Output Growth and Inflation Across Space and Time

by Erwin Diewert
University of British Columbia and
University of New South Wales

and

Kevin Fox
University of New South Wales

EMG Workshop 2015
University of New South Wales
December 4, 2015

Introduction

- **The present paper will use OECD data for the years 2000-2012 in order to study the following two measurement problems:**
- **How can estimates of OECD real GDP and associated measures of OECD inflation be constructed?**
- **How can the OECD annual PPP information be used in conjunction with member country data on real GDP growth to construct estimates of member country real GDP that are in principle comparable across space and time?**
- **The OECD provides annual PPPs so our strategy in solving the second problem will be to use OECD aggregate GDP growth rates along with the annual PPP information to produce Harmonized GDP series that are in principle, comparable across time and space.**

OECD Ordering of Countries

- **Our first task is to use the OECD data base to form country GDP volumes. We will use the OECD ordering of countries, which is as follows:**

- **1= Australia**
- **2= Austria**
- **3= Belgium**
- **4= Canada**
- **5= Chile**
- **6= Czech Republic**
- **7= Denmark**
- **8= Estonia**
- **9= Finland**
- **10= France**
- **11= Germany**
- **12= Greece**
- **13= Hungary**
- **14= Iceland**
- **15= Ireland**

OECD Ordering of Countries (cont)

- **16= Israel**
- **17= Italy**
- **18= Japan**
- **19= Korea**
- **20= Luxembourg**
- **21= Mexico**
- **22= Netherlands**
- **23= New Zealand**
- **24= Norway**
- **25= Poland**
- **26= Portugal**
- **27= Slovak Republic**
- **28= Slovenia**
- **29= Spain**
- **30= Sweden**
- **31= Switzerland**
- **32= Turkey**
- **33= United Kingdom**
- **34= United States.**

Country Domestic Price and Quantity Levels P_n^t and Q_n^t

- **The country values for nominal GDP in the national currencies for the years 2000-2012 can be obtained from the OECD electronic data base, OECD.Stat.**
- **Convert these estimates into billions and denote the estimate for country n in year t by V_n^t The corresponding volume estimates can be obtained from OECD.Stat TableB1-GE: Gross domestic product (GDP); National currency, constant prices, national base year, millions, annual data.**
- **Convert these estimates into billions and denote these volumes (or quantities) by Q_n^t for n = 1,...,34 and t = 2000, ...,2012.**
- **The corresponding country price level for country n in year t is defined as $P_n^t \equiv V_n^t/Q_n^t$ for n = 1,...,34 and t = 2000,...,2012**

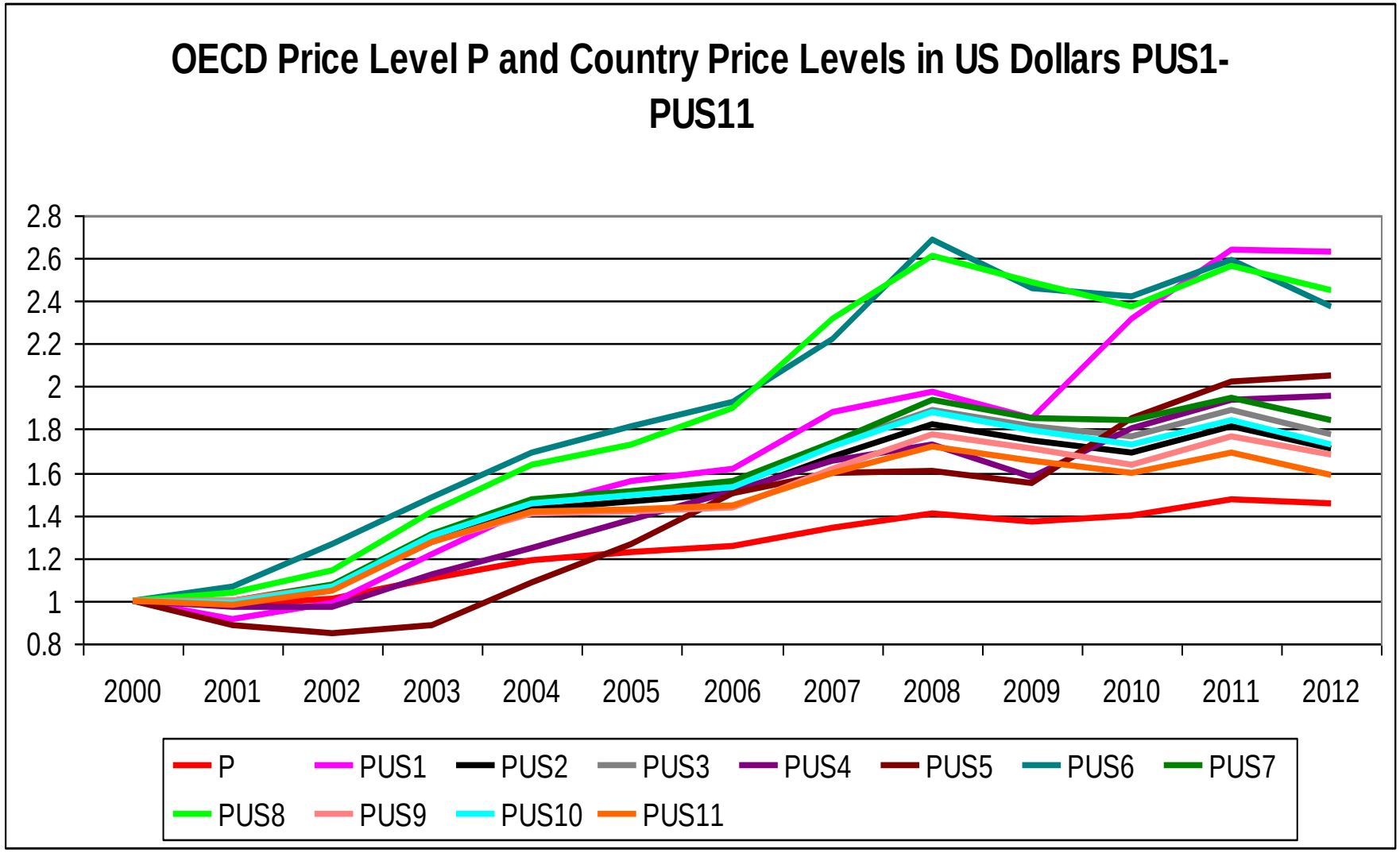
Definition of OECD Aggregate GDP Inflation

- **Since the country volumes Q_n^t are measured in domestic currency units (which are not comparable across countries), we need to convert the domestic nominal values of GDP into common currency units using the average exchange rates for each year.**
- **In principle, the numeraire country could be any of the 34 OECD countries but it seems reasonable to choose the largest country as the numeraire country.**
- **The OECD has conveniently done this for us, converting each country's nominal GDP into US dollars at the average market exchange rates for the given year. Convert these estimates into billions and denote the US dollar estimate for nominal GDP for country n in year t by v_n^t.**

