Honours projects in Pure Mathematics, 2022

Below is a list of some possible honours supervisors, their research areas and suggestions for possible honours projects. The options are far from limited to what is included here so have a look and see who does mathematics that you find interesting and chat to them. For a complete list of supervisors, see

https://www.maths.unsw.edu.au/currentstudents/honours-pure-mathematics

Haris Aziz (School of Computer Science and Engineering)

- Fair Division

  Fair Division (https://en.wikipedia.org/wiki/Fair_division) is a subfield of mathematics that was initiated by celebrated mathematicians such as Banach, Knaster, and Steinhaus. The field has received renewed focus as computer applications give rise to new combinatorial settings in which resources or tasks need to be allocated to competing agents in a fair and efficient manner. Fundamental questions include the following ones. For a given setting and a specific notion of fairness, does a fair allocation always exist for the setting? Is there a constructive algorithm to compute such an allocation? How many queries are required to identify a fair allocation? The focus of the project will be on these types of research questions. Students with interest in graph theory, discrete mathematics, and/or combinatorial topology will enjoy the project.

- Axiomatic Approach to Stable Matching

  In two-sided matching, members from two sides, such as students and schools are matched to each other based on their preferences. The goal is typically to match the participants in a stable manner (https://en.wikipedia.org/wiki/Stable_marriage_problem). Other than stability, several axioms are desirable for the problem. For example, one desirable axiom is strategyproofness: the matching algorithm should not provide any incentive for participants to misreport their preferences. In this project, the goal will be to understand the tradeoffs between various notions of stability and other desirable axioms such as strategyproofness. Research questions will be of the following type. Are a given set of axioms for stable matching compatible? What are the necessary and sufficient conditions for a given set of axioms to be compatible? Students with interest in graph theory and discrete mathematics will enjoy the project.

Thomas Britz

I am interested in most areas of combinatorics and graph theory, so let me know if anything related to these topics interests you too. Below are some suggestions for topics.

- **The combinatorics of partially ordered sets.**
  This topic would be hard but rewarding. It is about the combinatorics of posets, in particular the work by Dilworth, Fomin, Greene, Kleitman and others. The main theorem of interest is the Duality Theorem, a beautiful and deceptively simple-looking result discovered and proved independently by Fomin and Greene. Are there new and interesting applications of this result? Does it relate to other duality theorems in combinatorics and related areas?

- **How to count.**
  Counting might be the first and simplest mathematics that we learn - but it is also some of the most interesting and challenging mathematics. There are several powerful counting methods, such as the Inclusion-Exclusion Principle and Polya Counting, but many counting problems require particular ingenuity and creativity. The art of counting, Enumerative Combinatorics, is beautiful, fun, and challenging.

- **Algebraic Combinatorics.**
  Algebra and combinatorics frequently overlap and interact, and their intersection is Algebraic Combinatorics. This is a deep and wide field of mathematics that uses abstract algebra to address combinatorial problems and which uses combinatorial methods to yield algebraic results.

Arnaud Brothier

My research area is in operator algebra and its connections with group theory, ergodic theory, representation theory and conformal field theory. It is thus a mixed of functional analysis and algebra.

Operator algebra was introduced to study quantum mechanics and is thus a very important subject to get into if one wants to work in mathematical physic on the quantum side of it.

I would be happy to supervise Honour topics in any topics connected to my research. My most recent focus has been on representations of groups, properties of groups (analytical or topological), properties of tensor categories and constructions of physical models such as conformal field theory and Yang-Mills theory.

- **Functors give representations of groups and physical models.**

  In a series of recent papers Vaughan Jones produced a beautiful connection between subfactor theory and group theory. Giving a functor from the category of binary forests into the category of Hilbert spaces he built a representation of the famous Thompson's group.

  This is a widely unexplored framework at the border of group theory, category theory and functional analysis.

  Some aspects are really analytic and use single operator theory, C*-algebras and dynamical systems.

  One can study analytical properties for groups such as amenability, the Haagerup property, the Cowling-Haagerup constant, and Kazhdan property (T).

  Some other are much more algebraic and use advance representation theory such as tensor categories and the formalism of Jones' planar algebras.

  Finally, this connection provides a way to build physical models and is thus related to mathematical-physics.

