N S_n^{t-1} (q_n^t/q_n^{t-1})$, $Q_{P,link}^t \equiv [\sum_{n=1}^N S_n^t (q_n^t/q_n^{t-1})^{-1}]^{-1} = [\sum_{n=1}^N S_n^t (Q_n^t/Q_n^{t-1})^{-1}]^{-1}$ and $Q_{F,link}^t \equiv [Q_{L,link}^t Q_{P,link}^t]^{1/2}$. It can be seen that exchange rate fluctuations affect only the US dollar country shares (the S_n^t) and not the growth rates of country real GDP (the Q_n^t/Q_n^{t-1}) and thus these volume link indexes will be much more stable than their price counterparts. We note that the chained Fisher Q^t listed in Table 2 can be defined as follows: $Q^{2000} \equiv 26694.3$; $Q^t \equiv Q^{t-1} Q_{F,link}^t$; t =2001,...,2012.

Table A1: OECD Country Price Levels in National Currencies P_n^t

n	P_n^{2000}	P_n^{2001}	P_n^{2002}	P_n^{2003}	P_n^{2004}	P_n^{2005}	P_n^{2006}	P_n^{2007}	P_n^{2008}	P_n^{2009}	P_n^{2010}	P_n^{2011}	P_n^{2012}
1	1.000	1.028	1.057	1.089	1.131	1.186	1.244	1.301	1.366	1.378	1.462	1.484	1.474
2	1.000	1.019	1.031	1.043	1.061	1.082	1.103	1.125	1.144	1.162	1.179	1.203	1.224
3	1.000	1.021	1.041	1.062	1.084	1.110	1.136	1.163	1.188	1.202	1.227	1.251	1.275
4	1.000	1.011	1.022	1.056	1.090	1.126	1.156	1.192	1.241	1.218	1.253	1.294	1.316
5	1.000	1.042	1.083	1.136	1.221	1.309	1.474	1.546	1.553	1.613	1.755	1.815	1.847
6	1.000	1.046	1.074	1.084	1.128	1.124	1.130	1.167	1.189	1.217	1.197	1.186	1.205
7	1.000	1.025	1.049	1.066	1.091	1.122	1.146	1.172	1.222	1.230	1.282	1.292	1.321
8	1.000	1.065	1.115	1.160	1.211	1.285	1.398	1.560	1.645	1.647	1.652	1.702	1.758
9	1.000	1.030	1.043	1.036	1.041	1.046	1.055	1.086	1.118	1.135	1.139	1.170	1.204
10	1.000	1.020	1.043	1.064	1.081	1.102	1.126	1.155	1.184	1.193	1.204	1.220	1.238
11	1.000	1.011	1.026	1.037	1.048	1.055	1.058	1.075	1.083	1.096	1.108	1.121	1.138
12	1.000	1.031	1.066	1.108	1.141	1.173	1.201	1.241	1.300	1.330	1.345	1.359	1.348