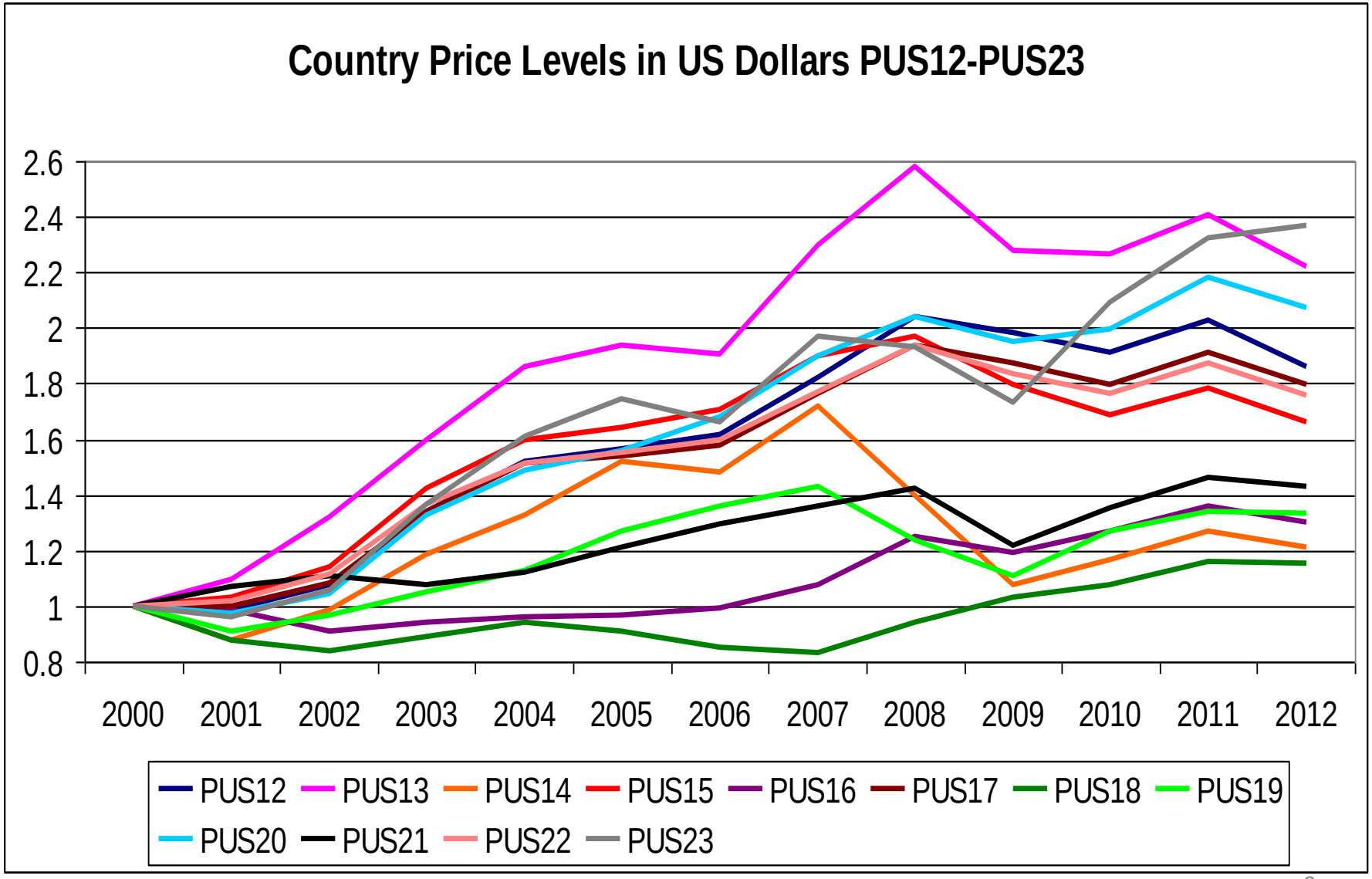
Definition of OECD Aggregate GDP Inflation (cont)

- The year t, country n US dollar price level for GDP, p_n^t, is initially defined as v_n^t/Q_n^t where the country volumes or real outputs Q_n^t have already been defined using national data.
- The resulting p_n^t were normalized so that $p_n^{2000} = 1$ for n = 1,...,34.
- The Q_n^t were then normalized in the opposite direction so that US dollar values were preserved. Denote the resulting normalized Q_n^t as q_n^t for n = 1,...,34 and t = 2000,...,2012.
- These US dollar price levels p_n^t and the corresponding volumes q_n^t are listed in Tables A3 and A4 in the Appendix.
- These price levels are comparable over time only; i.e., they are not comparable across countries.
- These US dollar (noncomparable across countries) price levels are plotted on the following 3 Charts.

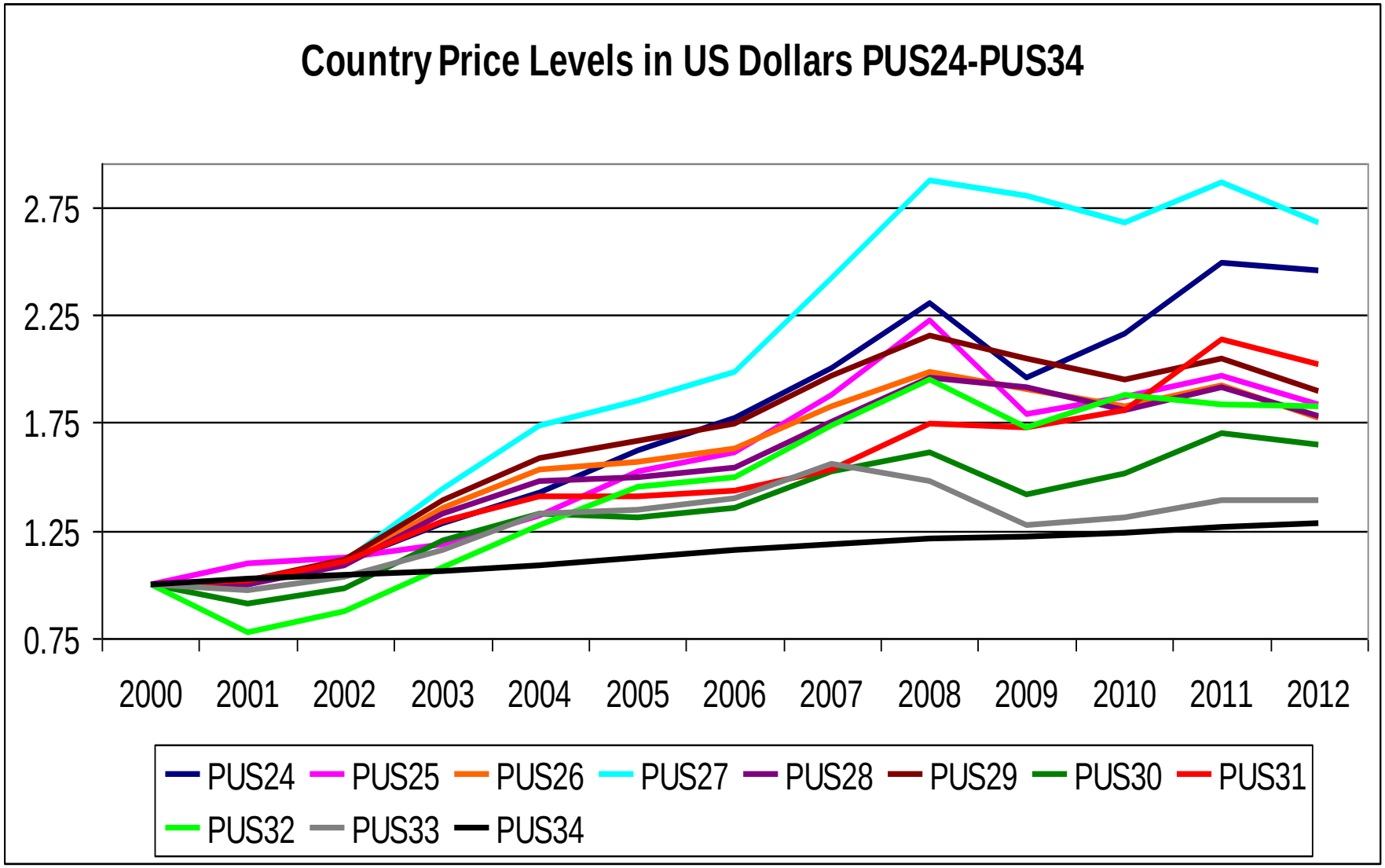
US Dollar Price Levels; OECD and Countries 1-11