- **Homotopy and homology for group theory**

  This project is about using invariants coming from algebraic topology for distinguishing groups. Given a group G, we say that (X, x) is a classifying space for G if it is a pointed path-connected CW complex satisfying that $\pi_1(X, x) \simeq G$ and all the other homotopy groups are trivial. A classifying space is unique up to homotopy equivalence. Although, two classifying spaces may not be isomorphic as complexes. Moreover, a classifying space may have exceptional properties such as having a finite n-skeleton for a certain natural number n. This project is about constructing concretely classifying spaces for specified groups that are simplicial complexes with exceptional properties. Establishing that these complexes are indeed classifying spaces (or similar invariants) will be proved using homotopy and or homology theory.

- **Subfactor theory**

  This theory initiated by Vaughan Jones studies inclusion of von Neumann algebras that have a trivial center.

  The algebraic data of such inclusions is very rich and provides a very sophisticate Galois theory in the framework of rigid C*-tensor categories.

  Moreover, using subfactors, one can define the famous Jones polynomial that is an invariant for knots.

  Subfactors can also be considered as abstract dynamical systems and thus carry analytical properties.

- **Mathematics for quantum physics**

  This project is about getting familiar with mathematical formalism used in quantum mechanics and in quantum field theory.

  Algebras of observable are described by C*-algebras and von Neumann algebras where states are given by linear functionals.

  One project is to understand better those mathematical objects and understand, using Tomita-Takesaki theory and Connes theory, how time evolution naturally appears from the mathematics.

  Another project is the study of algebraic quantum field theory using conformal nets. A conformal net is a family of von Neumann algebras (local algebras of fields) indexed by intervals of the circle on which the spatial diffeomorphism group acts. It carries a very rich representation theory. Their study is both analytic and algebraic.

- **Duality**

  This project is about studying one or several duality situations.

  The dual of a topological vector space (TVS) is the space of all continuous linear functional. A TVS is called reflexive when it is isomorphic under a canonical map to its double dual. One project is to study such spaces in particular when they are Banach spaces.

  Gelfand transform relates abelian C*-algebras and locally compact spaces by considering continuous functions on the space of characters. It realizes an equivalence of categories that commutes with the operation of adding a unit to the algebra and performing the compactification of Alexandroff. A particular case of this Gelfand transform gives Fourrier duality for locally compact abelian groups via the Fourrier transform implying Pontryagin duality: the double dual of a locally compact abelian group is isomorphic to itself via a canonical evaluation map.

  Going further one can adapt similar ideas for **non-**Abelian groups using quantum groups.

Daniel Chan

My research interests are in algebra and algebraic geometry, though I have also supervised honours theses in number theory and topology. To get an idea of some of the mathematics I am interested in, you can check my YouTube channel which is aimed at beginning honours students

http://www.youtube.com/c/DanielChanMaths

If anything there is of interest to you, we can find an appropriate honours project in that area. Below is a sample of some honours projects. You may also wish to check my webpage

http://web.maths.unsw.edu.au/~danielch

for other projects and past honours theses.

- **Kleinian singularities and the McKay correspondence.**
  Let G be a finite subgroup of $SL_2(\mathbb{C})$ which acts naturally on $\mathbb{C}^2$. Then the space of G-orbits $\mathbb{C}^2/G$ is a complex surface in $\mathbb{C}^3$ which is not smooth. These Kleinian singularities are interesting objects in mathematics. In this project we look at the McKay correspondence which relates the representation theory of G to the geometry of $\mathbb{C}^2/G$. The theory involves an interesting mix of geometry, group theory and ring theory.

- ***D*-modules.**
  The set of differential operators $\sum p_i(x)\frac{d^i}{dx^i}$ forms a non-commutative ring called the first Weyl algebra A_1. It arises naturally in the theory of differential equations. In this project, we look at higher dimensional analogues A_n and their rather subtle module theory. One highlight is Bernstein's spectacular purely algebraic solution to a question of Gelfand's in complex analysis.

Michael Cowling

- **Quasiconformal mappings.**
  Conformal mapping is a classical approach to questions in partial differential equations such as the Dirichlet problem. However, it is hard to find conformal mappings, and so quasiconformal mappings were suggested as a substitute. This project is to find out about quasiconformal mappings and their uses, and to look at some open problems in the theory.

