MATHEMATICS ENRICHMENT CLUB.
Problem Sheet 1, May 6, 2019

1. Express $0.284284284\ldots$ as a fraction in simplest terms.

2. Let x, y and z be integers. Show that if $x - y + 2z$ is divisible by 11, then so is $-12x + y - 13z$.

3. Anna and Boris move simultaneously towards each other from points A and B, respectively. Their speeds are constant, but not necessarily equal. Had Anna started 30 minutes earlier, they would have met 2 kilometres nearer to B. Had Boris started 30 minutes earlier instead, they would have met d kilometers nearer to A. Find d.

4. A four digit number and its square ends in the same four digits. Find the number.

5. A 3×3 magic square is a grid filled with the numbers 1 to 9 so that the sum of rows, column and diagonal are all equal. E.g

6	1	8
7	5	3
2	9	4

   Counting different orientations of the grid as the same magic square, prove that the above example is the only solution.

Senior Questions

1. (a) Prove the identity

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}.$$

   (b) Using the this result, show that the infinite series satisfies

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \tan^{-1}(x).$$

2. (a) For an integer n, show that $n(n+1)(n+2)(n+3)+1$ is a perfect square.

   (b) Thus evaluate $\sqrt{(31)(30)(29)(28)+1}$.