Explaining Total Factor Productivity

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Abstract

In this paper we investigate the relationship between a common measure of total factor productivity (TFP) and the concept of disembodied, factor-augmenting technological change. We propose a convenient way to compute the factor-augmenting rates of technological change from the estimates of an ordinary aggregate Translog production function. Using U.S. data as an illustration, we find that TFP is overwhelmingly explained by labor. Furthermore, technological change is anti-labor biased, in the sense that it tends to decrease the income share of labor. This is due to the relatively large Hicksian elasticity of complementarity between capital and labor. Nonetheless, technological change has a positive effect on the return of both capital and labor, although the impact on labor is less than what TFP or average labor productivity would suggest.

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Explaining Total Factor Productivity

"Needed: A Theory of Total Factor Productivity"

Edward C. Prescott (1998)

1. Introduction

Total Factor Productivity (TFP) has become the choice measure of productivity. TFP is often referred to as the Solow residual, and it is just that, namely a residual. Of course, TFP need not be derived from a Cobb-Douglas production function as it was in Solow's original work. There are today many more sophisticated indices available, such as the Fisher and Törnqvist superlative measures that are exact for flexible functional forms of the production function. Nonetheless, as suggested by the above quote, the fact remains that TFP is rather opaque as to the nature of the phenomena that it pertains to measure. TFP captures the effects of changes in technology, institutions, and other productivity shocks, but it gives little insights as to what takes place inside the black box of technology. In particular, it is difficult to reconcile TFP with various models of factor augmenting technological change. Is technological change neutral or is it biased? If it is neutral, is it neutral in the sense of Hicks, Harrod, or Solow? It is often assumed that increases in productivity, as captured by TFP, allow for increases in real wages, but must this really be the case? What about the real return on capital and the real rate of interest? Must they necessarily increase with productivity? The purpose of this paper is to sort out some of these questions.

As an illustration, we will report estimates of an aggregate Translog production function for the United States, allowing for factor augmenting disembodied technological change. We will then derive an index of TFP that is exact for this production function, and show how it can be decomposed into two components, one showing the contribution of labor and the other the contribution of capital. The impact of technological change on factor income shares and factor rental prices can be clearly established.

2. Total factor productivity: Index number approach

Total factor productivity can be defined as the part of output growth that cannot be explained by input growth. Assume that the technology counts one output and two inputs, capital and labor. We denote the quantity of output by y_t, and the quantities of capital and labor services by $x_{K,t}$ and $x_{L,t}$, respectively, all three quantities being measured at time *t*. The corresponding prices are given by p_t, $w_{K,t}$, and $w_{L,t}$. A state-of-the-art measure of TFP is given by the following index:

(1) $$T_{t,t-1} \equiv \frac{Y_{t,t-1}}{X_{t,t-1}}$$

where

(2) $$Y_{t,t-1} \equiv \frac{y_t}{y_{t-1}}$$

(3) $$X_{t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\ln\frac{x_{K,t}}{x_{K,t-1}} + \frac{1}{2}(s_{L,t} + s_{L,t-1})\ln\frac{x_{L,t}}{x_{L,t-1}}\right]$$

and

(4) $$s_{j,t} \equiv \frac{w_{j,t}x_{j,t}}{p_t y_t}, \quad j \in \{K, L\}$$

$Y_{t,t-1}$ is the output quantity relative, and $X_{t,t-1}$ is a Törnqvist index of input quantities. $T_{t,t-1}$ as given by (1) can thus be described as an implicit Törnqvist index of TFP.

Using the data of Kohli (2010) for the United States, 1970-2001, one finds that TFP has averaged about 1.09% per year. While this is useful information, it tells us nothing about the nature of technological change, and whether it benefitted capital or labor, or both.

3. The production function approach: Four views of total factor productivity

Assume that the aggregate technology can be represented by the following two-input, one-output production function:

(5) $$y_t = f(x_{K,t}, x_{L,t}, t)$$

Note that the production function itself is allowed to shift over time to account for technological change. We assume that the production function is linearly homogeneous, increasing, and concave with respect to the two input quantities. Assuming that firms are optimizing and that factors are mobile between firms, the usual first-order conditions hold:

(6) $$\frac{\partial f(\cdot)}{\partial x_{K,t}} = \frac{w_{K,t}}{p_t}$$

(7) $$\frac{\partial f(\cdot)}{\partial x_{L,t}} = \frac{w_{L,t}}{p_t}$$

Differentiation with respect to time furthermore yields:

(8) $$\frac{\partial f(\cdot)}{\partial t} = \mu_t y_t$$

where μ_t is the instantaneous rate of technological change. Note that Euler's Theorem together with (6) – (7) implies:

(9) $p_t y_t = w_{K,t} x_{K,t} + w_{L,t} x_{L,t}$

Following Diewert and Morrison (1986), we define the following index of TFP:

(10) $$T_{t,t-1} = \sqrt{\frac{f(x_{t-1},t)}{f(x_{t-1},t-1)}\frac{f(x_t,t)}{f(x_t,t-1)}}$$

So defined, TFP indicates the change in output that is made possible by the passage of time from t to t-1, holding inputs quantities constant. Since input quantities could equally well be held be held constant at their t-1 values or at their t values, Diewert and Morrison recommended taking the geometric mean of the two corresponding indices, which gives the index a Fisher form, so to say.

To make (10) operational, one must specify a particular functional form for the production function. Assume that it has the following Translog form:

(11) $$\begin{aligned} \ln y_t = \alpha_0 + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})^2 + \\ \beta_T t + \phi_{KT}(\ln x_{K,t} - \ln x_{L,t})\, t + \frac{1}{2}\phi_{TT} t^2 \end{aligned}$$

It can be seen that this function is not just flexible with respect to the quantities of capital and labor, but also with respect to time: it is thus TP flexible to use the terminology of Diewert and Wales (1992). The inverse input demand functions can then be derived in share form through logarithmic differentiation:

(12) $$s_{K,t} = \frac{\partial \ln f(\cdot)}{\partial \ln x_{K,t}} = \beta_K + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) + \phi_{KT} t$$

(13) $$s_{L,t} = \frac{\partial \ln f(\cdot)}{\partial \ln x_{L,t}} = (1-\beta_K) - \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) - \phi_{KT} t$$

Differentiation with respect to time yields the instantaneous rate of technological change:

(14) $$\mu_t = \frac{\partial \ln f(\cdot)}{\partial t} = \beta_T + \phi_{KT}(\ln x_{K,t} - \ln x_{L,t}) + \phi_{TT} t$$

Introducing (11) into (10) yields the following measure of TFP:

(15) $$\ln T_{t,t-1} = \beta_T + \frac{1}{2}\phi_{KT}(\ln x_{K,t} - \ln x_{L,t}) + \frac{1}{2}\phi_{KT}(\ln x_{K,t-1} - \ln x_{L,t-1}) + \frac{1}{2}\phi_{TT}(2t-1)$$

A second interpretation of TFP, in view of (14), is that (the logarithm of) $T_{t,t-1}$ is equal to the average of the instantaneous rates of technological change of time t-1 and time t:

(16) $\ln T_{t,t-1} = \frac{1}{2}(\mu_t + \mu_{t-1})$

Let $\bar{\mu}_{t,t-1}$ be average rate of technological change between times t-1 and t. By Diewert's (1976) quadratic approximation lemma $\bar{\mu}_{t,t-1} = (\mu_{t-1} + \mu_t)/2$, so this yields a third interpretation of TFP:

(17) $\ln T_{t,t-1} = \bar{\mu}_{t,t-1}$

Finally, there is a fourth way of interpreting TFP, based on the approach of Section 2, namely as mentioned in Section 2, that TFP is the output growth that cannot be explained by input growth. Indeed, introducing (11) into (2), we get:

$$
\begin{aligned}
\ln Y_{t,t-1} = \ln y_t - \ln y_{t-1} &= \beta_K (\ln x_{K,t} - \ln x_{K,t-1}) + (1-\beta_K)(\ln x_{L,t} - \ln x_{L,t-1}) + \\
&\quad \frac{1}{2}\phi_{KK}[(\ln x_{K,t} - \ln x_{L,t})^2 - (\ln x_{K,t-1} - \ln x_{L,t-1})^2] + \\
&\quad \phi_{KT}[(\ln x_{K,t} - \ln x_{L,t})t - (\ln x_{K,t-1} - \ln x_{L,t-1})(t-1)] + \\
&\quad \beta_T + \frac{1}{2}\phi_{TT}(2t-1) \\
&= \beta_K (\ln x_{K,t} - \ln x_{K,t-1}) + (1-\beta_K)(\ln x_{L,t} - \ln x_{L,t-1}) + \\
&\quad \frac{1}{2}\phi_{KK}[(\ln x_{K,t} - \ln x_{L,t}) + (\ln x_{K,t-1} - \ln x_{L,t-1})]\times \\
&\quad [(\ln x_{K,t} - \ln x_{L,t}) - (\ln x_{K,t-1} - \ln x_{L,t-1})] + \\
&\quad \frac{1}{2}\phi_{KT}[(\ln x_{K,t} - \ln x_{L,t}) + (\ln x_{K,t-1} - \ln x_{L,t-1})] + \\
&\quad \frac{1}{2}\phi_{KT}[(\ln x_{K,t} - \ln x_{L,t}) - (\ln x_{K,t-1} - \ln x_{L,t-1})](2t-1) + \\
&\quad \beta_T + \frac{1}{2}\phi_{TT}(2t-1)
\end{aligned}
$$

(18)
$$
\begin{aligned}
&= \left[\beta_K + \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1}) + \frac{1}{2}\phi_{KT}(2t-1)\right](\ln x_{K,t} - \ln x_{K,t-1}) + \\
&\quad \left[(1-\beta_K) - \frac{1}{2}\phi_{KK}(\ln x_{L,t} + \ln x_{L,t-1}) - \frac{1}{2}\phi_{KT}(2t-1)\right](\ln x_{L,t} - \ln x_{L,t-1}) + \\
&\quad \beta_T + \frac{1}{2}\phi_{KT}(\ln x_{K,t} - \ln x_{L,t}) + \frac{1}{2}\phi_{KT}(\ln x_{K,t-1} - \ln x_{L,t-1}) + \frac{1}{2}\phi_{TT}(2t-1)
\end{aligned}
$$

Next, making use of (12) – (13) in (3), we find:

$$\ln X_{t,t-1} = \left[\beta_K + \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1}) + \frac{1}{2}\phi_{KT}(2t-1)\right](\ln x_{K,t} - \ln x_{K,t-1}) + \tag{19}$$
$$\left[(1-\beta_K) - \frac{1}{2}\phi_{KK}(\ln x_{L,t} + \ln x_{L,t-1}) - \frac{1}{2}\phi_{KT}(2t-1)\right](\ln x_{L,t} - \ln x_{L,t-1})$$

so that we find again:

$$\begin{aligned} \ln Y_{t,t-1} - \ln X_{t,t-1} &= \beta_T + \frac{1}{2}\phi_{KT}(\ln x_{K,t} - \ln x_{L,t}) + \frac{1}{2}\phi_{KT}(\ln x_{K,t-1} - \ln x_{L,t-1}) + \\ &\quad \frac{1}{2}\phi_{TT}(2t-1) \\ &= \ln T_{t,t-1} \end{aligned} \tag{20}$$

That is, (1) is actually exact for the Translog functional form.

To sum up our results this far, we find that TFP can be interpreted in four different ways: (1) it is the change in output made possible by the passage of time, holding input quantities constant; (2) it is the average of the instantaneous rates of technological change of times t-1 and t; (3) it is the average rate of technological change between times t-1 and t; (4) it is the part of output growth that cannot be explained by input growth. We have shown that in the Translog case, all four interpretations are perfectly equivalent. This would obviously also be true for the Cobb-Douglas functional form since it is a special case of the Translog.

Estimates of the Translog production function for the United States, 1970-2001, are reported in Table 1, column 1.[1] The value of the logarithm of the likelihood function (LL), together with estimates of the 2001 instantaneous rate of technological change, of the 2001 capital share, and of the 2001 Hicksian elasticity of complementarity are also reported.[2] These can be used to compute TFP according to (15) – or equivalently (16), (17), or (20). We get an average yearly estimate of 1.02%, which is little different from the one obtained on the basis of the index number approach. Over the entire sample period, the compounded effect of TFP reaches about 37.1% of real output.

4. Impact of technological change on factor rental prices

An obvious advantage of the index number approach is that one does not need econometric estimates of the parameters of the production function to obtain an estimate of TFP: $T_{t,t-1}$ can be calculated on the basis of the data alone with the help of (1) – (4). On the other hand, as already pointed out in Section 2, this approach tells us nothing about the nature of technological change, or about its impact on income shares or on the two factor rental prices. The econometric approach is more revealing in this respect. Thus, looking at (12) – (13), it is clear that the sign ϕ_{KT} is essential in determining the impact of the passage of time on factor

[1] The estimates are drawn from Kohli (2010), and they were obtained by jointly estimating (11) and (13); for further details see Kohli (2010).
[2] See Section 7 below for a discussion of the role of the Hicksian elasticity of complementarity.

shares. Thus, if $\phi_{KT} > 0$, one can say that technological is pro-capital and anti-labor biased, in the sense that it increases the share of capital over time and reduces the share of labor. Capital is thus favored at the expense of labor. Looking at the estimate of ϕ_{KT} reported in Table 2, this indeed turns to be the case for the United States. What about factor rental prices, though?

Clearly, if technological change leads to an increase in output, for given factor endowments, and to an increase in the share of capital, it must increase the real return to capital. But what about labor?