13	1.000	1.113	1.207	1.272	1.338	1.372	1.420	1.497	1.576	1.632	1.671	1.714	1.770
14	1.000	1.086	1.147	1.155	1.183	1.217	1.324	1.399	1.564	1.694	1.811	1.870	1.924
15	1.000	1.060	1.118	1.160	1.187	1.215	1.256	1.277	1.241	1.193	1.175	1.183	1.191
16	1.000	1.018	1.059	1.053	1.054	1.064	1.084	1.082	1.099	1.152	1.166	1.194	1.234
17	1.000	1.029	1.062	1.095	1.121	1.142	1.161	1.189	1.219	1.244	1.249	1.266	1.288
18	1.000	0.988	0.973	0.956	0.943	0.931	0.921	0.912	0.901	0.896	0.877	0.860	0.853
19	1.000	1.039	1.072	1.110	1.144	1.152	1.150	1.174	1.208	1.249	1.295	1.314	1.327
20	1.000	1.001	1.022	1.082	1.102	1.155	1.233	1.279	1.284	1.294	1.387	1.445	1.489
21	1.000	1.058	1.132	1.229	1.338	1.395	1.490	1.573	1.673	1.742	1.812	1.920	1.989
22	1.000	1.051	1.091	1.115	1.123	1.150	1.171	1.192	1.218	1.219	1.229	1.243	1.259
23	1.000	1.037	1.039	1.067	1.102	1.124	1.162	1.216	1.249	1.258	1.318	1.338	1.327
24	1.000	1.017	0.999	1.028	1.089	1.186	1.291	1.330	1.475	1.396	1.483	1.584	1.624
25	1.000	1.035	1.058	1.062	1.106	1.135	1.152	1.197	1.235	1.280	1.299	1.340	1.373
26	1.000	1.036	1.075	1.107	1.134	1.163	1.195	1.229	1.248	1.260	1.268	1.271	1.267
27	1.000	1.050	1.091	1.149	1.216	1.245	1.282	1.296	1.333	1.317	1.324	1.345	1.362
28	1.000	1.087	1.169	1.234	1.274	1.295	1.323	1.378	1.435	1.482	1.467	1.484	1.487
29	1.000	1.042	1.087	1.133	1.178	1.230	1.280	1.322	1.354	1.355	1.356	1.356	1.356
30	1.000	1.024	1.039	1.058	1.061	1.071	1.091	1.121	1.157	1.180	1.190	1.206	1.218
31	1.000	1.013	1.019	1.027	1.036	1.038	1.061	1.088	1.118	1.113	1.117	1.121	1.122
32	1.000	1.529	2.101	2.589	2.910	3.117	3.407	3.619	4.054	4.268	4.510	4.897	5.229
33	1.000	1.023	1.048	1.071	1.096	1.118	1.150	1.176	1.214	1.241	1.279	1.309	1.331
34	1.000	1.023	1.039	1.059	1.088	1.123	1.158	1.189	1.212	1.221	1.236	1.260	1.282