US Dollar Price Levels; Countries 12-23



US Dollar Price Levels; Countries 24-34



Aggregate Measures of OECD GDP Growth and Inflation: First Approach

- **We are now in a position to calculate aggregate OECD real output and the corresponding OECD price level for the years 2000-2012 using the (US dollar) price and (domestic) volume data, p_n^t and q_n^t, as inputs into the Fisher chained index number formula.**
- **Denote the chained Fisher aggregate OECD volume level for year t by Q^t and the corresponding US dollar year t price level by P^t for t = 2000,...,2012. For t = 2001,...,2012, define the year t OECD Approach 1 *volume growth rate* γ^t and the corresponding OECD *US dollar inflation rate* ρ^t in percentage points as follows:**

$$\text{(10) } \gamma^t \equiv 100[(Q^t/Q^{t-1}) - 1] \text{ ;}$$

$$\text{(11) } \rho^t \equiv 100[(P^t/P^{t-1}) - 1] \text{ .}$$

Aggregate Measures of OECD GDP Growth and Inflation: First Approach (cont)

- The chained Fisher OECD aggregate price and volume levels, P^t and Q^t, for the years 2000-2012 are listed in Table 1 along with the corresponding percentage point annual growth rates, ρ^t and γ^t, for the years 2001-2012.
- For comparison purposes, we also calculated the aggregate OECD chained Laspeyres and Paasche indexes over the same period. The resulting Laspeyres and Paasche price levels, $P_L{}^t$ and $P_P{}^t$, are also listed in Table 1.
- It can be seen that the chained Fisher, Laspeyres and Paasche price levels are all very close to each other so that for this particular application, the choice of index number formula does not matter very much.
- Note the US dollar price level deflation in 2001, 2009 and 2012 and the large OECD GDP volume decline in 2009.

Table 1: OECD Annual Aggregate Volumes Q^t and Price Levels in US Dollars P^t and Percentage Point Changes

Year t	Q^t	P^t	P_L^t	P_P^t	γ^t	ρ^t	ρ_{EU}^t	P_{EU}^t	P_{ICP}^t
2000	26694.3	1.0000	1.0000	1.0000				1.0000	1.0000
2001	27022.9	0.9794	0.9793	0.9795	1.23	-2.06	0.84	1.0084	1.0301
2002	27432.9	1.0081	1.0079	1.0082	1.52	2.92	-2.14	0.9868	1.0549
2003	28007.3	1.1079	1.1079	1.1078	2.09	9.90	-8.35	0.9044	1.0796
2004	28896.6	1.1933	1.1934	1.1932	3.18	7.71	-2.10	0.8854	1.1067
2005	29670.9	1.2271	1.2271	1.2272	2.68	2.84	2.68	0.9091	1.1326
2006	30566.7	1.2560	1.2558	1.2561	3.02	2.35	1.46	0.9224	1.1610
2007	31374.2	1.3392	1.3388	1.3396	2.64	6.63	-2.27	0.9015	1.1893
2008	31410.0	1.4109	1.4104	1.4114	0.11	5.36	-1.56	0.8874	1.2173
2009	30267.1	1.3729	1.3726	1.3732	-3.64	-2.69	2.60	0.9105	1.2305
2010	31138.6	1.4013	1.4007	1.4019	2.88	2.07	7.06	0.9748	1.2477
2011	31688.5	1.4776	1.4770	1.4782	1.77	5.44	0.46	0.9793	1.2697
2012	32162.6	1.4534	1.4525	1.4543	1.50	-1.63	6.42	1.0422	1.2888

Discussion of Table 1

- **The sample average growth rate for OECD real GDP was 3.18% per year. The sample average OECD inflation rate (measured in US dollars at market exchange rates) was 3.24% per year.**
- **It can be seen that there was only one year where OECD real growth was negative: 2009 (–3.64%).**
- **What is somewhat surprising is that there were 3 years where OECD inflation was negative when measured in US dollars at market exchange rates: 2001 (–2.06%), 2009 (–2.69%) and 2012 (–1.63%). The deflation for 2012 is particularly surprising, given the fairly loose monetary policies across the OECD region in recent years.**
- **As mentioned before, the choice of Laspeyres, Paasche or Fisher aggregate GDP and price level indexes does not matter much for this application.**

More Discussion of Table 1

- **The principles used to construct our OECD aggregate measures of real GDP, Q^t, are the same principles used to construct country wide estimates of real GDP within a country.**
- **The country estimates of real GDP aggregate output growth over regions within the country use regional price levels as weights for the regional volumes. In constructing national estimates of real output, the national statistician does not assume that the quantities or volumes in each region are comparable across regions: all that is assumed is that whatever is being measured at the regional level is comparable over time.**
- **This is the same principle that is being used in Table 1 to construct OECD real output: there is no assumption that the country units of measurement are comparable across countries.**

And Even More Discussion of Table 1

- **There is one difference in our suggested method for constructing OECD real GDP as opposed to methods used to construct national estimates of real GDP: in order to construct OECD real GDP, we needed to convert national values of GDP into a common currency using annual average market exchange rates.**
- **We chose to make this conversion using US dollars as the numeraire currency. In principle, we could have chosen the numeraire currency to be the currency of any one of the 34 member countries.**
- **What would happen if we chose another currency to be the numeraire currency? The unit of measurement would change, but the overall OECD *growth rates* for real GDP would remain the same; i.e., the γ^t listed in Table 1 *would not change* but the inflation measures ρ^t would change; i.e., they are numeraire dependent. [Compare ρ^t with $\rho_{EU}{}^t$].**

OECD Growth and Inflation Measurement Using Annual PPP Information: Approach 2

- **For many purposes, it is useful to be able to compare the GDP of one country with the GDP of another country in comparable units of measurement.**
- **Thus the OECD (in close cooperation with Eurostat) produces an annual series of price indexes (or PPPs) that enable one to compare the real GDP of member countries with each other.**
- **The relevant table of PPPs for the 34 countries can be found in OECD.Stat, Table 4: PPPs and Exchange Rates; PPPGDP; Purchasing Power Parities for GDP; National currency per US dollar; Annual; 2000-2012.**
- **Our second approach to measuring OECD volumes and inflation uses these PPPs along with national growth rates.**

Country Shares of Annual OECD Real Output

- **Recall that the country n nominal value of GDP in year t in domestic currency was defined by V_n^t previously.**
- **Divide these nominal values by the corresponding country n year t PPP in order to obtain an estimate, r_n^t, of country n's *real GDP* in year t in volume units that are comparable across countries for year t:**

(12) $r_n^t \equiv V_n^t/PPP_n^t$; n = 1,...,34; t = 2000,...,2012.

- **Once the r_n^t have been calculated, they can be summed so that $r^t \equiv \sum_{n=1}^{34} r_n^t$ and then the *year t country n share of OECD real output* can be defined as follows:**

(13) $s_n^t \equiv r_n^t/r^t$; n = 1,...,34; t = 2000,...,2012.