- **Hausdorff dimension and other measures of size.**
  What do we mean by dimension? The Hausdorff dimension is a measure of the "thickness" of sets in euclidean spaces. It is possible to construct "strange" sets whose Hausdorff dimension is fractional. More recently, other ways of measuring dimension have been proposed. This project is to find out about Hausdorff dimension, and to see what can be said about other types of dimension.

Ian Doust

- **Metric embeddings.**
  Metric spaces, that is sets where there is some sensible way of saying how far apart two elements are, arise frequently both in theoretical and practical work. Such spaces are much easier to deal with if one can realize them as subsets of some well understood space, ideally something like a low dimensional Euclidean space. An important problem is to be able to determine when such metric space embeddings can be done. There are some beautiful classical results in this direction, but also much very recent work which could form the basis of an honours thesis.

Jie Du

- **Quivers and their representations.**
A quiver is a network which consists of vertices and oriented edges (or arrows) connecting vertices. A representation of a quiver is a collection of finite dimensional vector spaces indexed by the vertices together with a collection of linear transformations indexed by the arrows. This project investigates the representations of quivers, especially the classification of indecomposable ones. We will also look at some applications to Lie theory such as semisimple complex Lie algebras and quantum groups.

- **Symmetric groups and their representations/affine version.**
A symmetric group is the permutation group on n letters. It has many fascinating properties in both combinatorics and representation theory. This project will start with an understanding of some basic structure of the symmetric group. This includes the length function, Young subgroups and their shortest coset representatives, and the Robinson-Schensted correspondence. Then we move on looking at the representation theory of the symmetric group or the structure of affine symmetric group, an infinite but tameable version of the symmetric group, and their associated Hecke algebras.

- **The classical and quantum Schur-Weyl duality.**
For a finite dimensional space V over the complex number field $\mathbb{C}$, there are commutative actions on the r-fold tensor product $V^{\otimes r}$ of V by the general linear group $G = GL(V)$ and the symmetric group S_r on r letters. This induces two algebra homomorphisms
$$\phi : \mathbb{C}G \to \mathrm{End}(V^{\otimes r}) \quad \text{and} \quad \psi : \mathbb{C}S_r \to \mathrm{End}(V^{\otimes r}).$$
The classical Schur-Weyl duality shows that
$$\mathrm{im}(\phi) = \mathrm{End}_{\mathbb{C}S_r}(V^{\otimes r}) \quad \text{and} \quad \mathrm{im}(\psi) = \mathrm{End}_{\mathbb{C}G}(V^{\otimes r}).$$
If the field $\mathbb{C}$ is replaced by an arbitrary field k, then the duality continues to hold but the proof is much harder. This project will investigate this duality in the quantum case (which is already known) and in the affine case (which is partially known).

Jim Franklin

- **Extreme risks and expert opinion.**
Risks of rare events that “haven’t happened yet” are hard to assess from data; the risk of terrorist attacks, quarantine incursions etc are hard to quantify even approximately because of the lack of directly relevant data. So methods of combining small data sets with expert opinion are needed. The Basel II process for analysis of banks’ operational risk is a well-developed system for using a combination of methods like extreme value theory and scenario analysis. There are ideas that can be applied more widely.

- **Philosophy of applied mathematics.**
How is it possible to do mathematics “in the armchair”, just by thinking, yet find that it applies perfectly in the real world? A philosophy of modelling is needed, explaining what real-world properties mathematics studies. The project examines the types of mathematical properties that physical (and other) objects can have, such as ratio, symmetry and continuity. This “Aristotelian” approach is contrasted with the standard alternatives in the philosophy of mathematics, Platonism and nominalism.

Catherine Greenhill

My research interests are in probabilistic, algorithmic and asymptotic combinatorics. Feel free to talk to me (or email me) about possible Honours topics, or to suggest an idea of your own.

- **Pulling graphs apart.**
  Suppose you take a complete graph on n vertices, with all possible edges present. Is it possible to decompose the edge set into a union of disjoint triangles? Two obvious necessary conditions are that 3 must divide the number of edges, and that the degree of every vertex must be even (so n must be odd). But it turns out that these necessary condition are also sufficient. This is a *graph decomposition* problem, with many generalisations that have been studied by many authors. The proofs typically involve clever combinatorial constructions.

  Now suppose you take a random graph on n vertices such that every vertex has degree d, where d is some constant. There have been a sequence of results showing how to pull such a random regular graph apart into random regular factors of various kinds. These results are known as *contiguity arithmetic*, and shed light on the structure of random regular graphs. The proof involves a method known as *small subgraph conditioning*.

