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Decomposing Value Added Growth Over Sectors into Explanatory Factors

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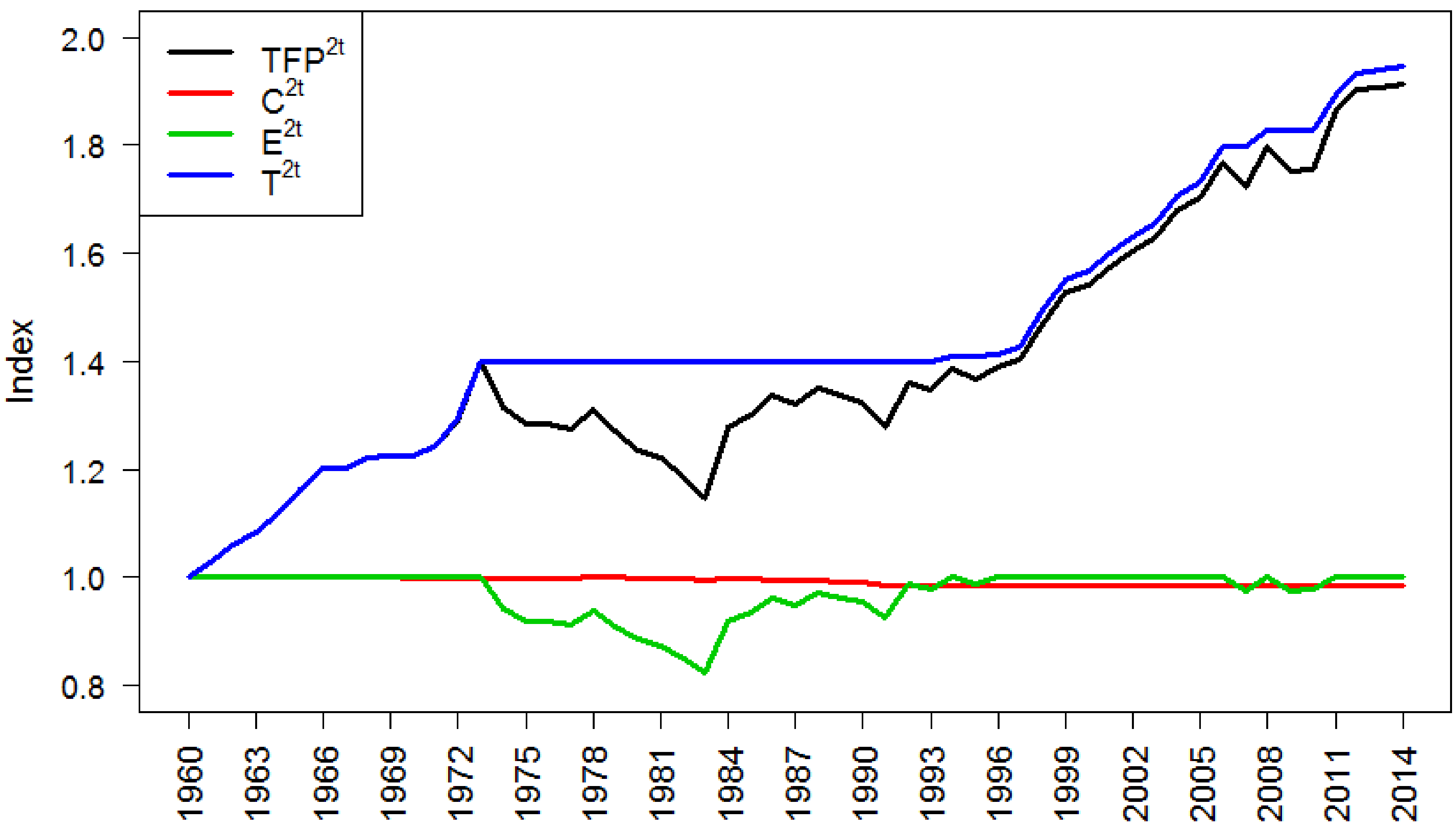
Summary

- Decompose nominal value added growth over multiple sectors into explanatory factors.

