The University of New South Wales
School of Mathematics and Statistics

Mathematics Drop–in Centre

POWER LAWS

It is important to be able to simplify or expand algebraic expressions involving powers. We begin with the following rules:

$$a^x a^y = a^{x+y} \ ; \quad \frac{a^x}{a^y} = a^{x-y} \ ; \quad (a^x)^y = a^{xy} \ . \qquad (*)$$

It is worth spending a little time in *understanding* these rules: if you try to memorise them without understanding you will find this topic very difficult later on. To understand the first, consider for example

$$a^8 a^3 = (aaaaaaaa)(aaa) = aaaaaaaaaaa = a^{11} = a^{8+3} \ .$$

The key step is the second: it should be easy to see that we can combine a group of eight as and a group of three into a group of eleven. Similarly, the last formula in $(*)$ is illustrated by

$$\begin{aligned}(a^2)^5 &= (a^2)(a^2)(a^2)(a^2)(a^2)\\ &= (aa)(aa)(aa)(aa)(aa) = aaaaaaaaaa = a^{10} = a^{2\times 5} \ ,\end{aligned}$$

where we have combined five groups of two as into a single group of ten.

Three more important rules are

$$a^0 = 1 \ ; \quad a^1 = a \ ; \quad a^{-y} = \frac{1}{a^y} \ .$$

To understand why the last of these is true, go back to the second formula in $(*)$ and substitute $x = 0$.

(Some of) the above formulae involve the same base to two different powers. There are also rules where we have an expression involving two bases to the same power:

$$a^x b^x = (ab)^x \ ; \quad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x \ .$$

Once again you should try to understand why these are true. For the first, we know that we can multiply numbers in any order we like without affecting the result; so, for example,

$$(ab)^5 = (ab)(ab)(ab)(ab)(ab) = (aaaaa)(bbbbb) = a^5 b^5 \ .$$

Note that expansions like $(a+b)^x$ are not so easy (**not** $a^x + b^x$!!) and are usually treated by means of the Binomial Theorem.

Although we have so far been thinking of the exponents x, y as integers, the same rules apply for any real numbers. A fractional power means a root: for example

$$a^{1/2} = \sqrt{a} \ ; \quad a^{1/3} = \sqrt[3]{a} \ ; \quad a^{4/5} = (a^4)^{1/5} = \sqrt[5]{a^4} \ .$$

It is harder to say precisely what is meant by an expression like a^π (remember that π is not a fraction) – follow calculus lectures for this. Finally, remember that a power expression may be undefined for certain values of a. For example, $a^{1/2}$ is meaningless if a is negative, and a^{-2} is meaningless if $a = 0$.

Examples.

- $\dfrac{a^2(ab)^3}{b^4} = \dfrac{a^2a^3b^3}{b^4} = a^5b^{-1} = \dfrac{a^5}{b}$.
- $(2c^2d^5)^3 = 2^3(c^2)^3(d^5)^3 = 8c^6d^{15}$.
- $(x^4y^5z^6y^7z^8)^{1/4} = (x^4y^{12}z^{14})^{1/4} = xy^3z^{7/2} = xy^3\sqrt{z^7}$.
- $(x^{1\cdot 2}y^{3\cdot 4})^2x^{-5\cdot 6} = x^{2\cdot 4}y^{6\cdot 8}x^{-5\cdot 6} = x^{-3\cdot 2}y^{6\cdot 8}$.

EXERCISES.

Please try to complete the following exercises. Remember that you **cannot** expect to understand mathematics without doing lots of practice! Please do not look at the answers before trying the questions. If you get a question wrong you should go through your working carefully, find the mistake and fix it. If there is a mistake which you cannot find, or a question which you cannot even start, please consult your tutor or the Mathematics Drop–in Centre.

1. Following the examples in the first paragraph, write powers in terms of multiplication in order to "explain" the identity $a^9/a^4 = a^{9-4}$.
2. Write the following expressions in terms of products of powers, where each pronumeral occurs once only:
   (a) $\dfrac{x^9(xy^5)^{-2}}{(x^4y)^3}$;
   (b) $(a^{2/3}b^{4/5})^6$;
   (c) $\dfrac{(x^5y^3)^{1\cdot 3}}{(x^{2\cdot 4}y^{3\cdot 1})^2}$;
   (d) $\dfrac{(abc^2)^3}{(b^2c)^5} \Big/ \dfrac{(a^3b)^6}{(a^4bc^2)^2}$.
3. Write the following radical expressions in terms of powers, and then simplify them:
   (a) $\sqrt{a^3y^7}\,\sqrt[3]{a^8y^{-10}}$;
   (b) $\dfrac{\sqrt[4]{x^5y^6}}{(\sqrt{x}\,\sqrt[3]{y})^7}$.
4. Write the following power expressions in terms of radicals (that is, square roots, cube roots etc):
   (a) $p^{1/6}q^{2/7}$;
   (b) $\dfrac{(x^{1/3}y^{1/4})^2}{x^{1/6}y^{3/5}}$.

ANSWERS.

1. $\dfrac{a^9}{a^4} = \dfrac{aaaaaaaaa}{aaaa} = aaaaa = a^5 = a^{9-4}$.
2. (a) $x^{-5}y^{-13}$, or $\dfrac{1}{x^5y^{13}}$;
   (b) $a^4b^{24/5}$;
   (c) $x^{1\cdot 7}y^{-2\cdot 3}$, or $\dfrac{x^{1\cdot 7}}{y^{2\cdot 3}}$;
   (d) $a^{-7}b^{-11}c^5$, or $\dfrac{c^5}{a^7b^{11}}$.
3. (a) $a^{25/6}b^{1/6}$;
   (b) $x^{-9/4}y^{-5/6}$.
4. (a) $\sqrt[6]{p}\,\sqrt[7]{q^2}$;
   (b) $\dfrac{\sqrt{x}}{\sqrt[10]{y}}$.