Table A2: OECD Country GDP Volumes Relative to the Corresponding 2000 Volumes, $Q_n{}^t/Q_n{}^{2000}$

n	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
1	1.000	1.039	1.072	1.116	1.152	1.187	1.232	1.278	1.299	1.327	1.359	1.405	1.456
2	1.000	1.009	1.026	1.035	1.061	1.087	1.127	1.168	1.185	1.140	1.160	1.193	1.203
3	1.000	1.008	1.022	1.030	1.064	1.082	1.111	1.143	1.155	1.122	1.148	1.169	1.167
4	1.000	1.018	1.048	1.067	1.101	1.134	1.166	1.192	1.200	1.167	1.204	1.234	1.256
5	1.000	1.033	1.061	1.101	1.178	1.250	1.322	1.390	1.436	1.421	1.503	1.591	1.679
6	1.000	1.031	1.053	1.093	1.145	1.222	1.308	1.383	1.426	1.361	1.395	1.420	1.406
7	1.000	1.007	1.012	1.016	1.039	1.064	1.101	1.118	1.109	1.046	1.061	1.072	1.068
8	1.000	1.063	1.133	1.221	1.298	1.413	1.556	1.672	1.603	1.377	1.412	1.547	1.608
9	1.000	1.023	1.042	1.063	1.106	1.139	1.189	1.252	1.256	1.149	1.187	1.220	1.210
10	1.000	1.018	1.028	1.037	1.064	1.083	1.110	1.135	1.134	1.098	1.117	1.140	1.140
11	1.000	1.015	1.015	1.011	1.023	1.030	1.068	1.103	1.115	1.058	1.100	1.137	1.145
12	1.000	1.042	1.078	1.142	1.192	1.219	1.286	1.332	1.329	1.287	1.223	1.136	1.064
13	1.000	1.037	1.084	1.126	1.180	1.226	1.274	1.276	1.287	1.200	1.213	1.232	1.211
14	1.000	1.039	1.041	1.066	1.150	1.233	1.291	1.368	1.384	1.293	1.240	1.274	1.291
15	1.000	1.050	1.107	1.148	1.196	1.269	1.339	1.405	1.375	1.287	1.274	1.301	1.303
16	1.000	0.998	0.998	1.012	1.062	1.114	1.179	1.248	1.299	1.314	1.379	1.443	1.489
17	1.000	1.019	1.023	1.023	1.040	1.050	1.073	1.091	1.079	1.019	1.037	1.042	1.016
18	1.000	1.004	1.007	1.023	1.048	1.061	1.079	1.103	1.091	1.031	1.079	1.073	1.094
19	1.000	1.040	1.114	1.145	1.198	1.246	1.310	1.377	1.409	1.413	1.503	1.558	1.590
20	1.000	1.025	1.067	1.085	1.132	1.192	1.251	1.333	1.323	1.250	1.288	1.313	1.311
21	1.000	1.000	1.007	1.021	1.063	1.098	1.153	1.192	1.207	1.134	1.195	1.241	1.288
22	1.000	1.019	1.020	1.024	1.046	1.068	1.104	1.147	1.168	1.125	1.142	1.153	1.139
23	1.000	1.037	1.089	1.133	1.175	1.214	1.235	1.278	1.256	1.274	1.276	1.304	1.346
24	1.000	1.020	1.035	1.045	1.087	1.115	1.141	1.171	1.172	1.152	1.158	1.172	1.208
25	1.000	1.012	1.027	1.066	1.123	1.164	1.237	1.320	1.388	1.411	1.465	1.532	1.561
26	1.000	1.020	1.028	1.018	1.034	1.042	1.057	1.082	1.082	1.051	1.071	1.058	1.024
27	1.000	1.035	1.082	1.134	1.191	1.271	1.377	1.521	1.609	1.529	1.597	1.644	1.674
28	1.000	1.029	1.069	1.100	1.149	1.195	1.264	1.352	1.398	1.287	1.303	1.313	1.279
29	1.000	1.037	1.065	1.098	1.134	1.174	1.222	1.265	1.276	1.227	1.224	1.225	1.205
30	1.000	1.013	1.038	1.062	1.107	1.142	1.191	1.231	1.223	1.162	1.238	1.274	1.286
31	1.000	1.012	1.014	1.015	1.039	1.067	1.107	1.150	1.175	1.152	1.186	1.207	1.220
32	1.000	0.943	1.001	1.054	1.153	1.249	1.336	1.398	1.407	1.339	1.462	1.590	1.625
33	1.000	1.022	1.045	1.087	1.121	1.157	1.189	1.230	1.221	1.157	1.177	1.190	1.191
34	1.000	1.010	1.027	1.056	1.096	1.133	1.163	1.184	1.181	1.147	1.176	1.198	1.231