- **The country shares of OECD real GDP are listed in Table 2.**

Harmonization Principles

- **Our suggested solution to the problem of harmonizing national growth rates of GDP with the country shares of OECD aggregate real GDP rests on two principles:**

- **The resulting harmonized estimates of country volumes must be consistent with the real annual cross country volume shares s_n^t listed in Table 2;**
- **OECD aggregate real GDP growth must be equal to the rates of aggregate growth generated by our recommended Fisher indexes Γ_F^t defined by (16) below.**

- **Using the above two principles, the comparable across time and space country GDP volumes will be uniquely determined (up to a scalar units of measurement factor).**

Aggregate OECD Volume Growth: Laspeyres

- **The *year t growth factor for country n* can be defined as Q_n^t/Q_n^{t-1} where Q_n^t is country n's GDP volume in year t. Thus define the *OECD Laspeyres type growth factor* (or chain link) for year t, Γ_L^t, as the following weighted average of the national growth factors:**

(14) $$\Gamma_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(Q_n^t/Q_n^{t-1}) ; \qquad t = 2001,...,2012.$$

- **The above measure of OECD GDP volume growth is the method used by the OECD to calculate their official annual measure of OECD volume growth.**
- **It certainly is a sensible measure, using country (one plus) growth rates going from year t–1 to year t, Q_n^t/Q_n^{t-1}, weighted by the country real volume shares s_n^{t-1} for year t–1, which were derived using PPPs.**

Aggregate OECD Volume Growth: Paasche and Fisher

- **The counterpart to the Laspeyres type formula defined by (14) is the following *Paasche type formula* (which applies the Laspeyres formula but reverses the role of time):**

(15) $\Gamma_P^t \equiv [\sum_{n=1}^{34} s_n^t (Q_n^t/Q_n^{t-1})^{-1}]^{-1}$; **t = 2001,...,2012.**

- **Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following *Fisher type formula* for OECD volume growth for year t:**

(16) $\Gamma_F^t \equiv [\Gamma_L^t \Gamma_P^t]^{1/2}$; **t = 2001,...,2012.**

- **The annual Fisher chain links defined by (16) are our preferred estimates for OECD volume growth using Approach 2.**

Aggregate OECD Volume Growth: Approach 2 Indexes

- **The growth factors (or chain link indexes) defined by (14)-(16) can be multiplied together to generate OECD aggregate volume levels. The growth factors can also be transformed into growth rates, γ_L^t, γ_P^t and γ_F^t (in percentage points), by using the following definitions for t = 2001,...,2012:**

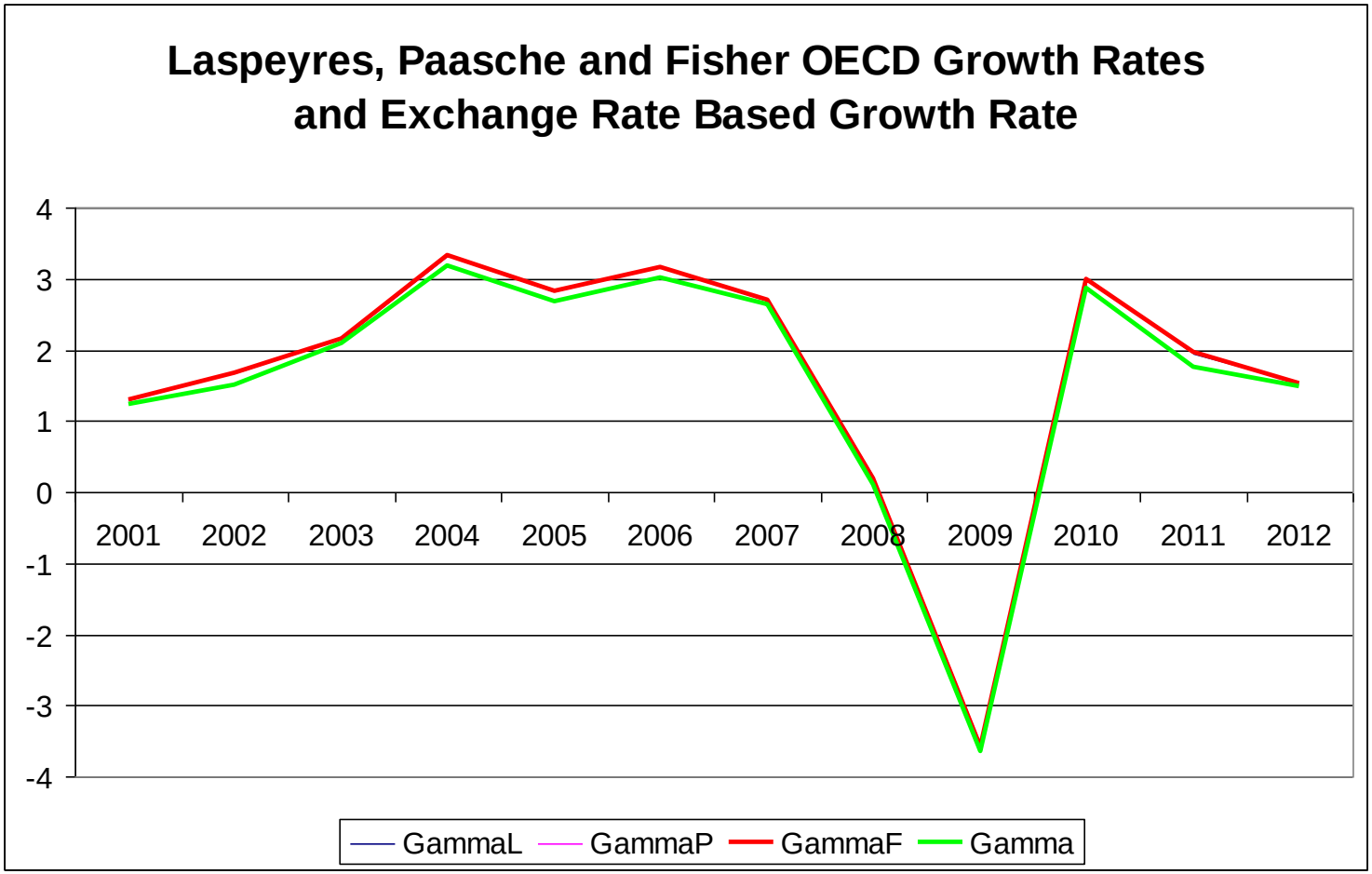
(17) $\gamma_L^t \equiv 100[\Gamma_L^t - 1]$; $\gamma_P^t \equiv 100[\Gamma_P^t - 1]$; $\gamma_F^t \equiv 100[\Gamma_F^t - 1]$.

- **The Approach 2 annual OECD volume growth measures defined by (17) as well as our earlier Approach 1 US dollar weighted measures γ^t are listed in Table 3 which is reproduced on the next slide.**

Alternative Aggregate OECD Volume Growth Measures

Year t	γ_L^t	γ_P^t	γ_F^t	γ^t
2001	1.2908	1.2964	1.2936	1.2313
2002	1.6832	1.6767	1.6799	1.5171
2003	2.1670	2.1610	2.1640	2.0937
2004	3.3269	3.3331	3.3300	3.1753
2005	2.8318	2.8311	2.8314	2.6796
2006	3.1525	3.1592	3.1558	3.0193
2007	2.7065	2.7074	2.7070	2.6415
2008	0.1912	0.1902	0.1907	0.1144
2009	-3.5714	-3.5736	-3.5725	-3.6386
2010	2.9946	3.0011	2.9978	2.8791
2011	1.9562	1.9629	1.9595	1.7662
2012	1.5428	1.5303	1.5365	1.4962
Average	1.6893	1.6897	1.6895	1.5812

Alternative Aggregate OECD Volume Growth Measures



Aggregate OECD Volume Growth Measures: Discussion

- **Our preferred Fisher measure of OECD growth in comparable units across countries, γ_F^t, grew on average about 1/10 of a percentage point more rapidly per year than our preferred measure of OECD GDP growth using US dollar weights (or more generally, using exchange rate weighted weights), γ^t, that was explained in the previous section.**
- **Which measure, γ_F^t or γ^t, is best?**
- **Both γ^t and γ_F^t are essentially share weighted averages of the national volume growth factors, Q_n^t/Q_n^{t-1}. However, the shares used in the construction of the γ^t are country shares of OECD nominal GDP constructed using exchange rates and the resulting shares are subject to "excessive" exchange rate fluctuations. On the other hand, the shares used to construct γ_F^t are real shares of OECD output constructed using PPPs and will be more stable.**