From (7) we can write the rental price of labor as follows:

$$(21) \qquad w_{L,t} = \frac{\partial f(x_{K,t}, x_{L,t}, t)}{\partial x_{L,t}} p_t = s_{L,t} \frac{f(x_{K,t}, x_{L,t}, t)}{x_{L,t}} p_t$$

Differentiating with respect to time and making use of (13), we get:

$$(22) \qquad \frac{\partial w_t}{\partial t} = \frac{\partial s_{L,t}}{\partial t} \frac{p_t y_t}{x_{L,t}} + \frac{s_{L,t}}{x_{L,t}} \frac{\partial f(\cdot)}{\partial t} p_t = -\phi_{KT} \frac{p_t y_t}{x_{L,t}} + w_{L,t} \mu_t$$

Or, after having divided through by $w_{L,t}$:

$$(23) \qquad \hat{w}_{L,t} = \mu_t - \frac{\phi_{KT}}{s_{L,t}}$$

where the hat (^) indicates a relative change. As long as the technology is progressing, the first term on the right hand side will be positive. The second term is an indicator of the bias of technological change.[3] As seen above, if ϕ_{KT} is positive, technological change is anti-labor biased. It might even be that $\phi_{KT} / s_{L,t} > \mu_t$, in which case technological change would be ultra anti-labor biased:[4] technological change would lead to an actual fall in the observed wage rate. As it turns out, $0 < \phi_{KT} / s_{L,t} < \mu_t$ in the case of the United States, so that technological change is anti-labor biased, but not ultra anti-labor biased. Nonetheless, the rate of increase of real wages is less than the rate of TFP. Over the entire sample period, real wages increased by about 46%, with about 27% explained by technological change, the rest being explained by capital deepening.[5] We thus can conclude that technological change accounted for nearly two thirds of real wage increases.[6]

Although the econometric approach yields much richer results than the index number approach of Section 2, the fact remains that neither approach teaches us much about the

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[3] See Kohli (1991, 1994).

[4] See Kohli (2010).

[5] See Kohli (2010), Table 3.

[6] If $AB = C$, with $A > 0$ and $B > 0$, and with $a = \ln(A)$, $b = \ln(B)$ and $c = \ln(C)$ so that $a + b = c$, then $A = C^{a/c}$ and $B = C^{b/c}$. We thus can take a/c and b/c as being the relative contributions of A and B to C.

nature of the process of technological change. In particular, it is not immediately obvious why technological change is anti-labor biased.

5. Disembodied, factor augmenting technological change

A more transparent approach, often used in the literature, is to assume that technological change is disembodied and factor augmenting.[7] Assume that technological progress leads to increases in the technical efficiency of existing capital and labor over time. Let $\gamma_{K,t}$ and $\gamma_{L,t}$ denote the capital and labor efficiency factors, respectively, and let $\tilde{x}_{K,t}$ and $\tilde{x}_{L,t}$ be the endowments of capital and labor measured in efficiency units. We then have:

$$\tilde{x}_{K,t} = \tilde{x}_K(x_{K,t},t) = x_{K,t}\,\gamma_{K,t} \tag{24}$$

$$\tilde{x}_{L,t} = \tilde{x}_L(x_{L,t},t) = x_{L,t}\,\gamma_{L,t} \tag{25}$$

Assume, for instance, that technological change increases the efficiency of capital and labor at constant rates, μ_K and μ_L. In that case:

$$\gamma_{K,t} = e^{\mu_K t} \tag{26}$$

$$\gamma_{L,t} = e^{\mu_L t} \tag{27}$$

and:

$$\tilde{x}_{K,t} = \tilde{x}_K(x_{K,t},t) = x_{K,t}\,e^{\mu_K t} \tag{28}$$

$$\tilde{x}_{L,t} = \tilde{x}_L(x_{L,t},t) = x_{L,t}\,e^{\mu_L t} \tag{29}$$

Let the production function again be Translog and expressed in terms of the efficiency units of capital and labor:

$$\ln y_t = \alpha_0 + \beta_K \ln\tilde{x}_{K,t} + (1-\beta_K)\ln\tilde{x}_{L,t} + \frac{1}{2}\phi_{KK}(\ln\tilde{x}_{K,t} - \ln\tilde{x}_{L,t})^2 \tag{30}$$

Making use of (28) and (29), we get:

$$\begin{aligned}\ln y_t &= \alpha_0 + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \beta_K\mu_K t + (1-\beta_K)\mu_L t \\ &\quad + \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})^2 + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L)t \\ &\quad + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2 t^2\end{aligned} \tag{31}$$

The capital and labor shares are again obtained by logarithmic differentiation:

[7] See Kohli (1975, 1981, 1991), for instance.

(32) $$s_{K,t} = \beta_K + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) + \phi_{KK}(\mu_K - \mu_L)t$$

(33) $$s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) - \phi_{KK}(\mu_K - \mu_L)t$$

The impact of the passage of time on the shares of capital and labor now depends on the relative size of μ_K and μ_L, and on the sign of ϕ_{KK}. As for the rate of technological change ($\mu \equiv \partial \ln y/\partial t$), we get:

(34) $$\mu_t = \beta_K\mu_K + (1-\beta_K)\mu_L + \phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t} - \ln x_{L,t}) + \phi_{KK}(\mu_K - \mu_L)^2 t$$

In view of (32) – (33), this can also be expressed as:

(35) $$\mu_t = s_{K,t}\mu_K + s_{L,t}\mu_L$$

Thus, the aggregate rate of technological change is a weighted mean of the rates of increase of efficiency of capital and labor.

Introducing (31) into (10), we get an expression for TFP:

(36) $$\begin{aligned}\ln T_{t,t-1} = {} & \beta_K\mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t} - \ln x_{L,t}) + \\ & \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t-1} - \ln x_{L,t.-1}) + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2(2t-1)\end{aligned}$$

Taking account of (34), we again may write:

(37) $$\ln T_{t,t-1} = \frac{1}{2}(\mu_t + \mu_{t-1})$$

Moreover, in view of (35), we get yet another interpretation of TFP – a fifth one –, namely that it is a moving geometric mean of the rates of factor augmentation:

(38) $$\ln T_{t,t-1} = \frac{1}{2}(s_{K,t} + s_{K,t-1})\mu_K + \frac{1}{2}(s_{L,t} + s_{L,t-1})\mu_L$$

Furthermore, if the true production function is given by (31), then (1) is again valid. Indeed, introducing (31) into (2) we get:

(39) $$\begin{aligned}\ln Y_{t,t-1} = {} & \beta_K(\ln x_{K,t} - \ln x_{K,t-1}) + (1-\beta_K)(\ln x_{L,t} - \ln x_{L,t-1}) + \\ & \frac{1}{2}\phi_{KK}[(\ln x_{K,t} - \ln x_{L,t})^2 - (\ln x_{K,t-1} - \ln x_{L,t-1})^2] + \\ & \phi_{KK}(\mu_K - \mu_L)[(\ln x_{K,t} - \ln x_{L,t})t - (\ln x_{K,t-1} - \ln x_{L,t-1})(t-1)] + \\ & \beta_K\mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2(2t-1)\end{aligned}$$