  This Honours topic can be tailored to the student's interest: it could be a study of graph decomposition problems, or of contiguity arithmetic results, or both. In particular, a very recent result by Delcourt and Postle lies in the intersection of these two areas, and could be a focus of study.

- **Combinatorial Markov chains.**
  A Markov chain is a simple stochastic process which is "memoryless", in that the next transition depends only on the current state, and not on the history of the chain. We consider discrete time Markov chains on finite state spaces which have exponentially large size (with respect to some parameter). If the Markov chain is ergodic then it converges to a unique stationary distribution. But how quickly does this convergence occur? There are a few methods for bounding this convergence rate, either to show that the chain "mixes rapidly" and hence can be used for efficient sampling, or that the chain "mixes torpidly" and converges exponentially slowly to the stationary distribution. The aim of the project is to survey the topic, learning about some of the results which have been proved and the methods used to prove them. (You don't need to be familiar with probability theory, as only simple discrete probability theory is required, which is essentially just counting.)

Pinhas Grossman

I am interested in a few related but distinct areas of research. Some areas that I can supervise projects in are the following.

- **von Neumann algebras**

  Operator algebras consist of linear operators on (usually infinite-dimensional) vector spaces. Unlike algebras of functions, these algebras are noncommutative. The two main types of operator algebras are C*-algebras, which can be thought of as noncommutative analogues of topological spaces; and von Neumann algebras, which can be thought of noncommutative analogues of measure spaces.

- **Modular tensor categories**

  A tensor category is a lot like a ring, but there are also morphisms between objects. Modular tensor categories are a particularly nice class of tensor categories which were originally discovered by physicists and play a central role in certain theories of quantum computation. Some of their structure is captured by pairs of unitary matrices called modular data, which satisfy a number of very nice algebraic, combinatorial, and number-theoretic properties.

- **Planar algebras**

  Planar algebras were introduced by Vaughan Jones in the 1990s as a new approach to studying tensor categories. One can represent morphisms in a tensor category as planar diagrams. A planar algebra is a collection of vector spaces on which certain planar pictures act. In addition to standard operations such as multiplication, there are many other interesting planar operations, such as rotation. Planar algebras can be presented by generators and relations. The relations are often skein relations of some sort, so planar algebras are also closely related to knot theory.

David Harvey

My research is mainly on algorithmic and computational problems in number theory.

I am happy to supervise projects on any topic in number theory (with or without a computational component), and some areas of algebraic geometry. Here are some examples:

- Zeta functions of algebraic curves and varieties
- p-adic numbers and applications, e.g., Hasse–Minkowski theorem
- Rational points on elliptic curves
- Theory of algebraic curves
- Higher reciprocity laws
- Gröbner bases
- Riemann zeta function, Dirichlet L-functions

Also, I usually have available some more specific computational problems that would be appropriate for an Honours student with a strong programming background.

Potential Honours students are welcome to drop in for a chat (best to email me first).

Jonathan Kress

- **Symmetry algebras of superintegrable systems.**
  Superintegrable systems are Hamiltonian systems with an excess of symmetries. These symmetries form non-trivial algebraic structures that have been studied in connection with the classical special functions, orthogonal polynomials and the recently discovered 'exceptional orthogonal polynomials. The aim of this project is to understand these symmetry algebras and how they are related to special functions.

- **Conformally equivariant quantisation.**
  An observable of a classical Hamiltonian system is a functions of the its position and momenta that does not change as the system evolves. Quantising such a system involves replacing the classical observables with differential operators that commute with the Hamiltonian operator. However, in general, there is no unique way to do this. Imposing additional structure, such as conformal covariance, on both the classical and quantum systems can provide a way to resolve this problem. The aim of this project is to understand what this means and study some specific examples.

Anita Liebenau

- **Extremal graph theory.**
  Typical questions in this area are the following. What is the largest number of edges that a graph can have without containing a triangle? What is the smallest d such that every graph G with minimum degree at least d contains a Hamilton cycle? What is the largest number of edges that a graph G can have without containing a cycle of length 8 as a subgraph? The first two are resolved and known as Mantel's and Dirac's theorem. We lack satisfying answers for the third one. A project in this area could have a focus on probabilistic methods, on algebraic or geometric constructions, or on algorithmic aspects.