Aggregate OECD Inflation Indexes: Approach 2

- **The OECD real output shares, s_n^t defined by (13), can be used as weights for national GDP inflation rates. Recall that the national currency GDP price deflator for country n in year t was defined as P_n^t. Recall also that (14)-(16) defined OECD Laspeyres, Paasche and Fisher volume link volume indexes, Γ_L^t, Γ_P^t and Γ_F^t. Use these definitions with the national inflation factors (P_n^t/P_n^{t-1}) replacing the national volume growth factors (Q_n^t/Q_n^{t-1}) in order to define the *OECD Laspeyres, Paasche and Fisher ICP based chain link price indexes*, Π_L^t, Π_P^t and Π_F^t for t = 2001,...,2012:**

$$(18)\ \Pi_L^t \equiv \sum_{n=1}^{34} s_n^{t-1}(P_n^t/P_n^{t-1}) ;$$

$$\Pi_P^t \equiv [\sum_{n=1}^{34} s_n^t(P_n^t/P_n^{t-1})^{-1}]^{-1} ;$$

$$\Pi_F^t \equiv [\Pi_L^t \Pi_P^t]^{1/2}.$$

Aggregate OECD Inflation Indexes: Approach 2

- **These chain link indexes can be multiplied together to generate the corresponding OECD aggregate price levels. The inflation growth factors can also be transformed into *growth rates*, ρ_L^t, ρ_P^t and ρ_F^t in percentage points, by using the following definitions for t = 2001,...,2012:**

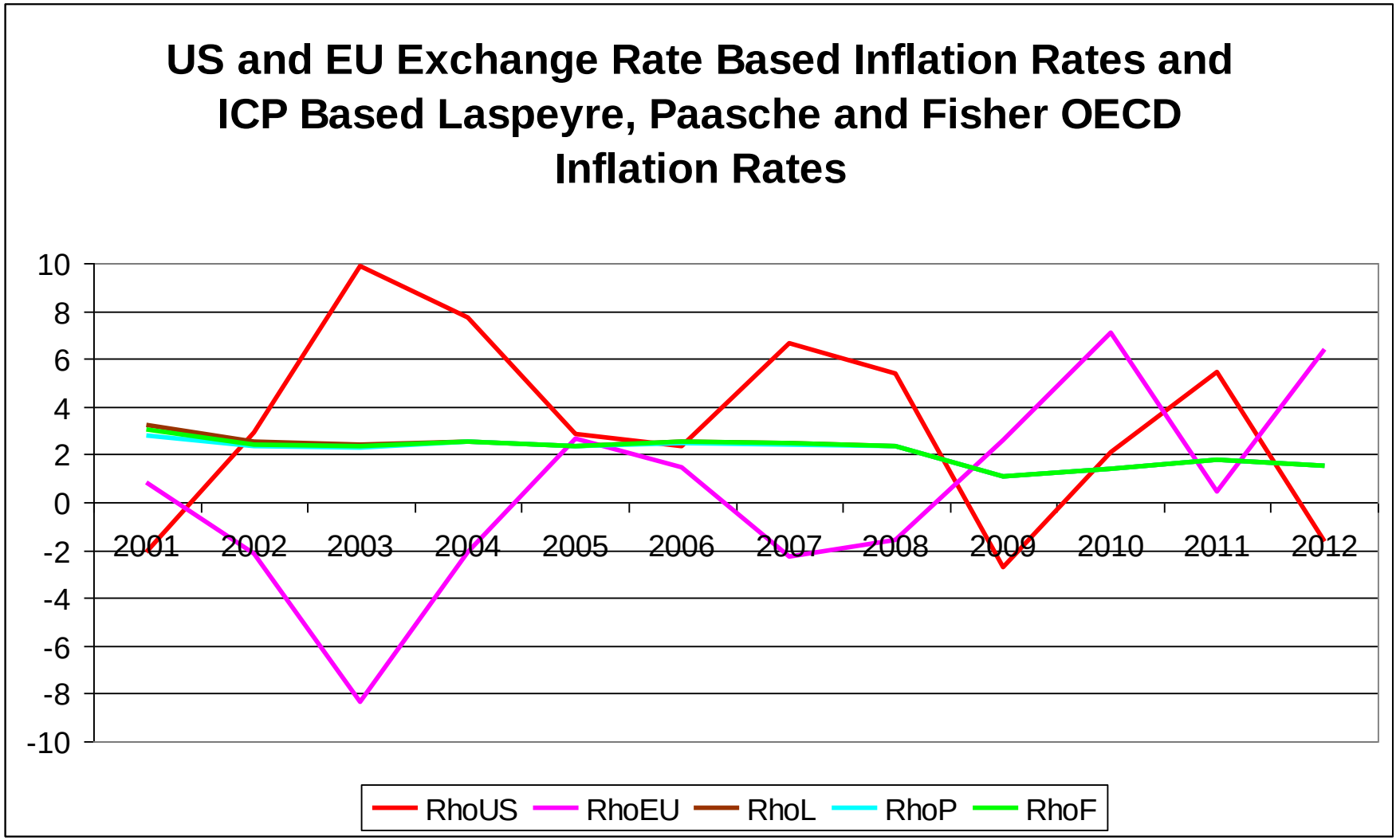
(19) $\rho_L^t \equiv 100[\Pi_L^t - 1]$; $\rho_P^t \equiv 100[\Pi_P^t - 1]$; $\rho_F^t \equiv 100[\Pi_F^t - 1]$.

- **The above ICP based inflation rates (in percentage points) are listed in the next slide along with the earlier US dollar and Euro based inflation estimates that were derived using US dollar and Euro estimates of country nominal GDP values and country volume estimates, $\rho^t = \rho_{US}^t$ and ρ_{EU}^t .**

Aggregate OECD Inflation Indexes: Approach 2

Year	ρ_{US}^{t}	ρ_{EU}^{t}	ρ_{L}^{t}	ρ_{P}^{t}	ρ_{F}^{t}
2001	-2.06	0.84	3.22	2.80	3.01
2002	2.92	-2.14	2.51	2.31	2.41
2003	9.90	-8.35	2.39	2.29	2.34
2004	7.71	-2.10	2.53	2.50	2.51
2005	2.84	2.68	2.34	2.34	2.34
2006	2.35	1.46	2.51	2.50	2.51
2007	6.63	-2.27	2.45	2.44	2.44
2008	5.36	-1.56	2.36	2.34	2.35
2009	-2.69	2.60	1.09	1.08	1.09
2010	2.07	7.06	1.41	1.39	1.40
2011	5.44	0.46	1.76	1.76	1.76
2012	-1.63	6.42	1.51	1.50	1.50

Aggregate OECD Inflation Indexes: Approach 2



New Problem: The Construction of Comparable Across Countries and Time GDP (Harmonized) Volumes

- Use the Fisher chain links defined by (16) to define Q_H^t as follows:

(18) $Q_H^{2000} \equiv 1$; $Q_H^t \equiv Q_H^{t-1} \Gamma_F^t$; $t = 2001,...,2012.$

- Now use the country shares of OECD real GDP s_n^t listed in Table 2 and the aggregate index Q_H^t to define the following preliminary *harmonized country volumes* for country n in year t, q_{Hn}^t, as follows:

(19) $q_{Hn}^t \equiv Q_H^t s_n^t$; $n = 1,...,34$; $t = 2000,...,2012.$

- For each year t, $\sum_{n=1}^{34} q_{Hn}^t = \sum_{n=1}^{34} Q_H^t s_n^t = Q_H^t (\sum_{n=1}^{34} s_n^t) = Q_H^t$ and so the harmonized volumes satisfy the two principles listed on slide 19 above. In principle, the country volumes defined by (19) are independent of country prices and exchange rates. [Consider the one good case.]

The Construction of Comparable Across Countries and Time GDP Volumes (cont)

- **Recall that the value of country n's nominal GDP converted into US dollars at market exchange rates for year t was defined as v_n^t. Thus the corresponding *harmonized US dollar price* of a unit of (comparable across countries) real GDP for country n in year t is defined as follows:**

(20) $p_{Hn}^t \equiv v_n^t / q_{Hn}^t$; $n = 1,...,34$; $t = 2000,...,2012$.