As for the Törnqvist input quantity index, making use of (32) – (33) in (3), we get:

$$
\begin{aligned}
\ln X_{t,t-1} &= \beta_K(\ln x_{K,t} - \ln x_{K,t-1}) + (1-\beta_K)(\ln x_{L,t} - \ln x_{L,t-1}) + \\
&\quad \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1}) \times \\
&\quad (\ln x_{K,t-1} - \ln x_{K,t-1} - \ln x_{L,t-1} + \ln x_{L,t-1}) + \\
&\quad \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)[(\ln x_{K,t} - \ln x_{L,t}) - (\ln x_{K,t-1} - \ln x_{L,t-1})]\,(2t-1) \\
&= \beta_K(\ln x_{K,t} - \ln x_{K,t-1}) + (1-\beta_K)(\ln x_{L,t} - \ln x_{L,t-1}) + \\
&\quad \frac{1}{2}\phi_{KK}[(\ln x_{K,t} - \ln x_{L,t})^2 - (\ln x_{K,t-1} - \ln x_{L,t-1})^2] + \\
&\quad \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)[(\ln x_{K,t} - \ln x_{L,t}) - (\ln x_{K,t-1} - \ln x_{L,t-1})]\,(2t-1)
\end{aligned}
\tag{40}
$$

We thus find, as expected:

$$
\begin{aligned}
&\ln Y_{t,t-1} - \ln X_{t,t-1} = \beta_K \mu_K + (1-\beta_K)\mu_L \\
&+ \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)[(\ln x_{K,t} - \ln x_{L,t}) + (\ln x_{K,t-1} - \ln x_{L,t-1})] \\
&+ \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2(2t-1) \\
&= \ln T_{t,t-1}
\end{aligned}
\tag{41}
$$

Estimates of (31) are shown in Table 1, column 2. These are also drawn from Kohli (2010). One finds that $\mu_K = 0.2\%$, whereas $\mu_L = 1.3\%$. Technological change in the United States thus comes close to being Harrod neutral.[8] TFP, computed on the basis of (36) – or equivalently (37), (38), or (41) – yields an average yearly rate of 1.02%, which is the same as the value reported in Section 3.

6. The decomposition of TFP between capital and labor

We now turn to the contributions of capital and labor to TFP. For this purpose, we can define the following indices that indicate the output effects of efficiency changes in either capital or labor, in a way similar to (10) above:

$$
T^K_{t,t-1} = \sqrt{\frac{f(x_{K,t}\gamma_{K,t}, \tilde{x}_{L,t})}{f(x_{K,t}\gamma_{K,t-1}, \tilde{x}_{L,t})}\frac{f(x_{K,t-1}\gamma_{K,t}, \tilde{x}_{L,t-1})}{f(x_{K,t-1}\gamma_{K,t-1}, \tilde{x}_{L,t-1})}}
\tag{42}
$$

$$
T^L_{t,t-1} = \sqrt{\frac{f(\tilde{x}_{K,t}, x_{L,t}\gamma_{L,t})}{f(\tilde{x}_{K,t}, x_{L,t}\gamma_{L,t-1})}\frac{f(\tilde{x}_{K,t-1}, x_{L,t-1}\gamma_{L,t})}{f(\tilde{x}_{K,t-1}, x_{L,t-1}\gamma_{L,t-1})}}
\tag{43}
$$

Making use of (26) – (27) and (30) in (42) – (43), we find:

[8] This confirms the findings of Kohli (1981, 1991).

$$\ln T_{t,t-1}^{K} = \beta_K \mu_K + \frac{1}{2}\phi_{KK}\mu_K[(\ln x_{K,t} - \ln x_{L,t}) + (\ln x_{K,t-1} - \ln x_{L,t-1})] + \frac{1}{2}\phi_{KK}(\mu_K^2 - \mu_K\mu_L)(2t-1) = \frac{1}{2}(s_{K,t} + s_{K,t-1})\mu_K \quad (44)$$

$$\ln T_{t,t-1}^{L} = (1-\beta_K)\mu_L - \frac{1}{2}\phi_{KK}\mu_L[(\ln x_{K,t} - \ln x_{L,t}) + (\ln x_{K,t-1} - \ln x_{L,t-1})] + \frac{1}{2}\phi_{KK}(\mu_L^2 - \mu_K\mu_L)(2t-1) = \frac{1}{2}(s_{L,t} + s_{L,t-1})\mu_L \quad (45)$$

so that, in view of (38):

(46) $T_{t,t-1} = T_{t,t-1}^{K} T_{t,t-1}^{L}$

$T_{t,t-1}^{K}$ and $T_{t,t-1}^{L}$ thus give a complete decomposition of TFP in the Translog case. As shown by Figure 1, the compounding of $T_{t,t-1}^{K}$ and $T_{t,t-1}^{L}$ over the entire sample period indicates that the contribution of capital added up to just 1.6%, whereas the contribution of labor reached 34.9%. This reflects the finding that technological change is mostly labor augmenting in the case of the United States.

7. Factor augmenting technological change and TP flexibility

It is apparent that production function (31) contains one *less* parameter than (11). It is therefore not TP flexible. Indeed, the logarithm of the likelihood function is somewhat less than for the model of Section 3.

To generalize the model, let us assume now that the capital and labor efficiency factors are quadratic functions of time. We thus replace (26) and (27) by the following:

(47) $\gamma_{K,t} = e^{\mu_K t + \frac{1}{2}\lambda_K t^2}$

(48) $\gamma_{L,t} = e^{\mu_L t + \frac{1}{2}\lambda_L t^2}$

so that:

(49) $\tilde{x}_{K,t} \equiv x_{K,t} e^{\mu_K t + \frac{1}{2}\lambda_K t^2}$

(50) $\tilde{x}_{L,t} \equiv x_{L,t} e^{\mu_L t + \frac{1}{2}\lambda_L t^2}$

Note that the instantaneous rates of factor augmentation ($\tau_{K,t}$ and $\tau_{L,t}$) are now functions of time:

(51) $$\tau_{K,t} \equiv \frac{\partial \ln \widetilde{x}_{K,t}(x_{K,t}, t)}{\partial t} = \mu_K + \lambda_K t$$

(52) $$\tau_{L,t} \equiv \frac{\partial \ln \widetilde{x}_{L,t}(x_{L,t}, t)}{\partial t} = \mu_L + \lambda_L t$$

Introducing (49) and (50) into (30), we get:

(53) $$\begin{aligned} \ln y_t = {} & \alpha_0 + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \beta_K \mu_K t + (1-\beta_K)\mu_L t \\ & + \frac{1}{2}\beta_K \lambda_K t^2 + \frac{1}{2}(1-\beta_K)\lambda_L t^2 + \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})^2 \\ & + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L)t + \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\lambda_K - \lambda_L)t^2 \\ & + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2 t^2 + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\lambda_K - \lambda_L)t^3 \\ & + \frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)^2 t^4 \end{aligned}$$

The inverse demand functions are now as follows:

(54) $$s_{K,t} = \beta_K + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) + \phi_{KK}(\mu_K - \mu_L)t + \frac{1}{2}\phi_{KK}(\lambda_K - \lambda_L)t^2$$

(55) $$s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln x_{K,t} - \ln x_{L,t}) - \phi_{KK}(\mu_K - \mu_L)t - \frac{1}{2}\phi_{KK}(\lambda_K - \lambda_L)t^2$$

As for the instantaneous rate of technological change, we get:

(56) $$\begin{aligned} \mu_t = {} & \beta_K \mu_K + (1-\beta_K)\mu_L + \beta_K \lambda_K t + (1-\beta_K)\lambda_L t + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L) \\ & + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\lambda_K - \lambda_L)t + \phi_{KK}(\mu_K - \mu_L)^2 t + \frac{3}{2}\phi_{KK}(\mu_K - \mu_L)(\lambda_K - \lambda_L)t^2 \\ & + \frac{1}{2}\phi_{KK}(\lambda_K - \lambda_L)^2 t^3 \end{aligned}$$

In view of (51) – (52) and (54) – (55), this can be expressed as:

(57) $$\mu_t = s_{K,t}\tau_{K,t} + s_{L,t}\tau_{L,t}$$

Thus, the aggregate instantaneous rate of technological change is again found to be a weighted mean of the instantaneous rates of factor augmentation.