- For a single sector, explanatory factors are
  - efficiency changes,
  - changes in output prices,
  - changes in primary inputs,
  - changes in input prices,
  - technical progress, and
  - returns to scale.

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology



Summary

- Need sector's best practice technology for the two periods under consideration.

- Could use econometric or nonparametric (DEA) techniques

- We use Free Disposal Hull approach – no convexity assumptions

- Our approach has the advantage that it does not involve econometric estimation, and involves only observable data.

- Simple enough to be implemented by statistical agencies

- If efficient in both periods, can use the index number techniques of Diewert-Morrison (1986)/Kohli (1990).

- Address the problem of aggregating over sectors.

Cost Constrained Value Added Function for a Sector

- A sector produces M net outputs, $y \equiv [y_1,\ldots,y_M]$, using N primary inputs $x \equiv [x_1,\ldots,x_N] \geq 0_N$.

- If $y_m > 0$, then the sector produces the mth net output during period t while if $y_m < 0$, then the sector uses the mth net output as an intermediate input.

- Strictly positive vector of net output prices $p \equiv [p_1,\ldots,p_M] >> 0_M$ and strictly positive vector of input prices $w \equiv [w_1,\ldots,w_N] >> 0_N$

- *Period t production possibilities set* for the sector S^t

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Cost Constrained Value Added Function for a Sector

S^t satisfies the following regularity conditions:

(i) S^t is a closed set;

(ii) for every $x \geq 0_N$, $(0_M,x) \in S^t$;

(iii) if $(y,x) \in S^t$ and $y^* \leq y$, then $(y^*,x) \in S^t$ (free disposability of net outputs);

(iv) if $(y,x) \in S^t$ and $x^* \geq x$, then $(y,x^*) \in S^t$ (free disposability of primary inputs);

(v) if $x \geq 0_N$ and $(y,x) \in S^t$, then $y \leq b(x)$ where the upper bounding vector b can depend on x (bounded primary inputs implies bounded from above net outputs).

Cost Constrained Value Added Function for a Sector

Period t cost constrained value added function:

$$R^t(p,w,x) \equiv \max_{y,z} \{p\cdot y : (y,z)\in S^t;\ w\cdot z \le w\cdot x\}$$

$R^t(p,w,x)$ is well defined even if there are increasing returns to scale in production; i.e., the constraint $w\cdot z \le w\cdot x$ leads to a finite value for $R^t(p,w,x)$.

If (y^*,z^*) solves this constrained maximization problem, then sectoral value added $p\cdot y$ is maximized subject to the constraints that (y,z) is a feasible production vector and primary input expenditure $w\cdot z$ is equal to or less than "observed" primary input expenditure $w\cdot x$.

Cost Constrained Value Added Function for a Sector

Observed value added, $p^t \cdot y^t$, may not equal the optimal value added.

Value added efficiency of the sector during period t:

$$e^t \equiv p^t \cdot y^t / R^t(p^t, w^t, x^t) \leq 1$$

The cost constrained valued added function has some interesting properties. If S^t is a cone, so that production is subject to constant returns to scale, can show that

$$R^t(p,w,x) \equiv w \cdot x / c^t(w,p)$$

where $c^t(w,p)$ is the unit cost function for producing a unit of value added.

Cost Constrained Value Added Function for a Sector

A brief digression to prove this result

Value added function:

$\Pi^t(p,x) \equiv \max_y \{p\cdot y: (y,x)\in S^t\}.$

The cost constrained value added function $R^t(p,w,x)$ has the following representation:

$$R^t(p,w,x) \equiv \max_{y,z} \{p\cdot y : (y,z)\in S^t;\ w\cdot z \leq w\cdot x;\}$$
$$= \max_z \{\Pi^t(p,z) : w\cdot z \leq w\cdot x;\ z \geq 0_N\}.$$

- Holding p constant, we can define the period t *"utility" function* $f^t(z) \equiv \Pi^t(p,z)$.