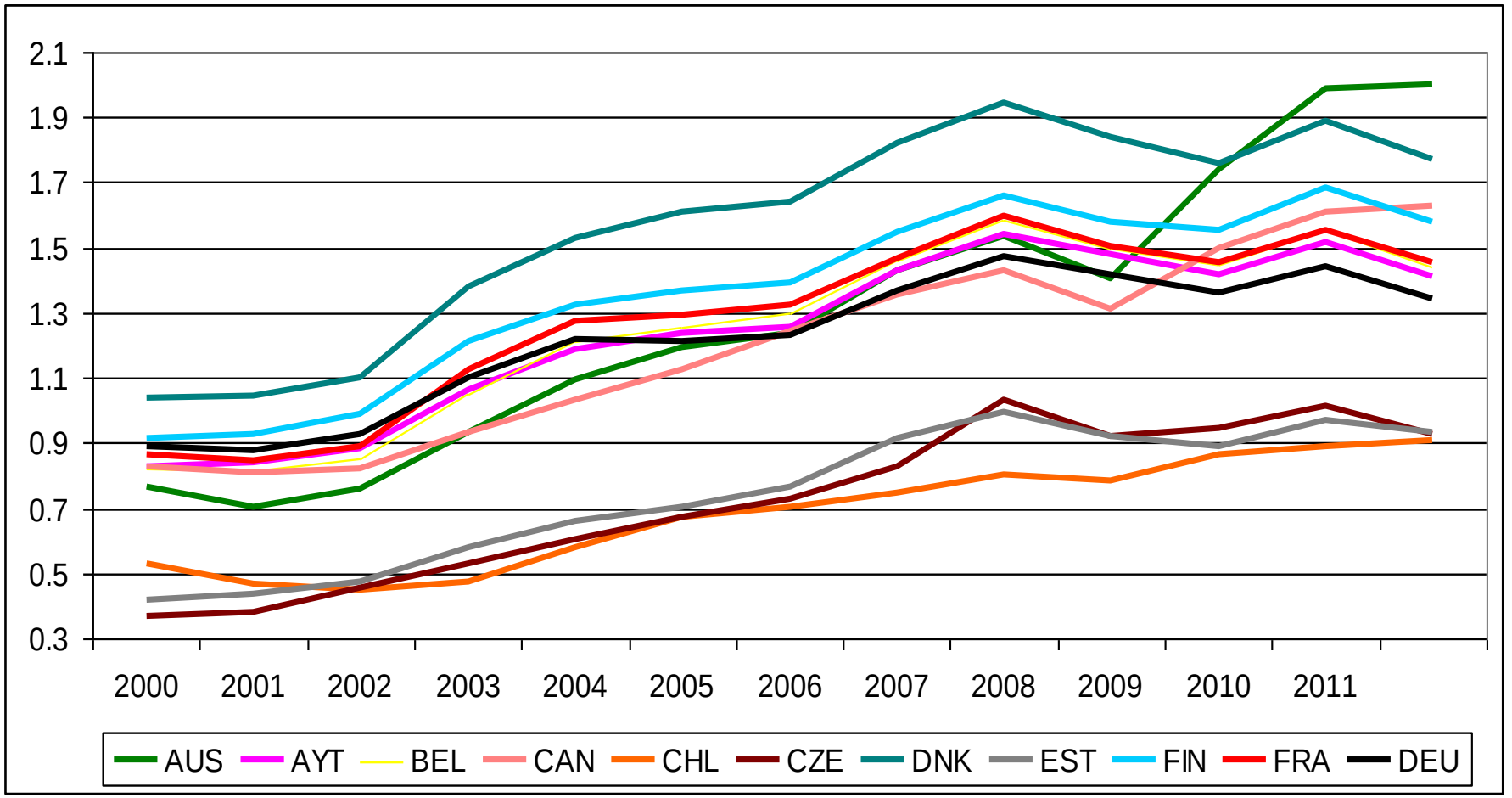
- **We impose a normalization on the prices defined by (20) that makes the price level for the US in 2000 equal to unity; i.e., we divide all prices defined by (20) by a constant that sets the resulting p_{H34}^{2000} equal to 1 and the quantities or volumes defined by (19) are all multiplied by this constant. The resulting normalized q_{Hn}^t and p_{Hn}^t are listed in Tables 4 and 5. [Compare with the q_n^t, p_n^t listed in Tables A3 and A4].**

Discussion of the Country Comparable Prices and Volumes Listed in Tables 4 and 5

- **Note that $q_{H34}^{2000} = q_{34}^{2000}$ and $p_{H34}^{2000} = p_{34}^{2000} = 1$ so that country GDP volumes are measured as multiples of a bundle of US GDP produced in the year 2000.**
- **Thus the price levels in Table 5 measure the US dollar value of constant bundle of GDP that is (in theory) comparable across countries.**
- **The price levels in Table 5 are comparable across space and time, whereas the price levels p_n^t listed in Table A3 of the Appendix are only comparable across time for each country.**
- **These comparable US dollar price levels are shown in the next 3 slides.**
- **They are (imperfect) indicators of a country's competiveness. [Imperfect because not all commodities are internationally traded plus there are errors in the PPPs].**

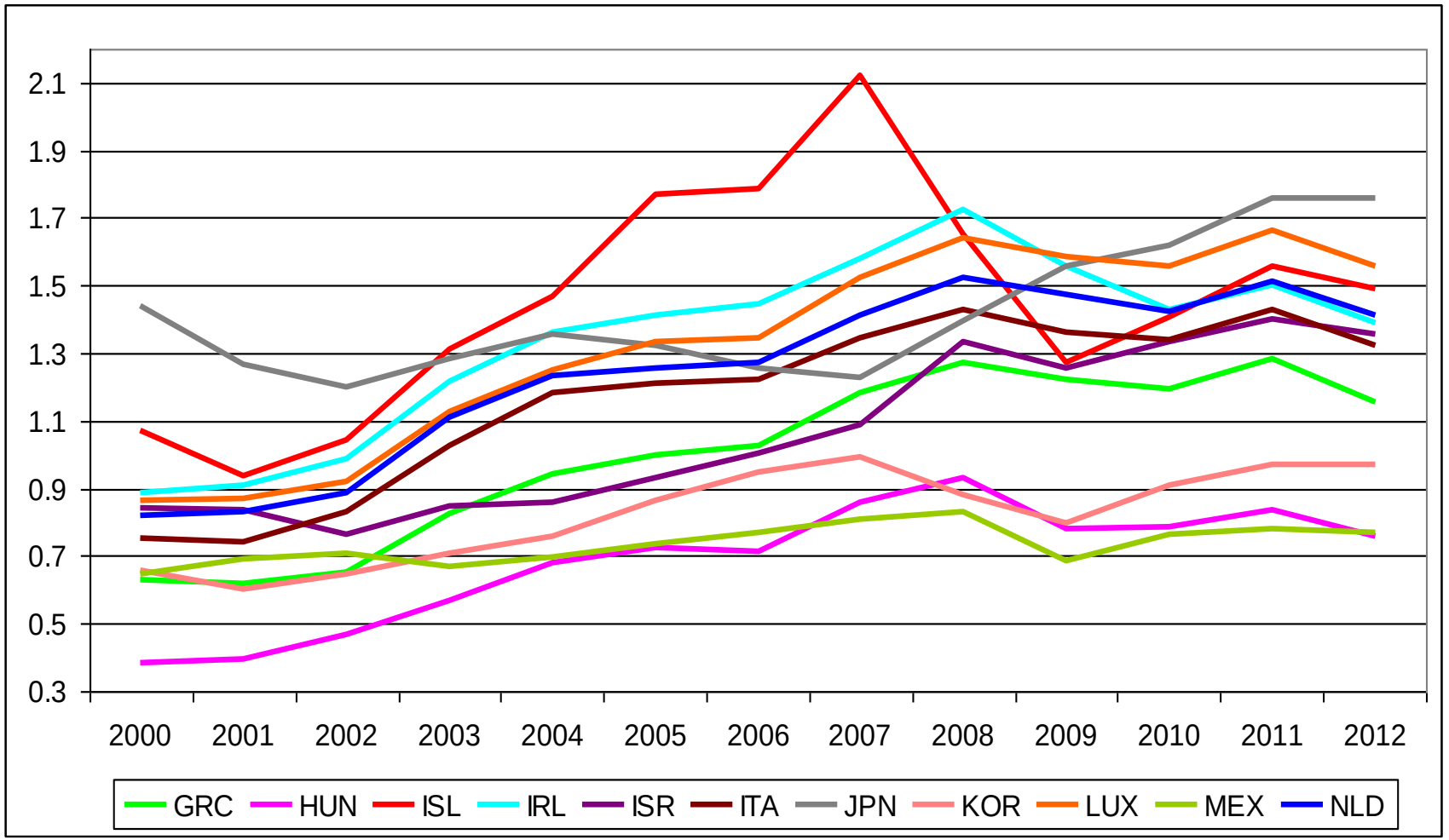
Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

Countries 1-11; US Price Level in 2000 = 1 [Convergence?]



