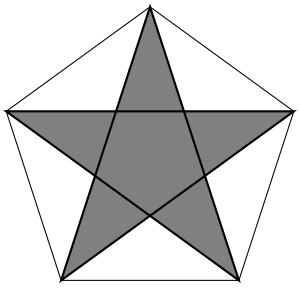
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 4, May 27, 2019[1]

1. (a) What is the remainder when 2^{2019} is divided by 7?
   (b) Find the last digit of 2^{2019}.

2. Gerald rolls 5 dice simultaneously. Each die has six faces labelled with the numbers $1, 2, 3, 4, 5$ and 6. What is the probability of getting 5 consecutive numbers as the outcome of the dice roll?

3. If $y = 2$ and
$$\sqrt{x + \sqrt{y + \sqrt{x + \sqrt{y + \dots}}}} = 7,$$
solve for x.

4. In a regular pentagon the diagonals are joined to form a star. What fraction of the pentagon does the star occupy?



5. Divide the numbers $24, 38, 39, 44, 45, 46, 48$ into two sets in such a way that the sum of the numbers in each set is prime. Show that this can only be done in one way.

6. Find a positive integer x, such that if x is increased by 10%, then we get another positive integer with the sum of digits decreased by 10%.

[1]Some problems from UNSW's publication *Parabola*, and the *Tournament of Towns in Toronto*

Senior Questions

1. Consider the points of intersection of the graphs $y = \cos x$ and $x = 100\cos(100y)$ for which both coordinates are positive. Let a be the sum of their x-coordinates and b be the sum of their y-coordinates. Determine the value of $\frac{a}{b}$.

2. Prove that $\log_a(x)\log_b(y) = \log_b(x)\log_a(y)$.

3. Find all solutions of the system of equations

$$\begin{aligned} x &= \frac{1}{2}\left(y + \frac{1}{y}\right) \\ y &= \frac{1}{2}\left(z + \frac{1}{z}\right) \\ z &= \frac{1}{2}\left(t + \frac{1}{t}\right) \\ t &= \frac{1}{2}\left(x + \frac{1}{x}\right). \end{aligned}$$

   Generalise to 2019 variables.