Introducing (53) into (10), we find:

$$
\begin{aligned}
\ln T_{t,t-1} &= \beta_K\mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\beta_K\lambda_K(2t-1) + \frac{1}{2}(1-\beta_K)\lambda_L(2t-1) \\
&+ \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L) + \frac{1}{4}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\lambda_K - \lambda_L)(2t-1) \\
&+ \frac{1}{2}\phi_{KK}(\ln x_{K,t-1} - \ln x_{L,t-1})(\mu_K - \mu_L) \\
&+ \frac{1}{4}\phi_{KK}(\ln x_{K,t-1} - \ln x_{L,t-1})(\lambda_K - \lambda_L)(2t-1) \\
&+ \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2(2t-1) + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\lambda_K - \lambda_L)(3t^2 - 3t + 1) \\
&+ \frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)^2(4t^3 - 6t^2 + 4t - 1)
\end{aligned} \tag{58}
$$

whereas in lieu of (44) and (45) we get:

$$
\begin{aligned}
\ln T^K_{t,t-1} &= \beta_K\mu_K + \frac{1}{2}\beta_K\lambda_K(2t-1) + \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\mu_K \\
&+ \frac{1}{4}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\lambda_K(2t-1) \\
&+ \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_K(2t-1) \\
&+ \frac{1}{4}\phi_{KK}(\mu_K - \mu_L)\lambda_K(4t^2 - 4t + 1) + \frac{1}{4}\phi_{KK}(\lambda_K - \lambda_L)\mu_K(2t^2 - 2t + 1) \\
&+ \frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)\lambda_K(4t^3 - 6t^2 + 4t - 1)
\end{aligned} \tag{59}
$$

$$
\begin{aligned}
\ln T^L_{t,t-1} &= (1-\beta_K)\mu_L + \frac{1}{2}(1-\beta_K)\lambda_L(2t-1) \\
&- \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\mu_L \\
&- \frac{1}{4}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\lambda_L(2t-1) \\
&- \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_L(2t-1) \\
&- \frac{1}{4}\phi_{KK}(\mu_K - \mu_L)\lambda_L(4t^2 - 4t + 1) - \frac{1}{4}\phi_{KK}(\lambda_K - \lambda_L)\mu_L(2t^2 - 2t + 1) \\
&- \frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)\lambda_L(4t^3 - 6t^2 + 4t - 1)
\end{aligned} \tag{60}
$$

We again find that:

$$
T_{t,t-1} = T^K_{t,t-1} T^L_{t,t-1} \tag{61}
$$

Estimation of (53) – jointly with (55) – yields the parameter estimates reported in column 3 of Table 1. We find that, although the estimates of μ_K and μ_L are little different from the ones reported in column 2, the estimates of λ_K and λ_L suggest that the rate of capital augmentation might be falling over time, whereas the rate of labor augmentation is increasing. We also find that the estimate of the Hicksian elasticity of complementarity is somewhat larger than what the other models suggested. We report in Figure 2 the cumulated values of $T^K_{t,t-1}$ and $T^L_{t,t-1}$ based on (59) and (60): they are little different from those depicted in Figure 1, and they show once again the overwhelming role of labor in explaining TFP.

8. A parsimonious and yet fully flexible model

We may note that (53) contains one *more* parameter than (11), i.e. one more parameter than needed for it to be TP flexible. A more parsimonious model is obtained by imposing the constraint $\lambda_K = \lambda_L (= \lambda)$. In that case, (53) – (56) and (58) – (60) simplify considerably. The production function, in particular, can now be written as:

$$
\begin{aligned}
\ln y_t = \alpha_0 &+ \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \frac{1}{2}\phi_{KK}(\ln x_{K,t} - \ln x_{L,t})^2 \\
&+ \beta_K \mu_K t + (1-\beta_K)\mu_L t + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L)t \\
&+ \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2 t^2 + \frac{1}{2}\lambda t^2
\end{aligned} \tag{62}
$$

whereas (54) and (55) become identical to (32) and (33), and the instantaneous rate of technological change becomes:

$$\mu_t = \beta_K \mu_K + (1-\beta_K)\mu_L + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(\mu_K - \mu_L) + [\phi_{KK}(\mu_K - \mu_L)^2 + \lambda]t \tag{63}$$

It turns out that the model of equation (62) is equivalent to (11), since there is a one to one correspondence between the two formulations, with:

$$\beta_T = \beta_K \mu_K + (1-\beta_K)\mu_L \tag{64}$$

$$\phi_{KT} = \phi_{KK}(\mu_K - \mu_L) \tag{65}$$

$$\phi_{TT} = \phi_{KK}(\mu_K - \mu_L)^2 + \lambda \tag{66}$$

or, expressing the parameters of (62) in terms of those of (11):

$$\mu_K = \beta_T + (1-\beta_K)\frac{\phi_{KT}}{\phi_{KK}} \tag{67}$$

$$\mu_L = \beta_T - \beta_K \frac{\phi_{KT}}{\phi_{KK}} \tag{68}$$

$$\lambda = \phi_{TT} - \frac{\phi_{KT}^2}{\phi_{KK}} \tag{69}$$

Relationships (67) – (69) provide a convenient guide to find out about the nature of technological change if one estimates (11), rather than (62).[9]

Estimation of (62) – jointly with (33) – yields the results reported in Table 1, column 4. It can be verified, in view of (67) – (69), that the estimates are identical to the ones contained in column 1. A maximum likelihood ratio test confirms that λ is statistically significantly different from zero (the test statistic is 5.22 for a critical χ^2 value of 3.85 for one degree of freedom at the 95% confidence level), but the estimates of μ_K and μ_L are little different from the ones reported for the non-TP flexible functional form (see the estimates in column 2). A maximum likelihood ratio test shows that the restriction $\lambda_K = \lambda_L$ cannot be rejected (the test statistic is 2.38 for a critical χ^2 value of once again 3.85). This indicates that the TP flexible Translog functional form – or alternatively formulation (62) – cannot be rejected in favor of the more general model given by (53).