Cost Constrained Value Added Function for a Sector

Then we have the following "utility" maximization problem:

$R^t(p,w,x) = \max_z \{f^t(z) : w \cdot z \le w \cdot x;\ z \ge 0_N\}$

where $w \cdot x$ is the consumer's "income".

For u in the range of $\Pi^t(p,z)$ over the set of nonnegative z vectors and for $w >> 0_N$, we can define the *cost function* $C^t(u,w)$ that corresponds to $f^t(z)$ as follows:

$C^t(u,w) \equiv \min_z \{w \cdot z : f^t(z) \ge u;\ z \ge 0_N\} = \min_z \{w \cdot z : \Pi^t(p,z) \ge u;\ z \ge 0_N\}$.

Cost Constrained Value Added Function for a Sector

If $\Pi^t(p,z)$ increases as all components of z increase, then $C^t(u,w)$ will be increasing in u and we can solve the following maximization problem for a unique u^*:

$$\max_u \{u: C^t(u,w) \leq w \cdot x\}$$

$$R^t(p,w,x) = u^* \text{ with } C^t(u^*,w) = w \cdot x.$$

The above formulae simplify considerably if S^t is a cone, so that production is subject to constant returns to scale:

- $\Pi^t(p,z)$ is linearly homogeneous in z and hence, so is $f^t(z) \equiv \Pi^t(p,z)$.

Cost Constrained Value Added Function for a Sector

Define the unit cost function c^t that corresponds to f^t as follows:

$$c^t(w,p) \equiv \min_z \{w \cdot z : \Pi^t(p,z) \geq 1;\ z \geq 0_N\}.$$

The total cost function, $C^t(u,w) = C^t(u,w,p)$ is now equal to $uc^t(w,p)$ and the solution to the utility maximization problem is the following u^*:

$$u^* = R^t(p,w,x) \equiv w \cdot x / c^t(w,p).$$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Change in value added efficiency

$$\varepsilon^t \equiv e^t/e^{t-1} = [p^t \cdot y^t/R^t(p^t,w^t,x^t)]/[p^{t-1} \cdot y^{t-1}/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})]$$

If $\varepsilon^t > 1$, then value added efficiency has *improved* going from period t–1 to t whereas it has *fallen* if $\varepsilon^t < 1$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Follow method of Konüs (1939) and Allen (1949) to define various *families of indexes* that vary only *one* of the four sets of variables, t, p, w and x, between the two periods under consideration and hold constant the other sets of variables.

Family of *output price indexes*:

$$\alpha(p^{t-1},p^{t},w,x,s) \equiv R^{s}(p^{t},w,x)/R^{s}(p^{t-1},w,x).$$

Two alternatives:

$\alpha_L{}^{t} \equiv \alpha(p^{t-1},p^{t},w^{t-1},x^{t-1},t-1) \equiv R^{t-1}(p^{t},w^{t-1},x^{t-1})/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})$;

$\alpha_P{}^{t} \equiv \alpha(p^{t-1},p^{t},w^{t},x^{t},t) \qquad \equiv R^{t}(p^{t},w^{t},x^{t})/R^{t}(p^{t-1},w^{t},x^{t}).$

Preferred overall measure of output price growth:

$\alpha^{t} \equiv [\alpha_L{}^{t}\,\alpha_P{}^{t}]^{1/2}$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family of *input quantity indexes*:

$$\beta(x^{t-1},x^t,w) \equiv w\cdot x^t/w\cdot x^{t-1}.$$

$\beta_L{}^t \equiv w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1}$;

$\beta_P{}^t \equiv w^t\cdot x^t/w^t\cdot x^{t-1}$.