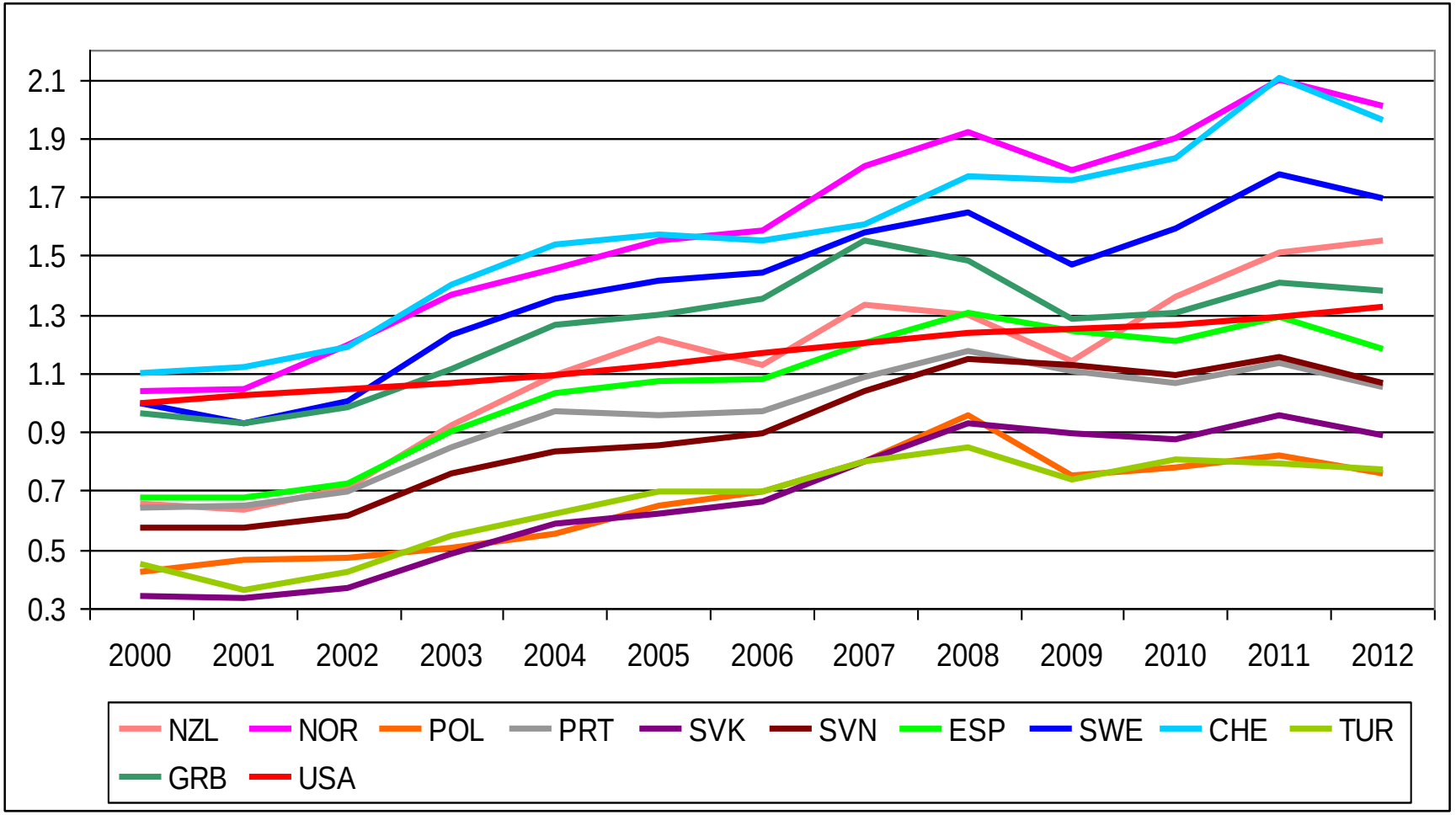
Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

Countries 12-22; US Price Level in 2000 = 1 [Note ISL!]



Harmonized OECD Country GDP Price Levels in Comparable US Dollar Units of Measurement p_{Hn}^t

Countries 23-34; US Price Level in 2000 = 1 [Red = USA]



Discussion of Harmonized GDP Price Levels in US $

- From Table 5, it can be seen that the countries with the lowest price levels (in US dollars) in 2012 are countries 13, 21, 25 and 32 (Hungary, Mexico, Poland and Turkey) with price levels in the 0.76 to 0.77 range.
- Countries with relatively high price levels in 2012 are countries 1 (Australia, $p_{H1}^{2012} = 2.00$), 4 (Canada, $p_{H4}^{2012} = 1.62$), 7 (Denmark, 1.77), 9 (Finland, 1.58), 18 (Japan, 1.76), 20 (Luxembourg, 1.56), 23 (New Zealand, 1.55), 24 (Norway, 2.01), 30 (Sweden, 1.69) and 31 (Switzerland, 1.96).
- Again, these price level estimates are (imperfect) indicators of the competiveness of the country on international markets, with lower price levels indicating greater competiveness.
- The Tables 4 and 5 country volumes and price levels are our preferred comparable across time and space estimates.

Section 4. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Base Period Shares of OECD Real GDP

- **In this section, we generated comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2000, the s_n^{2000} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix.**
- **This is a typical strategy in forming estimates of real GDP that rely on PPPs that are only produced infrequently: use the PPP based country shares of "world" GDP for a base period and project these shares forward using national GDP volume growth rates .**
- **How different the resulting estimates are from our preferred harmonized volume estimates, q_{Hn}^t, listed in Table 4 above?**

Section 4 Results

- **If we take each column in Table 5, subtract the corresponding entries in the same column of Table 7 and then take the absolute value of the differences, we find that the *average* absolute difference grows from 0 in 2000 to 9.4 percentage points in 2012.**
- **The maximum absolute difference grows from 0 in 2000 to 54.0 percentage points in 2012. These are massive differences in price levels, which translate into massive differences in GDP levels.**
- **The sequence of average absolute differences in percentage points over the 13 years is as follows: 0, 0.8, 1.6, 2.0, 2.2, 3.8, 5.2, 6.4, 7.6, 6.8, 8.0, 9.3, 9.4.**
- **The sequence of maximum absolute differences in percentage points over the 13 years is as follows: 0, 2.9, 5.2, 6.0, 5.4, 14.0, 25.1, 28.2, 46.9, 24.8, 34.3, 48.5, 54.0.**

Section 5. OECD Growth and Inflation Using Country Annual GDP Volume Growth Rates and Final Period Shares of OECD Real GDP