For TFP we now get:

$$
\begin{aligned}
\ln T_{t,t-1} = {} & \beta_K \mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t} - \ln x_{L,t}) + \\
& \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t-1} - \ln x_{L,t.-1}) + \frac{1}{2}[\phi_{KK}(\mu_K - \mu_L)^2 + \lambda]\,(2t-1)
\end{aligned}
\tag{70}
$$

This estimate of TFP is numerically identical to (15). The contributions of capital and labor can be obtained by setting $\lambda_K = \lambda_L (= \lambda)$ in (59) and (60):

$$
\begin{aligned}
\ln T^K_{t,t-1} = {} & \beta_K \mu_K + \frac{1}{2}\beta_K \lambda(2t-1) + \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\mu_K \\
& + \frac{1}{4}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\lambda(2t-1) \\
& + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_K(2t-1) + \frac{1}{4}\phi_{KK}(\mu_K - \mu_L)\lambda(3t^2 - 3t + 1)
\end{aligned}
\tag{71}
$$

$$
\begin{aligned}
\ln T^L_{t,t-1} = {} & (1-\beta_K)\mu_L + \frac{1}{2}(1-\beta_K)\lambda(2t-1) \\
& + \frac{1}{2}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\mu_L \\
& + \frac{1}{4}\phi_{KK}(\ln x_{K,t} + \ln x_{K,t-1} - \ln x_{L,t} - \ln x_{L,t-1})\lambda(2t-1) \\
& + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_L(2t-1) + \frac{1}{4}\phi_{KK}(\mu_K - \mu_L)\lambda(3t^2 - 3t + 1)
\end{aligned}
\tag{72}
$$

[9] This assumes that $\phi_{KK} \neq 0$. In the Cobb-Douglas case, when $\phi_{KK} = 0$, μ_K and μ_L cannot be identified separately, and it is not possible to distinguish between Harrod-neutral, Solow-neutral or Hicks-neutral technological change.

The decomposition of TFP between the contributions of capital and labor is shown in cumulated form in Figure 3. The overwhelming contribution of labor is clearly visible. Over the 1970-2001 sample period, the compounded contribution of labor reaches 34.7%, whereas the contribuition of capital only reaches 1.9%. One can thus conclude that about 94.6% of TFP in the United States is explained by labor. Note that, in view of the equivalence of (11) and (62), this conclusion is also valid for the TP-flexible Translog production function.

9. The impact of technological change on factor rental prices reexamined

We return to the question of the impact of technological change on factor rental prices. Now that we have estimates of μ_K, μ_L and λ we can gain some additional insights, and we bring forward the important role of Hicksian elasticity of complementarity between capital and labor in explaining why technological change is anti-labor biased in the case of the United States. As suggested by (22), technological progress must increase the real return of at least one factor, but not necessarily of both. Take the extreme case of Harrod-neutral technological progress, which is a reasonable approximation for the United States. In that case, technological progress leads to an increase in the endowment of labor measured in efficiency units. Output necessarily increases. The return to capital must increase as well since in the two-input case, the two inputs are necessarily Hicksian complements for each other. The return to labor per efficiency unit must necessarily decrease because of diminishing marginal returns. By how much depends on the size of the elasticity of complementarity. If capital and labor are strong Hicksian complements, the return to labor per efficiency unit will fall by a large amount, so that the return to labor per observed unit will decline, even though each unit of labor has become more efficient!

To investigate this more formerly, we begin by defining ε_{ij} as the inverse price elasticity of factor demand (the time subscript is omitted for clarity):

$$(73) \qquad \varepsilon_{ij} \equiv \frac{\partial \ln \widetilde{w}_i(\widetilde{x}_K, \widetilde{x}_L, p)}{\partial \ln \widetilde{x}_j}, \quad i, j \in \{K, L\}$$

Linear homogeneity of the production function implies:

$$(74) \qquad \varepsilon_{iK} + \varepsilon_{iL} = 0 \ , \quad i \in \{K, L\}$$

It is well known that:[10]

$$(75) \qquad \varepsilon_{KL} = \psi_{KL} s_L$$

$$(76) \qquad \varepsilon_{LK} = \psi_{KL} s_K$$

where ψ_{KL} is the Hicksian elasticity of complementarity between capital and labor:

[10] See Kohli (1991), for instance.

(77) $$\psi_{KL} \equiv \frac{f(\cdot)\dfrac{\partial^2 f(\cdot)}{\partial x_K \partial x_L}}{\dfrac{\partial f(\cdot)}{\partial x_K}\dfrac{\partial f(\cdot)}{\partial x_L}}$$

In the Translog case, it can be computed as:[11]

(78) $$\psi_{KL} = \frac{-\phi_{KK} + s_K(1-s_K)}{s_K(1-s_K)}$$

We thus get for the total change in the rental price of an efficiency unit of capital:

(79) $$\hat{\tilde{w}}_K = \varepsilon_{KK}\hat{\tilde{x}}_K + \varepsilon_{KL}\hat{\tilde{x}}_L = \varepsilon_{KL}(\mu_L - \mu_K) = \psi_{KL}s_L(\mu_L - \mu_K)$$

where the hats (^) again indicate relative changes. In terms of observed factor prices:

(80) $$\hat{w}_K = \hat{\tilde{w}}_K + \mu_K + \lambda = \psi_{KL}s_L(\mu_L - \mu_K) + \mu_K + \lambda$$

and similarly for labor:

(81) $$\hat{\tilde{w}}_L = \varepsilon_{LL}\hat{\tilde{x}}_L + \varepsilon_{LK}\hat{\tilde{x}}_K = \varepsilon_{LK}(\mu_K - \mu_L) = \psi_{KL}s_K(\mu_K - \mu_L)$$

(82) $$\hat{w}_L = \hat{\tilde{w}}_L + \mu_L + \lambda = \psi_{KL}s_K(\mu_K - \mu_L) + \mu_L + \lambda = \psi_{KL}s_K\mu_K + \lambda + (1-\psi_{KL}s_K)\mu_L$$

Looking at the results for the United States, it is clear that technological progress leads to an increase in the return to capital since all three right-hand-side terms in (80) are positive. For labor, looking at (82), the first two terms are positive, although they are close to zero given that technological change turns out to be almost Harrod-neutral and that λ is numerically small. The third term is positive as long as $\psi_{KL} < 1/s_K$, which indeed turns out to be the case.[12] So we can conclude that technological progress also increases the return to labor in the U.S. case. Note, however, that because the share of labor declines, the increase in real wages is less that the increase in the average productivity of labor, or of TFP for that matter.

Technological change in the United States is anti-labor biased because it is mostly labor augmenting, and because the Hicksian elasticity of complementarity between capital and labor is greater than one.[13] These two findings together explain why technological change has a negative impact on the share of labor.[14] This could not have been inferred from the mere finding that ϕ_{KT} is positive: technological change would also be anti-labor biased if it were Solow neutral and if the elasticity of complementarity were less than one.