Preferred overall measure of input quantity growth:

$\beta^t \equiv [\beta_L{}^t\, \beta_P{}^t]^{1/2}$.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family of *input mix indexes:*

$$\gamma(w^{t-1},w^t,p,x,s) \equiv R^s(p,w^t,x)/R^s(p,w^{t-1},x)$$

More accurate to say that $\gamma(w^{t-1},w^t,p,x,s)$ represents the hypothetical proportional change in cost constrained value added for the period s reference technology due to the effects of a change in the input price vector from w^{t-1} to w^t when facing the reference net output prices p and the reference vector of inputs x.

$$\gamma_{LPP}{}^t \equiv \gamma(w^{t-1},w^t,p^{t-1},x^t,t) \equiv R^t(p^{t-1},w^t,x^t)/R^t(p^{t-1},w^{t-1},x^t);$$

$$\gamma_{PLL}{}^t \equiv \gamma(w^{t-1},w^t,p^t,x^{t-1},t-1) \equiv R^{t-1}(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^{t-1},x^{t-1}).$$

$$\gamma^t \equiv [\gamma_{LPP}{}^t\gamma_{PLL}{}^t]^{1/2}.$$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family *of technical progress indexes*:

$$\tau(t-1,t,p,w,x) \equiv R^t(p,w,x)/R^{t-1}(p,w,x)$$

$\tau_L{}^t \equiv \tau(t-1,t,p^{t-1},w^{t-1},x^t) \equiv R^t(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^t)$.

$\tau_P{}^t \equiv \tau(t-1,t,p^t,w^t,x^{t-1}) \quad \equiv R^t(p^t,w^t,x^{t-1})/R^{t-1}(p^t,w^t,x^{t-1})$.

Recall, if the reference technologies in periods t and t–1 are cones, $R^t(p,w,x) = w \cdot x/c^t(w,p)$ and $R^{t-1}(p,w,x) = w \cdot x/c^{t-1}(w,p)$.

Thus in the case where the reference technology is subject to CRS, these "mixed" indexes of technical progress are independent of x and then true Laspeyres and Paasche type indexes.

Decomposing Value Added Growth for a Sector into Explanatory Factors

Family *of (global) returns to scale measures:*

$$\delta(x^{t-1},x^t,p,w,s) \equiv [R^s(p,w,x^t)/R^s(p,w,x^{t-1})]/[w\cdot x^t/w\cdot x^{t-1}].$$

$$\delta_L{}^t \equiv \delta(x^{t-1},x^t,p^{t-1},w^{t-1},t-1) \equiv R^{t-1}(p^{t-1},w^{t-1},x^t)/R^{t-1}(p^{t-1},w^{t-1},x^{t-1})]/[w^{t-1}\cdot x^t/w^{t-1}\cdot x^{t-1}];$$

$$\delta_P{}^t \equiv \delta(x^{t-1},x^t,p^t,w^t,t) \qquad \equiv [R^t(p^t,w^t,x^t)/R^t(p^t,w^t,x^{t-1})]/[w^t\cdot x^t/w^t\cdot x^{t-1}].$$

$$\delta^t \equiv [\delta_L{}^t\,\delta_P{}^t]^{1/2}$$

Decomposing Value Added Growth for a Sector into Explanatory Factors

Six explanatory growth factors:

1. Change in cost constrained value added efficiency: $\varepsilon^t \equiv e^t/e^{t-1}$
2. Change in output prices: $\alpha(p^{t-1},p^t,w,x,s)$
3. Change in input quantities: $\beta(x^{t-1},x^t,w)$
4. Change in input prices: $\gamma(w^{t-1},w^t,p,x,s)$
5. Changes due to technical progress: $\tau(t-1,t,p,w,x)$
6. Returns to scale measure: $\delta(x^{t-1},x^t,p,w,s)$

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Decomposing Value Added Growth for a Sector into Explanatory Factors