Table A3: OECD Country Price Levels in US Dollars at Market Exchange Rates p_n^t

n	p_n^{2000}	p_n^{2001}	p_n^{2002}	p_n^{2003}	p_n^{2004}	p_n^{2005}	p_n^{2006}	p_n^{2007}	p_n^{2008}	p_n^{2009}	p_n^{2010}	p_n^{2011}	p_n^{2012}
1	1.000	0.917	0.991	1.219	1.435	1.562	1.616	1.877	1.976	1.854	2.312	2.640	2.633
2	1.000	0.990	1.054	1.278	1.430	1.461	1.502	1.671	1.819	1.753	1.695	1.815	1.707
3	1.000	0.991	1.063	1.301	1.461	1.498	1.547	1.728	1.889	1.813	1.764	1.888	1.779
4	1.000	0.970	0.967	1.119	1.244	1.379	1.513	1.649	1.728	1.582	1.807	1.942	1.955
5	1.000	0.885	0.848	0.887	1.081	1.262	1.500	1.596	1.604	1.552	1.856	2.024	2.049
6	1.000	1.062	1.266	1.483	1.693	1.810	1.930	2.220	2.689	2.463	2.420	2.587	2.377
7	1.000	0.995	1.074	1.308	1.472	1.512	1.558	1.740	1.937	1.854	1.843	1.945	1.843
8	1.000	1.034	1.139	1.420	1.632	1.733	1.902	2.316	2.610	2.483	2.375	2.566	2.450
9	1.000	1.001	1.066	1.269	1.403	1.412	1.436	1.614	1.778	1.711	1.637	1.766	1.679
10	1.000	0.991	1.065	1.303	1.457	1.488	1.533	1.716	1.883	1.798	1.731	1.840	1.727
11	1.000	0.982	1.048	1.270	1.413	1.424	1.440	1.597	1.723	1.653	1.592	1.692	1.587
12	1.000	0.990	1.076	1.341	1.519	1.564	1.616	1.822	2.042	1.981	1.910	2.026	1.858
13	1.000	1.096	1.320	1.600	1.863	1.939	1.904	2.300	2.584	2.276	2.267	2.406	2.218
14	1.000	0.877	0.984	1.183	1.325	1.519	1.483	1.717	1.398	1.077	1.165	1.268	1.209
15	1.000	1.029	1.142	1.421	1.600	1.640	1.710	1.898	1.973	1.799	1.689	1.785	1.661
16	1.000	0.987	0.911	0.943	0.959	0.967	0.992	1.074	1.249	1.194	1.271	1.360	1.304
17	1.000	0.999	1.085	1.341	1.511	1.541	1.581	1.766	1.938	1.876	1.795	1.910	1.796
18	1.000	0.876	0.836	0.889	0.939	0.911	0.853	0.835	0.939	1.032	1.077	1.162	1.152
19	1.000	0.910	0.969	1.054	1.130	1.272	1.362	1.429	1.240	1.107	1.266	1.341	1.332
20	1.000	0.972	1.044	1.325	1.485	1.558	1.679	1.900	2.041	1.951	1.993	2.180	2.076
21	1.000	1.071	1.109	1.077	1.121	1.211	1.292	1.361	1.421	1.219	1.356	1.461	1.428
22	1.000	1.021	1.115	1.366	1.514	1.553	1.594	1.771	1.936	1.838	1.767	1.875	1.756
23	1.000	0.959	1.057	1.364	1.607	1.743	1.659	1.968	1.932	1.730	2.091	2.327	2.366
24	1.000	0.996	1.102	1.278	1.422	1.621	1.772	1.997	2.302	1.953	2.160	2.488	2.457
25	1.000	1.099	1.127	1.187	1.314	1.524	1.613	1.880	2.227	1.784	1.872	1.966	1.832
26	1.000	1.006	1.098	1.356	1.529	1.570	1.627	1.826	1.985	1.900	1.822	1.918	1.767
27	1.000	1.000	1.108	1.438	1.735	1.848	1.987	2.416	2.872	2.796	2.679	2.858	2.675
28	1.000	0.997	1.083	1.326	1.474	1.496	1.541	1.752	1.953	1.913	1.805	1.916	1.775
29	1.000	1.012	1.111	1.387	1.588	1.660	1.743	1.964	2.152	2.043	1.949	2.046	1.891
30	1.000	0.908	0.978	1.199	1.323	1.312	1.355	1.520	1.608	1.413	1.513	1.702	1.647
31	1.000	1.013	1.104	1.288	1.407	1.408	1.429	1.531	1.743	1.728	1.809	2.132	2.021
32	1.000	0.780	0.871	1.079	1.277	1.450	1.491	1.737	1.947	1.722	1.876	1.828	1.820
33	1.000	0.973	1.038	1.156	1.326	1.343	1.398	1.555	1.475	1.277	1.306	1.386	1.389
34	1.000	1.023	1.039	1.059	1.088	1.123	1.158	1.189	1.212	1.221	1.236	1.260	1.282