[11] See Kohli (1991).
[12] See the estimates reported in Table 1, at the bottom of column 2.
[13] See Kohli (2010). Note that in the two-input case, the Hicksian elasticity of complementarity is the inverse of the Allen-Uzawa elasticity of substitution ; Kohli (1991). Thus, a Hicksian elasticity of complementarity greater than one implies an Allen-Hicks elasticity of substitution less than one.
[14] This is actually offset by the increase in capital intensity over time, which has the opposite effect in view of the relatively large elasticity of complementarity

10. Generalization to an arbitrary number of inputs

Assume now that there are J inputs. Taking the linear homogeneity restrictions into account, the TP-flexible Translog production function can be written as follows:

$$
\begin{aligned}
\ln y_t = \alpha_0 + \ln x_{J,t} + \sum_{j=1}^{J-1} \beta_j (\ln x_{j,t} - \ln x_{J,t}) + \beta_T t \\
+ \frac{1}{2} \sum_{j=1}^{J-1} \sum_{k=1}^{J-1} \phi_{jk} (\ln x_{j,t} - \ln x_{J,t})(\ln x_{k,t} - \ln x_{J,t}) \\
+ \sum_{j=1}^{J-1} \phi_{jT} (\ln x_{j,t} - \ln x_{J,t}) t + \frac{1}{2} \phi_{TT} t^2
\end{aligned}
\tag{83}
$$

The same function expressed in terms of disembodied factor augmenting technological change becomes:

$$
\begin{aligned}
\ln y_t = \alpha_0 + \ln x_{J,t} + \mu_J t + \frac{1}{2} \lambda t^2 + \sum_{j=1}^{J-1} \beta_j (\ln x_{j,t} - \ln x_{J,t}) + \sum_{j=1}^{J-1} \beta_j (\mu_j - \mu_J) t \\
+ \frac{1}{2} \sum_{j=1}^{J-1} \sum_{k=1}^{J-1} \phi_{jk} (\ln x_{j,t} - \ln x_{J,t})(\ln x_{k,t} - \ln x_{J,t}) \\
+ \sum_{j=1}^{J-1} \sum_{k=1}^{J-1} \phi_{jk} (\ln x_{j,t} - \ln x_{J,t})(\mu_k - \mu_J) t \\
+ \frac{1}{2} \sum_{j=1}^{J-1} \sum_{k=1}^{J-1} \phi_{jk} (\mu_j - \mu_J)(\mu_k - \mu_J) t^2
\end{aligned}
\tag{84}
$$

This implies the following corresponding relationships between the parameters of the two functions:

$$\mu_J + \sum_{j=1}^{J-1} \beta_j (\mu_j - \mu_J) = \beta_T \tag{85}$$

$$\frac{1}{2} \sum_{k=1}^{J-1} \phi_{jk} (\mu_k - \mu_J) = \phi_{jT}, \quad j = 1, \ldots, J-1 \tag{86}$$

$$\lambda + \sum_{j=1}^{J-1} \sum_{k=1}^{J-1} \phi_{jk} (\mu_j - \mu_J)(\mu_k - \mu_J) = \phi_{TT} \tag{87}$$

As long as the $(J-1)\times(J-1)$ matrix of the ϕ_{jk}'s is not singular,[15] (86) can be solved for the $J-1$ factor augmenting differentials $\mu_j - \mu_J$; μ_J, and thus all the μ_j's, can then be obtained from (85), and λ by (87).

11. Conclusions

In this paper we investigated the relationship between a common measure of TFP and the concept of disembodied, factor augmenting technological change. This led us to come up with five different interpretations of TFP: (1) it is the part of output growth that cannot be explained by input growth; (2) it is the change in output made possible by the passage of time, holding input quantities constant; (3) it is the average of the instantaneous rates of technological change of times *t*-1 and *t*; (4) it is the average rate of technological change between times *t*-1 and *t*; (5) it is a moving geometric mean of the rates of factor augmentation. In the Translog case, all five interpretations are equivalent.

We have shown that in the case of a TP-flexible Translog production function TFP can always be interpreted as the outcome of disembodied, factor augmenting technological change. Indeed, we have proposed a convenient way to derive the factor-augmenting rates of technological change from the estimates of such a Translog production function.

Based on our data sample, we have found that technological change is almost Harrod-neutral in the case of the United States, so that TFP is overwhelmingly explained by labor. Furthermore, technological change is anti-labor biased, in the sense that it tends to decrease the income share of labor. This is due to the relatively large Hicksian elasticity of complementarity between capital and labor. Nonetheless, technological change has a positive effect on the return of both capital and labor, although the benefit to labor is less than what TFP or average labor productivity would suggest.

[15] This will normally be the case, unless flexibility has been intentionally restricted; see Diewert and Wales (1988).

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Table 1

Parameter estimates

	(11)	*(31)*	*(53)*	*(62)*
α_0	8.38851	8.39259	8.38944	8.38851
	(3522.9)	(4874.6)	(3417.6)	(3522.9)
β_K	0.27365	0.27405	0.27464	0.27365
	(189.6)	(188.6)	(181.0)	(189.6)
ϕ_{KK}	-0.15322	-0.13598	-0.20418	-0.15322
	(-3.75)	(-3.31)	(-4.16)	(-3.75)
ϕ_{KT}	0.00171			
	(5.19)			
β_T	0.01026			
	(24.25)			
ϕ_{TT}	0.00008			
	(1.93)			
μ_K		0.00182	0.00291	0.00217
		(1.75)	(4.93)	(2.57)
μ_L		0.01337	0.01302	0.01331
		(27.75)	(39.59)	(31.96)
λ				0.00010
				(2.38)
λ_K			-0.00011	
			(-0.95)	
λ_L			0.00015	
			(3.23)	
LL	226.70	224.09	227.89	226.70
μ_{2001}	0.01158	0.01008	0.01116	0.01158
$s_{K,2001}$	0.28452	0.28454	0.28007	0.28452
$\psi_{KL,2001}$	1.75267	1.66797	2.01263	1.75267