Exact decompositions of observed value added:

$$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \, \alpha_P{}^t \, \beta_L{}^t \, \gamma_{LPP}{}^t \, \delta_L{}^t \, \tau_L{}^t \, ;$$

$$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \, \alpha_L{}^t \, \beta_P{}^t \, \gamma_{PLL}{}^t \, \delta_P{}^t \, \tau_P{}^t .$$

Take the geometric mean of both sides of the above equations to get our preferred decomposition:

$$p^t \cdot y^t / p^{t-1} \cdot y^{t-1} = \varepsilon^t \, \alpha^t \, \beta^t \, \gamma^t \, \delta^t \, \tau^t \, .$$

Can re-organise to get:

$$TFPG^t \equiv \{[p^t \cdot y^t / p^{t-1} \cdot y^{t-1}] / \alpha^t\} / \beta^t = \varepsilon^t \, \gamma^t \, \delta^t \, \tau^t$$

A Nonparametric Approximation to the Cost Constrained Value Added Function

Assume that the production unit's period t production possibilities set S^t is the conical free disposal hull of the period t actual production vector and past production vectors. LP problem:

$$
\begin{aligned}
R^t(p,w,x) &\equiv \max_{\lambda} \{p\cdot(\Sigma_{s=1}^{t}\, y^s\lambda_s) \;;\; w\cdot(\Sigma_{s=1}^{t}\, x^s\lambda_s) \leq w\cdot x \;;\; \lambda_1 \geq 0 ,\ldots, \lambda_t \geq 0\} \\
&= \max_{s} \{p\cdot y^s\, w\cdot x/w\cdot x^s : s = 1,2,\ldots,t\} \\
&= w\cdot x \max_{s} \{p\cdot y^s/w\cdot x^s : s = 1,2,\ldots,t\} \\
&= w\cdot x/\min_{s} \{w\cdot x^s/p\cdot y^s : s = 1,2,\ldots,t\} \\
&= w\cdot x/c^t(w,p)
\end{aligned}
$$

where $c^t(w,p)$ is the *period t nonparametric unit cost function*

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National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

Sectoral value added decomposition, for each sector k:

$$v^{kt}/v^{k,t-1} = \alpha^{kt}\,\beta^{kt}\,\gamma^{kt}\,\delta^{kt}\,\varepsilon^{kt}\,\tau^{kt}$$

Period t share of national value added for sector k: $s^{kt} \equiv v^{kt}/v^{t}$

Can use period t-1 or period t shares to aggregate:

$$v^{t}/v^{t-1} = \Sigma_{k=1}^{K}\, s^{k,t-1}\alpha^{kt}\,\beta^{kt}\,\gamma^{kt}\,\delta^{kt}\,\varepsilon^{kt}\,\tau^{kt}$$

$$v^{t}/v^{t-1} = [\Sigma_{k=1}^{K}\, s^{kt}\,(\alpha^{kt}\,\beta^{kt}\,\gamma^{kt}\,\delta^{kt}\,\varepsilon^{kt}\,\tau^{kt})^{-1}]^{-1}$$

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

Nice! But these exact decompositions don't lead to simple decompositions into *national* explanatory factors.

Define (logarithms of) weighted *national explanatory factors:*

$\ln \alpha^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \alpha^{kt}$;

$\ln \beta^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \beta^{kt}$;

$\ln \gamma^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \gamma^{kt}$;

$\ln \delta^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \delta^{kt}$;

$\ln \varepsilon^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \varepsilon^{kt}$;

$\ln \tau^{t\bullet} \equiv \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln \tau^{kt}$.

