The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

SURDS

The main algebraic property of surds is that if $x, y \geq 0$ then

$$\sqrt{xy} = \sqrt{x}\sqrt{y} \ .$$

For example, we can simplify

$$\sqrt{108} \ = \ \sqrt{36 \times 3} \ = \ \sqrt{36}\sqrt{3} \ = \ 6\sqrt{3} \ .$$

If you didn't notice the best way to factorise 108, you could always have done the calculation one step at a time:

$$\begin{aligned} \sqrt{108} \ = \ \sqrt{9 \times 12} \ &= \ \sqrt{9}\sqrt{12} \ = \ 3\sqrt{12} \\ &= \ 3\sqrt{4 \times 3} \ = \ 3\sqrt{4}\sqrt{3} \ = \ 3 \times 2\sqrt{3} \ = \ 6\sqrt{3} \ . \end{aligned}$$

Another example, this time involving division:

$$\frac{15}{\sqrt{5}} \ = \ \frac{15}{\sqrt{5}}\,\frac{\sqrt{5}}{\sqrt{5}} \ = \ \frac{15\sqrt{5}}{5} \ = \ 3\sqrt{5} \ .$$

Note that similar formulae **do not** hold for addition: in general,

$$\sqrt{x+y} \ \neq \ \sqrt{x} + \sqrt{y} \ .$$

In particular, $\sqrt{x^2+y^2}$ is **not** equal to $x+y$: equating these two expressions is a very common mistake! Occasionally we can do something like

$$\sqrt{18} + \sqrt{50} \ = \ \sqrt{9 \times 2} + \sqrt{25 \times 2} \ = \ 3\sqrt{2} + 5\sqrt{2} \ = \ 8\sqrt{2} \ ,$$

but in most cases there is **no useful simplification** of $\sqrt{x} + \sqrt{y}$. We can, however, use basic algebra to multiply out sums and differences involving the same surd, for example,

$$\big(3 - \sqrt{5}\big)\big(1 + 2\sqrt{5}\big) \ = \ 3 + 6\sqrt{5} - \sqrt{5} - 2 \times 5 \ = \ -7 + 5\sqrt{5} \ .$$

We can simplify certain expressions involving surds by **rationalising the denominator**. First notice that using the "difference of two squares" formula we have

$$\big(a + b\sqrt{c}\big)\big(a - b\sqrt{c}\big) \ = \ a^2 - b^2 c \ .$$

We can then do calculations like

$$\begin{aligned} \frac{4 - \sqrt{7}}{1 + 2\sqrt{7}} \ &= \ \frac{4 - \sqrt{7}}{1 + 2\sqrt{7}}\,\frac{1 - 2\sqrt{7}}{1 - 2\sqrt{7}} \\ &= \ \frac{18 - 9\sqrt{7}}{-27} \\ &= \ \frac{-2 + \sqrt{7}}{3} \end{aligned}$$

and

$$\frac{\sqrt{3} - 1}{\sqrt{3} + 1} \ = \ \frac{\sqrt{3} - 1}{\sqrt{3} + 1}\,\frac{\sqrt{3} - 1}{\sqrt{3} - 1} \ = \ \frac{4 - 2\sqrt{3}}{2} \ = \ 2 - \sqrt{3} \ .$$

You do not *always* need to remove surds from the denominator. For example, we can write

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \ ,$$

but the right hand side is not really any simpler than the left.

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Simplify the following:

$$\sqrt{28}\ ,\quad \sqrt{45}\ ,\quad \sqrt{48}\ ,\quad \sqrt{1100}\ ,\quad \sqrt{180}\ ,\quad \sqrt{567}\ .$$

2. Combine into a multiple of a single surd, as in the example at the bottom of page 1:

$$\sqrt{27}+\sqrt{300}\ ,\quad \sqrt{98}-\sqrt{8}\ ,\quad \sqrt{63}+4\sqrt{175}\ ,$$
$$\sqrt{245}+\sqrt{500}\ ,\quad \sqrt{10}+\sqrt{40}+\sqrt{90}\ .$$

3. Simplify:

$$\frac{1+2\sqrt{2}}{3-\sqrt{2}}\ ,\quad \frac{1+2\sqrt{5}}{3-\sqrt{5}}\ ,\quad \frac{7+2\sqrt{6}}{5+2\sqrt{6}}\ ,\quad \frac{5+\sqrt{11}}{7-2\sqrt{11}}\ .$$

4. Simplify the following (the first step is given as a hint):

$$\frac{2\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\,\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \ ?$$

Then use similar ideas to simplify

$$\frac{2\sqrt{5}-5\sqrt{2}}{\sqrt{5}+\sqrt{2}} \quad \text{and} \quad \frac{3\sqrt{7}+4\sqrt{6}}{\sqrt{7}-\sqrt{6}}\ .$$

ANSWERS.

1. $2\sqrt{7}$, $3\sqrt{5}$, $4\sqrt{3}$, $10\sqrt{11}$, $6\sqrt{5}$, $9\sqrt{7}$.
2. $13\sqrt{3}$, $5\sqrt{2}$, $23\sqrt{7}$, $17\sqrt{5}$, $6\sqrt{10}$.
3. $1+\sqrt{2}$, $\dfrac{13+7\sqrt{5}}{4}$, $11-4\sqrt{6}$, $\dfrac{57+17\sqrt{11}}{5}$.
4. $\dfrac{13+3\sqrt{15}}{2}$, $\dfrac{20-7\sqrt{10}}{3}$, $45+7\sqrt{42}$.