Table A4: OECD Country GDP Volumes in US Dollar Units of Measurement q_n^t

n	q_n^{2000}	q_n^{2001}	q_n^{2002}	q_n^{2003}	q_n^{2004}	q_n^{2005}	q_n^{2006}	q_n^{2007}	q_n^{2008}	q_n^{2009}	q_n^{2010}	q_n^{2011}	q_n^{2012}
1	409.8	425.9	439.3	457.5	472.1	486.5	504.9	523.9	532.5	543.7	556.9	575.6	596.8
2	192.1	193.7	197.0	198.7	203.9	208.7	216.4	224.4	227.6	218.9	222.8	229.1	231.1
3	232.7	234.6	237.7	239.7	247.5	251.8	258.6	266.0	268.6	261.1	267.2	271.9	271.5
4	724.9	737.8	759.4	773.7	797.8	821.9	845.1	863.7	869.7	845.6	872.8	894.8	910.1
5	78.0	80.6	82.7	85.9	91.9	97.5	103.1	108.4	112.0	110.8	117.2	124.1	131.0
6	58.8	60.6	61.9	64.3	67.3	71.9	76.9	81.3	83.8	80.0	82.0	83.5	82.7
7	160.1	161.2	162.0	162.6	166.3	170.4	176.2	179.0	177.6	167.5	169.8	171.6	171
8	5.7	6.0	6.4	6.9	7.4	8.0	8.8	9.5	9.1	7.8	8.0	8.8	9.1
9	121.8	124.6	126.9	129.4	134.8	138.7	144.8	152.5	153	139.9	144.6	148.6	147.3
10	1326.3	1350.7	1363.2	1375.5	1410.5	1436.3	1471.7	1505.3	1504.1	1456.8	1481.9	1511.9	1512.1
11	1886.4	1915	1915.2	1908	1930.1	1943.3	2015.2	2081.1	2103.7	1995.4	2075.5	2144.7	2159.4
12	125.9	131.2	135.7	143.8	150.1	153.5	162.0	167.7	167.3	162.1	154.1	143.1	134.0
13	46.4	48.1	50.3	52.2	54.7	56.9	59.1	59.2	59.7	55.7	56.2	57.1	56.2
14	8.7	9.0	9.1	9.3	10.0	10.7	11.2	11.9	12.0	11.2	10.8	11.1	11.2
15	97.3	102.2	107.7	111.7	116.4	123.5	130.3	136.8	133.8	125.3	124.0	126.6	126.8
16	124.1	123.9	123.8	125.7	131.8	138.3	146.3	154.9	161.3	163.1	171.2	179.1	184.8
17	1104.0	1124.6	1129.6	1129.1	1148.7	1159.4	1184.9	1204.8	1190.9	1125.4	1144.8	1150.3	1121.2
18	4731.2	4748.0	4761.8	4842	4956.3	5020.9	5105.9	5217.8	5163.5	4878.1	5105.0	5075.9	5175.2
19	533.4	554.6	594.2	610.9	639.1	664.4	698.8	734.5	751.4	753.8	801.4	830.9	847.9
20	20.3	20.8	21.6	22.0	22.9	24.2	25.3	27.0	26.8	25.3	26.1	26.6	26.6
21	636.7	636.5	641.4	650.4	676.8	699.0	734.3	759.0	768.3	722.2	760.7	790.5	820.4
22	385.1	392.5	392.8	394.1	402.9	411.2	425.1	441.8	449.8	433.3	439.9	444.0	438.5
23	53.8	55.8	58.6	61.0	63.2	65.3	66.4	68.7	67.5	68.5	68.6	70.1	72.4
24	168.3	171.6	174.2	175.9	182.9	187.6	191.9	197.0	197.2	193.9	194.9	197.2	203.3
25	171.3	173.3	175.8	182.6	192.4	199.4	211.8	226.1	237.7	241.6	251.0	262.3	267.4
26	117.3	119.6	120.5	119.4	121.3	122.2	124.0	126.9	126.9	123.2	125.6	124.1	120.1
27	20.4	21.1	22.1	23.1	24.3	25.9	28.1	31.0	32.8	31.2	32.6	33.6	34.2
28	20.0	20.6	21.4	22.0	22.9	23.9	25.3	27.0	27.9	25.7	26.0	26.2	25.6
29	580.3	601.6	617.9	637.0	657.8	681.4	709.1	733.8	740.4	712	710.6	710.9	699.2
30	247.3	250.4	256.6	262.6	273.7	282.4	294.5	304.3	302.4	287.2	306.0	315.0	318.0
31	256.0	259.2	259.7	259.8	266.0	273.2	283.5	294.4	300.7	294.9	303.6	309.1	312.3
32	266.6	251.4	266.9	280.9	307.2	333.0	356.0	372.6	375.1	357.0	389.7	423.8	433.0
33	1493.6	1526.2	1561.2	1622.9	1674.4	1728.5	1776.2	1837.0	1822.9	1728.6	1757.3	1777	1779.2
34	10289.7	10387.3	10571.8	10866.9	11279.6	11657.6	11968.5	12182.7	12147.3	11806.9	12102.9	12326.5	12669.0

Table A5: Annual Purchasing Power Parities PPP_n^t for OECD Countries 2000-2012, National Currencies per US Dollar

