



**UNSW**  
SYDNEY

**UNSW SCIENCE**

**School of Mathematics and  
Statistics**

**Course outline**

**Term 1, 2023**

**MATH5175**

**Special Topics (Applied Mathematics)  
Integrable Systems. Classical Theory and  
Modern Applications**

## Staff

| Position           | Name                      | Email  | Room    |
|--------------------|---------------------------|--|---------|
| Lecturer-in-charge | Professor Wolfgang Schief | <a href="mailto:w.schief@unsw.edu.au">w.schief@unsw.edu.au</a> | RC-4069 |

Please refer to your Timetable on MyUNSW for your Lecture Tut, Lab enrolment days and times. Timetable link: <http://timetable.unsw.edu.au/2023/MATH5175.html>

## Administrative Contacts

Please visit the School of Mathematics and Statistics website for a range of information on School Policies, Forms and Help for Students.

For information on Courses, please go to “Current Students” and either Undergraduate and/or Postgraduate”, Course Homepage” for information on all course offerings,

The “Student Notice Board” can be located by going to the “Current Students” page; Notices are posted regularly for your information here. Please familiarise yourself with the information found in these locations. The School of Mathematics and Statistics web page is: <https://www.maths.unsw.edu.au>

If you cannot find the answer to your queries on the web you are welcome to contact the Student Services Office directly by phone.

By email      Postgraduate      [pg.mathsstats@unsw.edu.au](mailto:pg.mathsstats@unsw.edu.au)

By phone:      9385 7053

Should we need to contact you, we will use your official UNSW email address of in the first instance. **It is your responsibility to regularly check your university email account. Please state your student number in all emails.**

## Course Description / Aims

The discovery of the Inverse Scattering Transform (IST) method and its application to the celebrated Korteweg-de Vries (KdV) equation in 1967 by Gardner, Green, Kruskal and Miura is regarded as one of the most important developments in mathematical physics in the past 50 years. It followed the numerical discovery in 1965 by Kruskal and Zabusky of a groundbreaking nonlinear phenomenon, namely the soliton interaction. The IST method is a nonlinear analogue of the Fourier transform method for privileged systems of nonlinear partial differential equations which are known as soliton equations or integrable systems. The latter term refers to the fact that these systems have properties which generalise those of classical integrable systems in the sense of classical Hamiltonian mechanics.

Remarkably, in the past 50 years, it has been demonstrated that the applicability of integrable systems is not confined to the original physical setting of the important Fermi-Pasta-Ulam problem. Indeed, the ubiquitous nature of integrable systems is reflected in their (apparent and disguised) presence in a wide range of both mathematical fields (e.g. partial differential and difference equations, differential and algebraic geometry, Galois theory, representation theory, special functions, quantum and cluster algebras, number theory, Nevanlinna theory in complex analysis) and physical fields (e.g. general relativity, twistor theory, field theory, (continuum) mechanics, nonlinear optics, Josephson junctions, Bose-Einstein condensates, biophysics, surface and water waves, plasma physics).

By means of specific examples, this course serves as a gentle introduction to the world of integrable systems. Topics may include:

- Classical Liouville-Arnold integrability of ODEs based on Hamiltonian/Lagrangian mechanics, symmetries, conservation laws (Noether's theorem)
- Integrability of infinite-dimensional systems: PDEs, soliton theory, Lax pairs, Backlund transformations, (differential) geometric aspects
- Basic theory of (linear) difference equations
- Discrete (nonlinear) integrable systems: (algebraic) geometry and features of integrable maps and partial difference equations, discrete Painlevé equations, integrability detectors
- Applications in mathematics and physics.

