The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

QUADRATICS, PART 2

In the worksheet *Quadratics, part 1* you have revised solving quadratics of the type $ax^2+bx+c=0$ by inspection. This method is very quick and easy for some cases, but does not work well for others: we were unable to solve

$$x^2+8x-15=0\ . \tag{1}$$

As a general rule we recommend that you try solving by inspection first, and if necessary, move on to the following methods.

Completing the square involves taking an expression a^2x^2+bx and adding a constant $(b/2a)^2$ so as to make a perfect square:

$$a^2x^2+bx+\left(\frac{b}{2a}\right)^2=\left(ax+\frac{b}{2a}\right)^2\ .$$

Using this to solve the quadratic (1), we begin by taking the constant term to the right hand side:

$$\begin{aligned}
x^2+8x-15=0 \quad &\Rightarrow \quad x^2+8x=15\\
&\Rightarrow \quad x^2+8x+16=31\\
&\qquad\qquad [\text{adding } (\tfrac{8}{2\times1})^2 \text{ to each side}]\\
&\Rightarrow \quad (x+4)^2=31\\
&\Rightarrow \quad x+4=\pm\sqrt{31}\\
&\qquad\qquad [\text{don't forget the } \pm \text{ sign}]\\
&\Rightarrow \quad x=-4\pm\sqrt{31}\ .
\end{aligned}$$

To solve

$$2x^2-6x-15=0 \tag{2}$$

we could proceed as above with $a=\sqrt{2}$. However, in order to avoid messy arithmetic it is better to multiply both sides of the given equation by 2 and then take $a=2$. We have

$$\begin{aligned}
2x^2-6x-15=0 \quad &\Rightarrow \quad 4x^2-12x-30=0\\
&\Rightarrow \quad 4x^2-12x=30\\
&\Rightarrow \quad 4x^2-12x+9=39\\
&\qquad\qquad [\text{adding } (\tfrac{12}{2\times2})^2 \text{ to each side}]\\
&\Rightarrow \quad (2x-3)^2=39\\
&\Rightarrow \quad 2x-3=\pm\sqrt{39}\\
&\Rightarrow \quad x=\frac{3\pm\sqrt{39}}{2}\ .
\end{aligned}$$

Alternatively, we could begin this example by dividing both sides by 2: this will lead to more fractions in the calculation, but will give the same answer as long as you are careful.

The quadratic formula for the solutions of $ax^2+bx+c=0$ is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\ .$$

If b^2-4ac is negative this solution will involve *complex numbers*, which you will have met in school or in first year lectures.

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Solve the following quadratics. First try solving by inspection; if this seems too hard, either complete the square or use the formula. Where possible, simplify any surds in your answer.
   (a) $x^2 + 10x + 2 = 0$;
   (b) $x^2 - 8x + 5 = 0$;
   (c) $x^2 - 11x + 28 = 0$;
   (d) $x^2 + x - 22 = 0$;
   (e) $2x^2 + 10x = 3$;
   (f) $3x^2 + 8x - 11 = 0$;
   (g) $x^2 - 11x - 30 = 0$;
   (h) $x^2 + 6x - 11 = 0$;
   (i) $x^2 + 7x - 18 = 0$;
   (j) $5x^2 + 6x - 7 = 0$;
   (k) $x^2 - 2x = 17$.
2. Solve the general quadratic $ax^2 + bx + c = 0$, where $a \neq 0$, by the method of completing the square. After simplification, you should find that your answer is just the quadratic formula.

ANSWERS.

1. In the answers to this question we have suggested what we think is the easiest method; but of course the important thing is to get the correct solution, and it doesn't matter all that much which method you use.
   (a) $x = -5 \pm \sqrt{23}$ by completing the square;
   (b) $x = 4 \pm \sqrt{11}$ by completing the square;
   (c) $x = 4$, $x = 7$ by inspection;
   (d) $x = \dfrac{-1 \pm \sqrt{89}}{2}$ by formula;
   (e) $x = \dfrac{-5 \pm \sqrt{31}}{2}$ by completing the square or formula;
   (f) $x = 1$, $x = -\dfrac{11}{3}$ by inspection (maybe!) or formula;
   (g) $x = \dfrac{11 \pm \sqrt{241}}{2}$ by formula;
   (h) $x = -3 \pm 2\sqrt{5}$ by completing the square;
   (i) $x = 2$, $x = -9$ by inspection;
   (j) $x = \dfrac{-3 \pm 2\sqrt{11}}{5}$ by completing the square or formula;
   (k) $x = 1 \pm 3\sqrt{2}$ by completing the square.