Figure 1

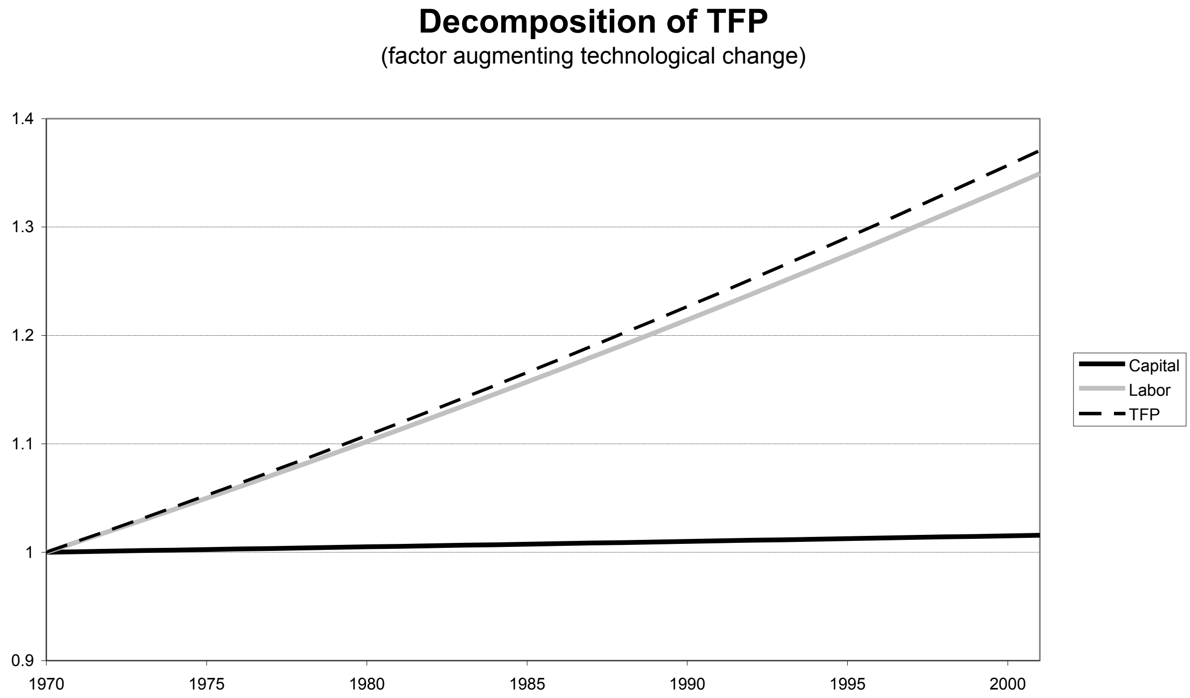


Figure 2

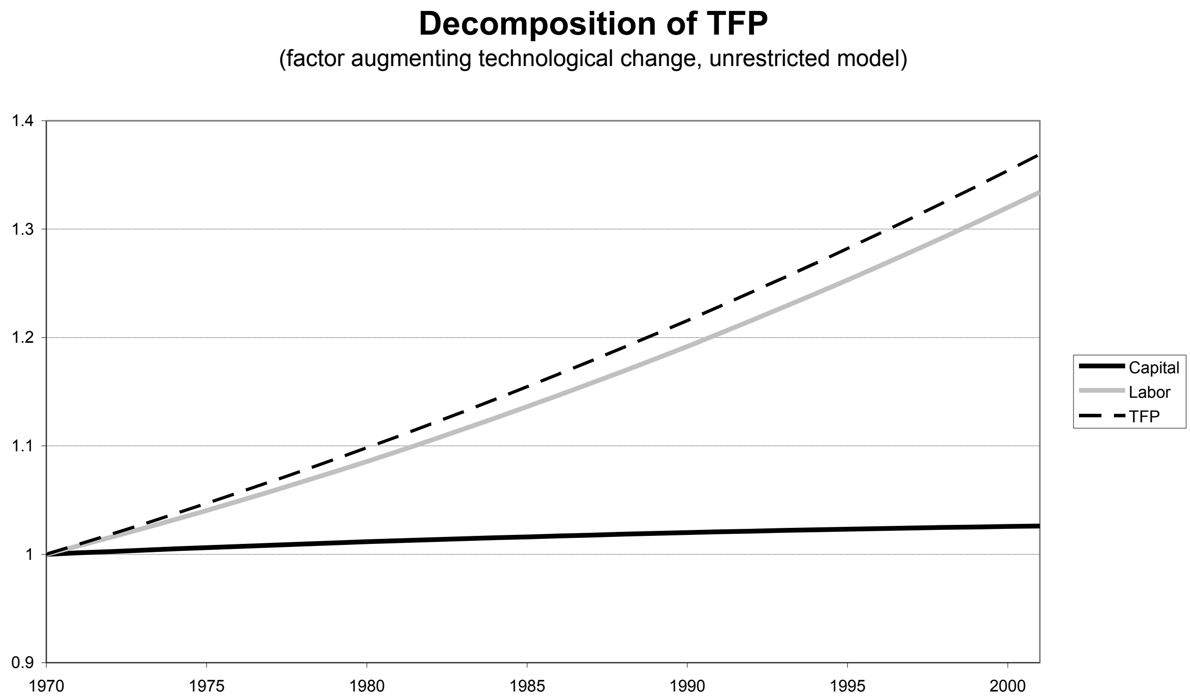
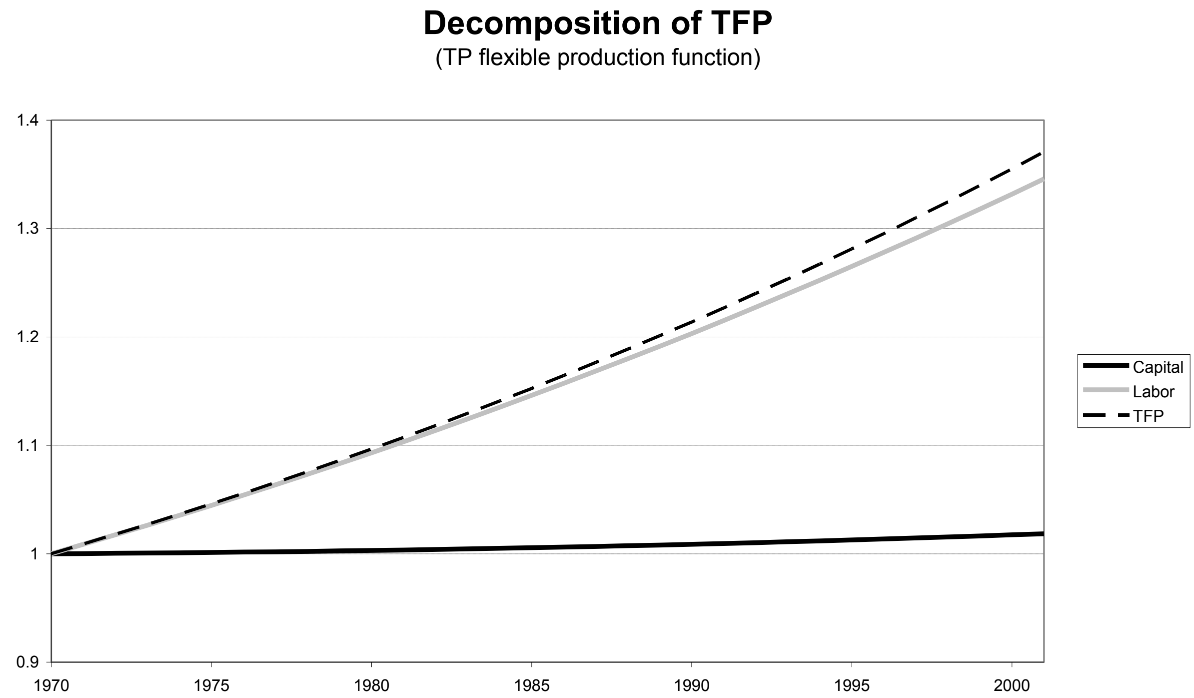


Figure 3


Decomposition of TFP
(TP flexible production function)
1.4
1.3
1.2
1.1
1
0.9
1970
1975
1980
1985
1990
1995
2000
Capital
Labor
TFP