## Assessment and Deadlines

| Assessment   | Week        | Weighting % | Due date if applicable |
|--------------|-------------|-------------|------------------------|
| Assignment 1 | Week 7      | 20          | 27 March 2023          |
| Assignment 2 | Week 10     | 20          | 17 April 2023          |
| Final Exam   | Exam period | 60          |                        |

## Course Schedule

The course will include material taken from some of the following topics. This is should only serve as a guide as it is not an extensive list of the material to be covered and the timings are approximate. The course content is ultimately defined by the material covered in lectures; refer to Moodle for Lecture notes.

| Weeks | Topic  |
|-------|--|
| 1     | Linear and nonlinear wave equations  |
| 2     | Soliton interaction, Lax pairs   |
| 3     | Conservations laws. Conserved quantities, Hamiltonian systems (ODEs)                                   |
| 4     | Classical Liouville-Arnold integrability of Hamiltonian systems, Hamiltonian densities and functionals |
| 5     | Bi-Hamiltonian PDEs, hierarchies of commuting flows  |
| 7     | Darboux transformations, application to integrable PDEs  |
| 8     | Backlund transformations, nonlinear superposition principles   |
| 9     | Hirota formalism, tau-functions  |
| 10    | Integrable lattice equations and geometric aspects   |

## Late Submission of Assessment Tasks

A late penalty of 5% of the maximum mark for the task will be applied per day or part day any assessment task is submitted more than 1 hour late. (Where "late" in this context means after any extensions granted for Special Consideration or Equitable Learning Provisions.) For example, an assessment task that was awarded 75% would be given 65% if it was 1-2 days late. Any assessment task submitted after 5 days will not be accepted.

Note that the penalty does not apply to

- Assessment tasks worth less than 5% of the total course mark, e.g. weekly quizzes, weekly class participation, or weekly homework tasks.
- Examinations and examination-style class tests
- Pass/Fail Assessments

## Text Books

Literature references will be given during the lectures.

## Moodle

Log in to Moodle to find announcements, general information, notes, lecture slide, classroom tutorial and assessments etc.

<https://moodle.telt.unsw.edu.au>

## School of Mathematics and Statistics and UNSW Policies

The School of Mathematics and Statistics has adopted a number of policies relating to enrolment, attendance, assessment, plagiarism, cheating, special consideration etc. These are in addition to the Policies of The University of New South Wales. Individual courses may also adopt other policies in addition to or replacing some of the School ones. These will be clearly notified in the Course Initial Handout and on the Course Home Pages on the School of Maths Stats web site.

Students in courses run by the School of Mathematics and Statistics should be aware of the School and Course policies by reading the appropriate pages on the Maths Stats web site starting at:

<https://www.maths.unsw.edu.au/currentstudents/assessment-policies>

The School of Mathematics and Statistics will assume that all its students have read and understood the School policies on the above pages and any individual course policies on the Course Initial Handout and Course Home Page. Lack of knowledge about a policy will not be an excuse for failing to follow the procedure in it.

## **Academic Integrity and Plagiarism**

UNSW has an ongoing commitment to fostering a culture of learning informed by academic integrity. All UNSW staff and students have a responsibility to adhere to this principle of academic integrity. Plagiarism undermines academic integrity and is not tolerated at UNSW. *Plagiarism at UNSW is defined as using the words or ideas of others and passing them off as your own.*

The **UNSW Student Code** provides a framework for the standard of conduct expected of UNSW students with respect to their academic integrity and behaviour. It outlines the primary obligations of students and directs staff and students to the Code and related procedures.

In addition, it is important that students understand that it is not permissible to buy essay/writing services from third parties as the use of such services constitutes plagiarism because it involves using the words or ideas of others and passing them off as your own. Nor is it permissible to sell copies of lecture or tutorial notes as students do not own the rights to this intellectual property.

If a student breaches the Student Code with respect to academic integrity, the University may take disciplinary action under the **Student Misconduct Procedure**.

The UNSW Student Code and the Student Misconduct Procedure can be found at:

<https://student.unsw.edu.au/plagiarism>

An online Module "[Working with Academic Integrity](https://student.unsw.edu.au/aim)" (<https://student.unsw.edu.au/aim>) is a six- lesson interactive self-paced Moodle module exploring and explaining all of these terms and placing them into your learning context. It will be the best one-hour investment you've ever made.