n	PPP_n^{00}	PPP_n^{01}	PPP_n^{02}	PPP_n^{03}	PPP_n^{04}	PPP_n^{05}	PPP_n^{06}	PPP_n^{07}	PPP_n^{08}	PPP_n^{09}	PPP_n^{10}	PPP_n^{11}	PPP_n^{12}
1	1.316	1.330	1.336	1.351	1.363	1.388	1.408	1.423	1.479	1.437	1.498	1.493	1.458
2	0.900	0.917	0.896	0.885	0.874	0.886	0.857	0.868	0.852	0.849	0.845	0.846	0.831
3	0.891	0.886	0.865	0.879	0.897	0.900	0.884	0.887	0.874	0.863	0.864	0.867	0.849
4	1.232	1.218	1.229	1.226	1.231	1.214	1.208	1.211	1.234	1.199	1.218	1.233	1.227
5	285.108	289.501	296.891	307.807	321.76	333.69	320.257	323.512	339.271	350.588	347.852	334.241	334.206
6	14.212	14.222	14.319	14.034	14.291	14.316	14.053	13.945	14.262	13.977	14.243	13.899	13.700
7	8.409	8.468	8.302	8.537	8.404	8.59	8.336	8.235	8.012	7.877	7.821	7.857	7.736
8	0.455	0.477	0.477	0.481	0.486	0.502	0.521	0.555	0.549	0.527	0.532	0.541	0.550
9	0.995	1.012	1.003	1.011	0.975	0.977	0.951	0.941	0.918	0.908	0.925	0.938	0.929
10	0.939	0.919	0.905	0.938	0.940	0.923	0.904	0.893	0.882	0.866	0.869	0.866	0.857
11	0.967	0.955	0.942	0.917	0.897	0.867	0.838	0.831	0.812	0.814	0.811	0.803	0.789
12	0.678	0.671	0.660	0.689	0.696	0.714	0.700	0.719	0.701	0.701	0.713	0.714	0.678
13	107.885	110.652	114.88	120.516	126.307	128.594	128.637	131.336	129.429	126.256	129.005	130.345	128.453
14	84.311	88.930	91.342	94.484	94.248	99.078	107.307	113.108	117.421	125.692	136.066	139.737	140.967
15	0.962	0.993	1.004	1.014	1.006	1.01	0.985	0.958	0.952	0.897	0.853	0.836	0.818
16	3.443	3.426	3.463	3.629	3.535	3.717	3.836	3.720	3.867	3.947	3.943	3.885	3.942
17	0.817	0.808	0.845	0.854	0.873	0.867	0.834	0.817	0.789	0.784	0.800	0.796	0.776
18	155.113	149.857	143.774	139.824	134.161	129.552	124.864	120.216	116.846	116.348	112.418	108.812	105.972
19	746.206	757.829	769.772	794.282	795.998	788.92	774.815	768.65	785.718	811.664	829.897	833.034	826.191
20	0.940	0.948	0.934	0.942	0.923	0.953	0.915	0.925	0.906	0.912	0.929	0.926	0.915
21	6.099	6.311	6.554	6.815	7.217	7.127	7.181	7.370	7.470	7.409	7.604	7.532	7.668
22	0.893	0.906	0.902	0.927	0.909	0.896	0.869	0.857	0.842	0.846	0.850	0.843	0.829
23	1.442	1.473	1.469	1.497	1.510	1.535	1.486	1.506	1.491	1.454	1.492	1.481	1.446
24	9.129	9.180	9.111	9.112	8.988	8.896	8.701	8.776	8.752	9.006	9.058	9.095	8.824
25	1.841	1.861	1.829	1.841	1.861	1.869	1.846	1.843	1.857	1.875	1.852	1.877	1.868
26	0.700	0.706	0.708	0.706	0.716	0.684	0.663	0.660	0.649	0.637	0.636	0.633	0.618
27	0.526	0.522	0.528	0.555	0.573	0.566	0.556	0.546	0.533	0.514	0.523	0.531	0.522
28	0.532	0.565	0.588	0.615	0.611	0.612	0.608	0.629	0.634	0.648	0.652	0.644	0.625
29	0.734	0.740	0.733	0.753	0.759	0.765	0.736	0.728	0.720	0.713	0.721	0.718	0.695
30	9.135	9.349	9.352	9.335	9.105	9.378	9.094	8.886	8.773	8.965	9.067	8.935	8.668
31	1.851	1.840	1.771	1.776	1.754	1.743	1.660	1.601	1.549	1.527	1.506	1.448	1.389
32	0.283	0.428	0.613	0.773	0.812	0.831	0.848	0.864	0.890	0.917	0.954	1.030	1.044
33	0.636	0.627	0.628	0.641	0.633	0.636	0.627	0.645	0.651	0.660	0.667	0.679	0.661
34	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

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