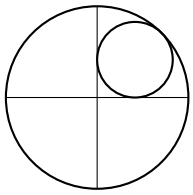
MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 17, September 24, 2019

1. *AMC 2010 Senior Division, Q16.*
   The 5-digit number $a986b$, where a is the first digit and b is the units digit, is divisible by 72. What is the value of $a + b$?

2. *AMC 2010 Senior Division, Q19.*
   A circle is inscribed in a quadrant of a larger circle. What is the ratio of the area of the inner circle to that of the quadrant?



3. *AMC 2010 Senior Division, Q24.*
   What is the smallest n such that no matter how n points are placed inside or on the surface of a cube of side length 16 units, there are at least two of these points which are closer than 14 units to each other?

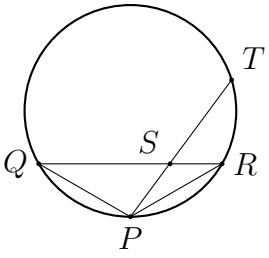
4. A sequence of real numbers, $\{x_1, x_2, x_3, \ldots\}$, is defined by

$$x_1 = \sqrt{2}, \qquad x_2 = \sqrt{3},$$
$$x_n = x_{n-1} - x_{n-2} \quad \text{for } n \geq 3.$$

   What is the value of x_{2019}?

5. *AMC 2010 Senior Division, Q28.*
   In the triangle PQR, $PQ = PR = 40$ cm and S is a point on QR such that $PS = 25$ cm. The extension of PS meets the circle through PQR at T.

   What is the length in centimetres of PT?



Senior Questions

1. The numbers x and y are positive integers that satisfy

$$3x^2 - 8y^2 + 3x^2y^2 = 2008.$$

   Find all possible values of x and y.

2. *AMC 2010 Senior Division, Q26.*
   A polynomial f is given. All we know about f is that all its coefficients are non-negative integers, $f(1) = 6$ and $f(7) = 3438$.

   What is the value of $f(3)$?

3. *AMC 2008 Senior Division, Q29.*
   A point O is inside an equilateral triangle PQR and the perpendiculars OL, OM and ON are drawn to the sides PQ, QR and RP respectively.

   The ratios of the lengths of the perpendiculars $OL : OM : ON$ is $1 : 2 : 3$. If

$$\frac{\text{area of } LONP}{\text{area of } \triangle PQR} = \frac{a}{b},$$

   where a and b are integers with no common factors, what is the value of $a + b$?