### **Plagiarism**

Plagiarism is presenting another person's work or ideas as your own. Plagiarism is a serious breach of ethics at UNSW and is not taken lightly. So how do you avoid it? A one-minute video for an overview of how you can avoid plagiarism can be found <https://student.unsw.edu.au/plagiarism>.

## **Additional Support**

### **ELISE (Enabling Library and Information Skills for Everyone)**

ELISE is designed to introduce new students to studying at UNSW. Completing the

ELISE tutorial and quiz will enable you to:

- analyse topics, plan responses and organise research for academic writing and other assessment tasks
- effectively and efficiently find appropriate information sources and evaluate relevance to your

- needs
- use and manage information effectively to accomplish a specific purpose
- better manage your time
- understand your rights and responsibilities as a student at UNSW
- be aware of plagiarism, copyright, UNSW Student Code of Conduct and Acceptable Use of UNSW ICT Resources Policy
- be aware of the standards of behaviour expected of everyone in the UNSW community
- locate services and information about UNSW and UNSW Library

Some of these areas will be familiar to you, others will be new. Gaining a solid understanding of all the related aspects of ELISE will help you make the most of your studies at UNSW.

The *ELISE* training webpages:

<https://subjectguides.library.unsw.edu.au/elise/aboutelise>

## **Equitable Learning Services (ELS)**

If you suffer from a chronic or ongoing illness that has, or is likely to, put you at a serious disadvantage, then you should contact the Equitable Learning Services (previously known as SEADU) who provide confidential support and advice.

They assist students:

- living with disabilities
- with long- or short-term health concerns and/or mental health issues
- who are primary carers
- from low SES backgrounds
- of diverse genders, sexes and sexualities
- from refugee and refugee-like backgrounds
- from rural and remote backgrounds
- who are the first in their family to undertake a bachelor-level degree.

Their web site is: <https://student.unsw.edu.au/els/services>

Equitable Learning Services (ELS) may determine that your condition requires special arrangements for assessment tasks. Once the School has been notified of these, we will make every effort to meet the arrangements specified by ELS.

Additionally, if you have suffered significant misadventure that affects your ability to complete the course, please contact your Lecturer-in-charge in the first instance.

## **Academic Skills Support and the Learning Centre**

The Learning Centre offers academic support programs to all students at UNSW Australia. We assist students to develop approaches to learning that will enable them to succeed in their academic study. For further information on these programs please go to:

<http://www.lc.unsw.edu.au/services-programs>

## Applications for Special Consideration for Missed Assessment

Please adhere to the Special Consideration Policy and Procedures provided on the web page below when applying for special consideration.

<https://student.unsw.edu.au/special-consideration>

Please note that the application is not considered by the Course Authority, it is considered by a centralised team of staff at the Nucleus Student Hub.

The School will contact you (via student email account) after special consideration has been granted to reschedule your missed assessment, for a *lab test or paper-based test* only.

For applications for special consideration for *assignment extensions*, please note that the new submission date and/or outcome will be communicated through the special consideration web site only, no communication will be received from the School.

For Dates on Final Term Exams and Supplementary Exams please check the “Key Dates for Exams” ahead of time to avoid booking holidays or work obligations.

<https://student.unsw.edu.au/exam-dates>

If you believe your application for Special Consideration has not been processed, you should email [specialconsideration@unsw.edu.au](mailto:specialconsideration@unsw.edu.au) immediately for advice.

## Course Evaluation and Development (MyExperience)

Student feedback is very important to continual course improvement. This is demonstrated within the School of Mathematics and Statistics by the implementation of the UNSW online student survey *myExperience*, which allows students to evaluate their learning experiences in an anonymous way. *myExperience* survey reports are produced for each survey. They are released to staff after all student assessment results are finalised and released to students. Course convenor will use the feedback to make ongoing improvements to the course.