- **Graph Ramsey theory.**
  Ramsey theory can be described as finding order in big-enough chaos. The driving question of the field is the following: What is the smallest number n such that every red/blue-colouring of the edges of the complete graph on n vertices contains a monochromatic copy of the complete graph on k vertices? This number is called the Ramsey number of K_k. The known lower and upper bounds are unsatisfyingly far apart. Moreover, no deterministic construction reaches the lower bound given by the probabilistic method. That is, if we draw a graph uniformly at random from all n-vertex graphs then this graph is a good candidate for a lower bound with a very high probability. Yet, we have no idea how to construct such a graph deterministically.

  There are many directions to take here. One project could be to look at the deterministic constructions for lower bounds and see whether those graphs have other interesting properties. Another project would study the following question. What is the smallest number m of edges needed to guarantee that every graph G with m edges and every 2 colouring of the edges of G contains a monochromatic copy of a given tree T. How does the answer change if we replace "tree T" by "cycle C"?

- **Positional games on graphs.**
  In a Maker-Breaker game played on the edge set of a graph G two players claim edges of G alternately. Maker wins the game if at the end the graph consisting of Maker's edges has some predefined property (e.g. to contain a perfect matching). There are interesting connections to random graphs. A project in this area would start with a literature review of the area to learn existing methods and then apply them to a game where the answers are not known yet.

Daniel Mansfield

I am interested in understanding mathematical documents from ancient Mesopotamia.

- **Computation and Errors** How did the ancient Mesopotamians perform multiplication? Did they use an abacus, and if so what did it look like? Is their system big endian or little endian? Computational errors provide valuable information that helps answer these questions. In this project you will analyse the errors from ancient mathematical and administrative texts, and hypothesise how the Mesopotamians were able to perform multiplication. This project would be suitable for students with a computer science background.

- **Applied Geometry in Mesopotamia** There are many examples of cadastral field plans made by professional and student surveyors from ancient Mesopotamia. These plans display an advanced understanding of geometry for the time. In this project, you will analyse these field plans to understand how geometry was understood and used at the time.

Alina Ostafe

- **Arithmetic dynamics and unlikely intersections.**
  Dynamical systems generated by iteration of polynomials andrational functions is a classical area of mathematics with a rich history and a widevariety of results and applications. Recently, there has been substantial interest in arithmetical dynamicalsystems, meaning the iteration of rational functions over fields of number-theoretic interest.

  This project will focus on arithmetic properties of orbits, and in particular, in studying intersections of orbits with different structural sets such as the set of some special integers, subgroups, subfields, etc. This is a deep and very active area of research at the cross-roads of Diophantine and algebraic geometry and number theory.

- **Algebraic aspects of polynomial dynamical systems over finite fields.**
  This project focuses on the study of non-classical dynamical systems over finite fields andthe study of atypical behaviour which is not present in standard constructions from complexdynamical systems. For example, some aspects of algebraicdynamical systems (ADS) to be studied within this project are:degree growth, irreducibility of iterates, trajectory length, etc. Such problems in ADSare of an intricate algebraic and number theoretic flavour, including the use of toolsfrom number theory and algebra. Research into ADS has not only a high theoretical value, but also applied significance thanks tothe great number of potential applications to many different areas of modern cryptography (for example in the construction of pseudorandom number generators, coding theory, Monte Carlo simulations, physics, etc. 3. Constructing good pseudorandom number generators: a cryptographic challenge

  For many cryptographic schemes it is very important to be able to generaterandom numbers as, for example, creating cryptographic keys which usuallyshould be generated at random from a given keyspace. The algorithm forgenerating numbers that approximate the properties of random numbers iscalled a pseudo-random number generator (PRNG).

  PRNG's are deterministic algorithms with relatively few parameters thatcan easily be implemented on a computer. As such, PRNG's have beensubjected to a rigorous theoretical analysis.

Usually PRNGs use recursive procedures and yield sequences that areultimately periodic. Some desirable properties of a sequence of pseudorandomnumbers, depending on the application, are: relatively large period length, it should have little intrinsic structure (linear complexity), it should have good statistical properties (uniform distribution), the generating algorithms should be efficient.

The aim of this project is to study such properties of classical and new recursivePRNGs. The area is an exciting mix of algebra, number theory and cryptography.

