MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 14, September 3, 2019

1. A drawer contains an unsorted collection of black, white, pink, and blue coloured socks. If socks are picked at random, one at a time, what is the minimum number which must be taken to be certain of obtaining five matching pairs?

2. Using each of the digits $1, 2, 3, 4$ and 5 exactly once to form 5-digit numbers, how many are divisible by 12?

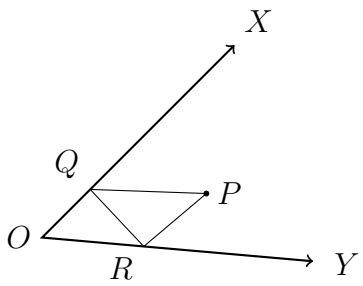
3. In a wrestling tournament, there are 100 participants, all of different strengths. The stronger wrestler always wins over the weaker opponent. Each wrestler fights twice and those who win both of their fights are given awards. What is the least possible number of awardees?

4. (a) Find the highest power of 2 that divides 33!.
   (b) How many zeros are there at the end of 2019!?

5. Let p be a prime number and x, y non-negative integers. Find all possible solutions to $p^x = y^4 + 4$.

6. Each of 11 weights is weighing an integer number of grams. No two weights are equal. It is known that if all these weights or any group of them are placed on a balance then the side with a larger number of weights is always heavier. Prove that at least one weight is heavier than 35 grams.

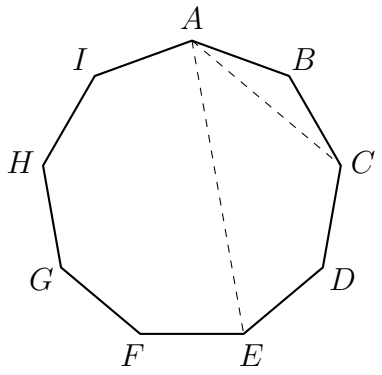
7. Suppose that $\angle XOY$ is an acute angle, and P is an interior point, as shown in the diagram.



   Explain how to find points Q and R, lying on OX and OY, respectively, so that the perimeter of $\triangle PQR$ is minimized.

Senior Questions

1. Prove that the difference between the longest and the shortest diagonal of a regular nonagon is equal to the length of the side.



2. There are five distinct real positive numbers. It is known that the total sum of their squares and the total sum of their pairwise products are equal.

   (a) Prove that we can choose three numbers such that it would not be possible to make a triangle with side lengths equal to these numbers.

   (b) Prove that the number of ways to form the triples satisfying (a) is at least six (triples which consist of the same numbers in different order are considered the same).