- **In this section, we generated comparable country GDP volume estimates for OECD countries covering the period 2000-2012 by using the real GDP country volume shares for 2012, the s_n^{20012} listed in Table 2 above, along with the national growth rates of country real GDP relative to 2000, the Q_n^t/Q_n^{2000} listed in Table A2 of the Appendix.**
- **This method for forming comparable country GDP volumes is used by the World Bank when the International Comparisons Project produces a new set of PPPs.**
- **The methodology is straightforward and follows the approach used in the previous section except that the 2012 country volume shares are used in place of the 2000 shares.**

Section 5 Results

- **Take each column in Table 5, subtract the corresponding entries in the same column of Table 8 and then take the absolute value of the differences.**
- **The *average* absolute difference for 2000 over the 34 countries is 6.0 percentage points, which increases to 7.9 percentage points for 2005 and then gradually decreases to 4.2 percentage points in 2012. Over all observations, the *maximum* absolute deviation is 35.6 percentage points.**
- **The sequence of average absolute differences over the 34 countries in percentage points over the 13 years is as follows: 6.0, 6.1, 6.2, 7.0, 7.8, 7.9, 6.2, 6.8, 5.9, 5.4, 5.1, 5.2, 4.2.**
- **The sequence of maximum absolute differences in percentage points over the 13 years is as follows: 24.6, 25.9, 32.3, 35.6, 33.1, 27.1, 20.5, 22.0, 16.0, 24.7, 18.7, 12.6, 6.3.**
- **Conclusion: extrapolation does not work well over 13 years**

Section 6: OECD Growth and Inflation Using Adjusted Country Annual GDP Volume Growth Rates and OECD Shares of Real GDP for Two Benchmark Years

- **The OECD provides annual PPPs so that estimates of relative GDP volumes can be constructed for all member countries for each year.**
- **However, the World Bank's ICP PPPs are only available at infrequent intervals. The question arises: how exactly should two widely separated benchmark real GDP shares be used along with annual growth rate information to interpolate between the benchmark years?**
- **We will model this situation using our OECD data base but will use only the benchmark GDP shares for the years 2000 and 2012 along with information on national GDP growth rates in order to interpolate between the benchmark years.**

The Section 6 Methodology

- **We will propose an interpolation method that leads to country shares of real GDP that are exactly consistent with the shares s_n^{2000} for the year 2000 and the shares s_n^{2012} for the year 2012.**
- **Step 1: Construct country measures of real GDP that jump from the year 2000 to the year 2012.**
- **The *long term growth factor for country n* can be defined as Q_n^{2012}/Q_n^{2000} where Q_n^t is country n's GDP volume in year t.**
- **Now use these long term growth factors along with the year 2000 country shares of OECD real GDP, s_n^{2000}, in order to define the *OECD Laspeyres type long term growth factor*, Γ_L, as the following weighted average of the national long term growth factors:**

(27) $\Gamma_L \equiv \sum_{n=1}^{34} s_n^{2000}(Q_n^{2012}/Q_n^{2000})$; **t = 2001,...,2012.**

The Section 6 Methodology (cont)

- **The counterpart to the Laspeyres type formula defined by (27) is the following *Paasche type formula* that uses the shares of 2012 and reciprocal long term growth rates:**

(28) $\Gamma_P \equiv [\sum_{n=1}^{34} s_n^{2012}(Q_n^{2012}/Q_n^{2000})^{-1}]^{-1}$; $t = 2001,...,2012.$

- **Since both indexes have the same logical foundation, it seems best to take a symmetric average of the two indexes, which leads to the following Fisher type formula for *OECD long term volume growth* going from the year 2000 to the year 2012:**

(29) $\Gamma_F \equiv [\Gamma_L \Gamma_P]^{1/2}$; $t = 2001,...,2012.$

- **(29) defines a direct comparison of the data of 2000 with the data of 2012 whereas in section 3, we used chained Fisher type indexes to go from 2000 to 2012.**
- **The chained Fisher index for 2012 relative to 2000 is equal to 1.2203, which is very close to its direct counterpart, 1.2208.**

The Section 6 Methodology: Step 2

- **Preliminary estimates of country GDP volumes in comparable units for the years 2000 and 2012, q_{In}^{2000} and q_{In}^{2012} (the index I indicates that these are interpolated estimates), are defined as follows:**

(30) $q_{In}^{2000} \equiv s_n^{2000}$; $q_{In}^{2012} \equiv \Gamma_F \, s_n^{2012}$; **n = 1,...,34.**

- **The volumes defined by (30) will be imposed as constraints on our interpolation scheme. Define the *implied long term growth factor* over the years 2000-2012 for country n, g_n, that is implied by the estimates of country levels given by equations (30):**

(31) $g_n \equiv q_{In}^{2012}/q_{In}^{2000}$; **n = 1,...,34.**

The Section 6 Methodology: Step 2 (cont)

- **These growth factors are not necessarily equal to the national growth factors G_n that are implied by the national growth rates defined as:**

(32) $G_n \equiv Q_n^{2012}/Q_n^{2000}$; n = 1,...,34.

- **Thus for each country n, there is an "error" factor or discrepancy, $E_n \equiv g_n/G_n$ between the implied growth rates g_n defined by (31) and the national growth rates between 2000 and 2012, G_n defined by (32).**
- **We will distribute these errors in a proportional manner and use the resulting adjusted national growth rates to interpolate between the two benchmark observations. Thus define the *country n proportional discrepancy factor*, α_n, as follows:**

- **(33) $\alpha_n \equiv [g_n/G_n]^{1/12}$; n = 1,...,34.**

The Section 6 Methodology: Step 2 (concluded)

- **The comparable interpolated country volumes q_{In}^{t} for nonbenchmark years t can now be defined as follows:**

(34) $q_{In}^{t} \equiv q_{In}^{t-1}(Q_n^t/Q_n^{t-1})\alpha_n$; **n = 1,...,34 ; t = 2001,....,2011.**

- **Once the q_{In}^{t} have been defined, the corresponding US dollar price levels p_{In}^{t} are defined in the usual way:**

(35) $p_{In}^{t} \equiv v_n^t/q_{In}^{t}$; **n = 1,...,34 ; t = 2001,....,2011.**

- **In order to make the volumes and prices defined by (34) and (35) comparable to the harmonized country prices and volumes expressed in US dollars that are listed Tables 4 and 5 in section 3, we impose a normalization on the prices defined by (35) that makes the price level for the US in 2000 equal to unity.**

Section 6 Results

- **The discrepancies between the new interpolated price levels and our preferred harmonized price levels are much reduced as compared to the discrepancies when only one benchmark set of PPPs is used.**
- **The overall sample *average* absolute discrepancy is now only 1.9 percentage points.**
- **Over all observations, the *maximum* absolute deviation is 12.5 percentage points.**
- **The sequence of *average* absolute differences over the 34 countries in percentage points over the 13 years is as follows: 0, 1.0, 1.8, 2.0, 2.3, 3.2, 3.0, 3.2, 2.8, 2.6, 1.7, 1.2, 0.06.**
- **The sequence of *maximum* absolute differences in percentage points over the 13 years is as follows: 0, 3.9, 9.7, 11.9, 9.4, 10.7, 8.3, 12.5, 12.3, 9.9, 6.1, 3.2, 0.08.**
- **Conclusion: the interpolation method works fairly well!**

Overall Conclusions

- **The results listed in sections 3-5 show that it is very hazardous for analysts interested in comparative levels of GDP across countries to use national growth rates and a *single* cross country comparison of real GDP levels.**
- **If PPP computations for a group of countries are only done on an infrequent, then the interpolation method explained in section 6 may prove to be a useful method for obtaining comparable GDP levels that are consistent with the GDP relative levels for the two benchmark years.**
- **The results in section 6 also indicate that different interpolation methods can generate very different results.**
- **If a group of countries want to construct comparable across countries and time estimates of real GDP (or any other macroeconomic aggregate), then in order to reduce measurement errors, it will be necessary to undertake PPP exercises on a fairly frequent basis; say every 3-6 years.**