Denis Potapov

- **Noncommutative Integration Theory and Multiple Operator Integrals.**
  Multiple Operator Integrals (MOI) is the modern tool in analysis which proved very efficient recently. The analysis research team at UNSW has recently succeeded resolving a few long standing and open problems with the help of MOI methods. I was pointed out by many of my colleagues and research collaborators that the MOI theory has reached the level of maturity which requires a good reference/graduate text book.

  In this project, I suggest to lay the foundation for a future book on Multiple Operator Integrals.

- **Dirac operators in modern analysis.**
  Differentiation operator has been the main example of the operator from Physics where the Multiple Operator Integration theory has been tested. Dirac operators on the other hand, are the higher dimensional version of the differentiation operator. In this project, I suggest to investigate a possibility of extension of some known applications of MOI methods to Dirac operator setting. Even though some results seems easily transferable to this setting; the others may bring some surprises.

Igor Shparlinski

- **Rational numbers in Cantor's set.**
  The project is about understanding the structure of rationals p/q which fall in Cantor's set. It includes counting the number such rationals with denominators $q \leq Q$, investigating their arithmetic structure and several other questions.

- **Pseudopowers and pseudosquares.**
  There are integers which "pretend" to be powers g^n of a given integer $g \geq 2$ or perfect squares n^2 modulo all primes $p \leq x$. Such numbers are called x-pseudopowers and x-pseudosquares, respectively. Besides being interesting objects in their own rights, they also appear in several other number theoretic and algorithmic problems. The goal is investigate their properties and distribution. This project will allow its participants to learn a diverse range of number theoretic techniques.

- **Dynamical systems of number theoretic origins.**
  Recently, there has been active interest in dynamical systems generated by iterations of various number theoretic functions reduced modulo a prime p. The following functions $x \mapsto 2^x \pmod p$ or $x \mapsto x^x \pmod p$ or $x \mapsto x! \pmod p$ are representative examples of such functions. The aim of the project is to study, theoretically and numerically, some natural properties of these maps, such as the number of fixed points, the distribution of orbit lengths and propagation from the origin.

- **Distribution of solutions of Diophantine equations and congruences in many variables.**
The project is about studying the distribution of solutions to equations $f(x_1, \ldots, x_n) = 0$ and similar congruences with a polynomial $f \in Z[X_1, \ldots, X_n]$. This is a classical question of analytic number theory. The aim of the project is to study several versions of the above question in which the variables $(x_1, \ldots, x_n)$ are from domains of $\mathbb{R}^n$ with various geometric properties, such as boxes or domains with a smooth boundary or convex domains.

John Steele

- **Extending solutions of the Einstein equations.**
Standard techniques for solving the field equations in General Relativity usually provide only a part of the space-time manifold. This project will study how we find the full manifold and what that might mean.

- **Generating the Kerr Black Hole**
In 1964 Newman and Janis came up with a way of generating Kerr's spinning black hole solution from Schwarzschild's static black hole solution of the Einstein Equations. This idea can be used in other contexts, but its physical interpretation is still a matter of debate. This project will study the Newman-Janis trick, other methods of adding rotation to solutions of the Einstein equations in GR and how the symmetries of the solutions behave as the solutions are generated.

- **The Carroll Group**
The Lorentz group of special relatively has a well known limit for the case where c, the speed of light, is sent to ∞, namely the group of Galilean transformations. Less well known is another limit due to Levy-Leblond and called by him the Carroll group (after Lewis Carroll), in which (in a certain sense) $c \to 0$. This project will consider the two groups, their structure, how they are related and some of their physical consequences.

- **Generating Solutions of the Einstein Equations.**
Starting from one known solution of a differential equation it is sometimes possible to create others. Similar ideas apply to the Einstein Field Equations and this project will consider some of the approaches used.

Fedor Sukochev

- **Noncommutative integration theory.**
This topic covers noncommutative integration theory both with respect to the trace and with respect to a state/weight. These projects involve studying geometry of various bimodules associated with von Neumann algebras (like noncommutative L_p-spaces).

- **Noncommutative Probability Theory.**
This topic studies noncommutative probability theory, both with respect to the trace and "free version of probability" a la Voiculescu.

- **Classical probability theory.**
This project involves studying norms of sums of independent and conditionally independent random variables in various spaces of measurable functions.

- **Noncommutative Geometry.**
  More precisely, this topic involves the part of noncommutative geometry which deals with singular traces and their applications. These projects may require a student also to delve into the theory of ideals of compact operators and their geometry.

- **Banach Space geometry.**
  This topic involves the study of Banach space geometry and related parts of noncommutative analysis, including differentiating operator-valued functions and applications to Mathematical Physics. Sometimes, these projects may also be relevant to some problems from perturbation theory and their applications in Mathematical Physics (e.g. Krein's spectral shift function, Koplienko's spectral shift function and their higher dimensional analogues).

Mircea Voineagu

There are possible projects in the areas of algebraic topology, category theory, algebraic geometry and homological algebra. For example:

- **Equivariant homotopy theory** In this project we investigate different models of doing homotopy theory on a topological space that has a finite group action. The main invariant of these theories is Bredon cohomology which is an object introduced in 80' and this invariant will be the subject of the project. Many of the possible properties of Bredon cohomology are still to be proved as extensions from the classical case, so the project can touch areas of the current research.

- **Applied Algebraic Topology** This is a project in the very new and exciting field of applications of algebraic topology in data analysis. In this project we analyze theoretically (and practically) different models of invariants on topological sets associated to concrete data sets. For example persistent homology, Mapper etc. represents way to understand data derived from biology in topological terms of connected components, 1- dimensional holes etc. For example, a 1-dimensional hole in a biological data (a circle) can be viewed as a possible clustering set around a hole and may have biological significance. The practical component of this project is analyzing neurodegenerative diseases such as autism etc from the point of view of topology. In collaboration with BABS we interpret the possible results from the biological point of view. For the practical component of this project a basic knowledge of programming language R is a plus. For the theoretical part one can analyze zig-zag persistence and its stability or be interested in an equivariant persistence invariant.

- **K-theory and algebraic geometry.**
  This is a project in algebraic geometry and algebraic K-theory. K-theory of an algebraic variety X is a set of algebraic invariants $K_i(X)$ based on the category of locally free sheaves on X. The Chow ring of an algebraic variety is constructed using all irreducible subvarieties of X with a product given by their intersection. A famous theorem of Grothendieck asserts that, with $\mathbb{Q}$-coefficients, $K_0(X)$ is ring-isomorphic with the Chow ring of X. This theorem was recently generalized to all higher K-groups by the introduction of motivic cohomology ring that vastly generalizes the Chow ring. In this project we will investigate various constructions, ideas and recent applications of motivic cohomology groups in algebraic geometry. It will be a deep interplay between algebraic geometry and other fields like algebraic topology and homological algebra.

Norman Wildberger

I am currently interested in a wide variety of geometrical developments, from classical Euclidean geometry to relativistic geometries and chromogeometry, and across to projective metrical geometries, namely hyperbolic and elliptic geometries. I am interested in number theoretical aspects of the subject, which naturally arise when looking at some topics over finite fields and indeed over the rational numbers, and in the differential geometrical aspects relating to Lie groups and representation theory.

For a quick introduction to my favourite subject, Universal Hyperbolic Geometry, see the YouTube series UnvHypGeom at user: njwildberger (about 40 videos so far). It is very beautiful!

My broad orientation to Honours: it should expose the student to a wide spectrum of important, interesting and useful undergraduate mathematics, allow for some independent explorations, and should be generally fun.

I would also encourage an Honours project in the exciting new realm of rational trigonometry, which has been recently shown to connect with the history of Old Babylonian mathematics.

Lee Zhao

My main research interest lies in the field of analytic number theory; that is, using analytic tools to answer number theoretic questions. The following are some potential honours project problems on which I will be happy to supervise students.

- **Spacing of special Farey sequences** A Farey sequence of level Q is the set of rational numbers in $[0, 1]$ with denominators not exceeding Q. Knowledge of the spacing of these fractions becomes scarce when we restrict to special denominators, for example squares. We will look at some conjectures and the current knowledge of spacing problems of these kinds of special Farey fractions. We will approach these problems numerically and develop estimates more precise than those previously conjectured.
- **Exponential and character sums** Exponential and character sums are extremely useful in analytic number theory as many problems can be reduced to the estimations of these sums. We will study the classical theory of these sums and how they relate to problems in number theory.

I will also be happy to discuss other potential projects in number theory.