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

Use some approximations (drawing on Schlömilch's inequality) to write:

$$\ln v^t/v^{t-1} \approx \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln(v^{kt}/v^{k,t-1})$$

$$= \Sigma_{k=1}^{K} (1/2)(s^{kt} + s^{k,t-1})\ln(\alpha^{kt}\,\beta^{kt}\,\gamma^{kt}\,\delta^{kt}\,\varepsilon^{kt}\,\tau^{kt})$$

$$= \ln \alpha^{t\bullet} + \ln \beta^{t\bullet} + \ln \gamma^{t\bullet} + \ln \delta^{t\bullet} + \ln \varepsilon^{t\bullet} + \ln \tau^{t\bullet}$$

National Total Factor Productivity Growth:

$$\mathrm{TFPG}^t \equiv [v^t/v^{t-1}]/\alpha^{t\bullet}\,\beta^{t\bullet} \approx \gamma^{t\bullet}\,\delta^{t\bullet}\,\varepsilon^{t\bullet}\,\tau^{t\bullet}$$

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National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

- Assume that the technology of each sector can be represented by a translog value added function with the restrictions on technical progress that are described in Diewert and Morrison (1986) and Kohli (1990).
- These papers also assumed constant returns to scale and competitive profit maximizing behavior. Under these assumptions:

$$v^{kt}/v^{k,t-1} = \alpha^{kt}\,\beta^{kt}\,\tau^{kt}$$

where α^{kt} turns out to be the period t Törnqvist value added output price index for sector k and β^{kt} is the period t Törnqvist primary input quantity index for sector k.

$$v^{t}/v^{t-1} \approx \alpha^{t\bullet}\,\beta^{t\bullet}\,\tau^{t\bullet}\ ;$$

Can be implemented using index numbers; i.e. not necessary to have estimates for sectoral best practice functions.

National Value Added Growth Decompositions: The Sectoral Weighted Average Approach

- This is a “bottom up” approach; start at the sector level and aggregate up to the national level.

- Not clear that the correct definition of national $TFPG^t \equiv [v^t/v^{t-1}]/\alpha^{t\bullet}\ \beta^{t\bullet}$ is correct.

- Now look at a “top down” approach.

National Value Added Growth Decompositions: The National Cost Constrained Value Added Function Approach

Sector k share of national best practice value added in period t:

$\sigma^{kt} \equiv R^{kt}(p^{kt},w^{kt},x^{kt})/R^{t}(p^{t},w^{t},x^{t})$

National efficiency Level:

$e^{t} \equiv v^{t}/R^{t}(p^{t},w^{t},x^{t})$

$= \Sigma_{k=1}^{K} \sigma^{kt} e^{kt}$

National efficiency change:

$\varepsilon^{t} \equiv e^{t}/e^{t-1} = [\Sigma_{k=1}^{K} \sigma^{kt} e^{kt}]/[\Sigma_{k=1}^{K} \sigma^{k,t-1} e^{k,t-1}]$

National Value Added Growth Decompositions: The National Cost Constrained Value Added Function Approach

Using a similar approach for other components, and similar definitions as for the explanatory components as before, we get the following exact decomposition of national value added growth:

$$v^t/v^{t-1} = \alpha^t \beta^t \gamma^t \delta^t \varepsilon^t \tau^t$$

Can derive approximations to all six national explanatory factors, so that we get:

$$v^t/v^{t-1} = \alpha^t \beta^t \gamma^t \delta^t \varepsilon^t \tau^t \approx \alpha^{t\bullet} \beta^{t\bullet} \gamma^{t\bullet} \delta^{t\bullet} \varepsilon^{t\bullet} \tau^{t\bullet}$$

Which is the same decomposition that we had for the "bottom up" approach.

TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

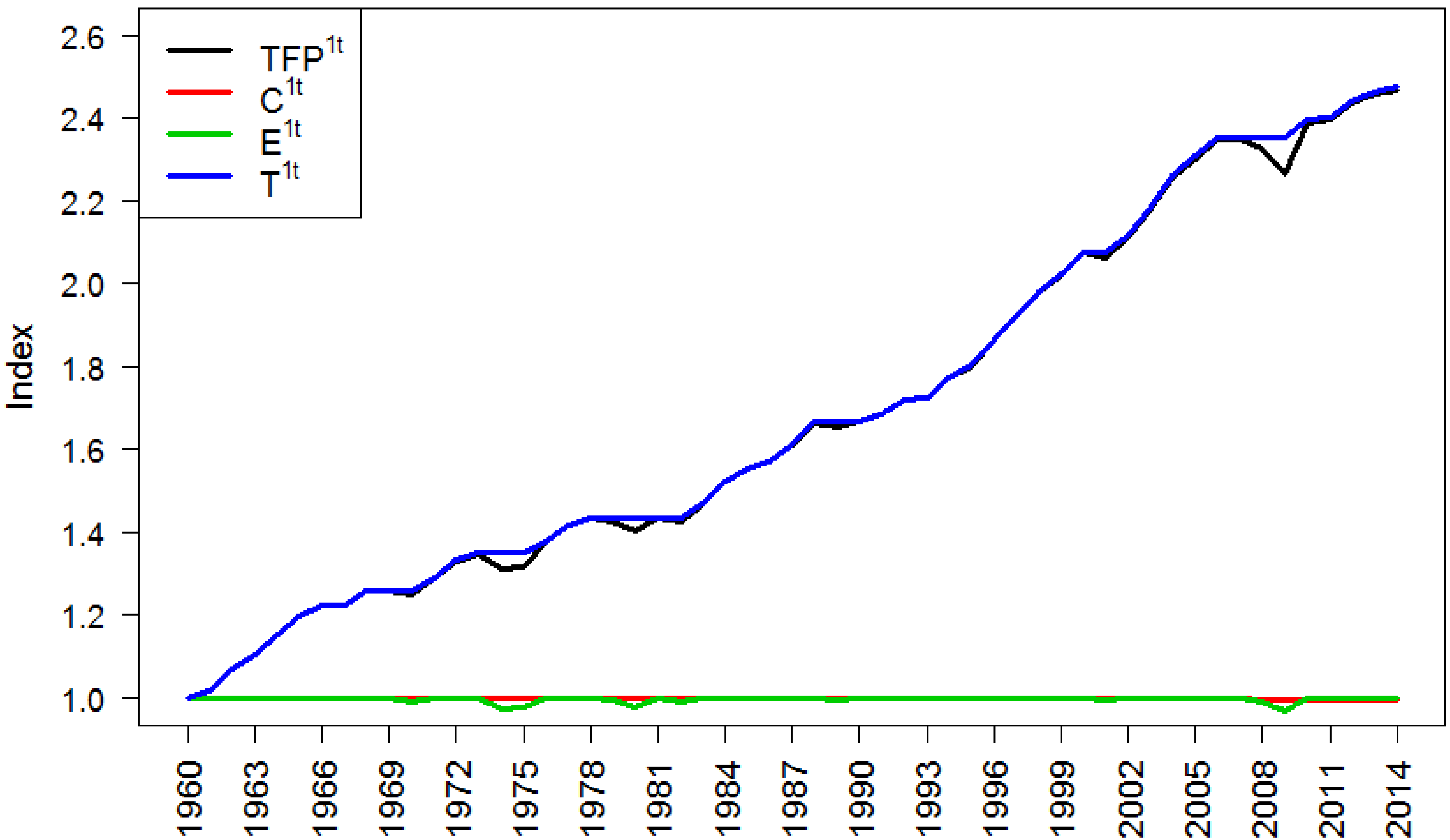
Use the (BEA, BLS, Fed Reserve) Integrated Macroeconomic Accounts to construct a data set for two major sectors of the U.S. economy in Diewert and Fox (2016) :

"Alternative User Costs, Rates of Return and TFP Growth Rates for the US Nonfinancial Corporate and Noncorporate Business Sectors: 1960-2014"

Sector 1: US Corporate Nonfinancial Sector

Sector 2: US Noncorporate Nonfinancial Sector

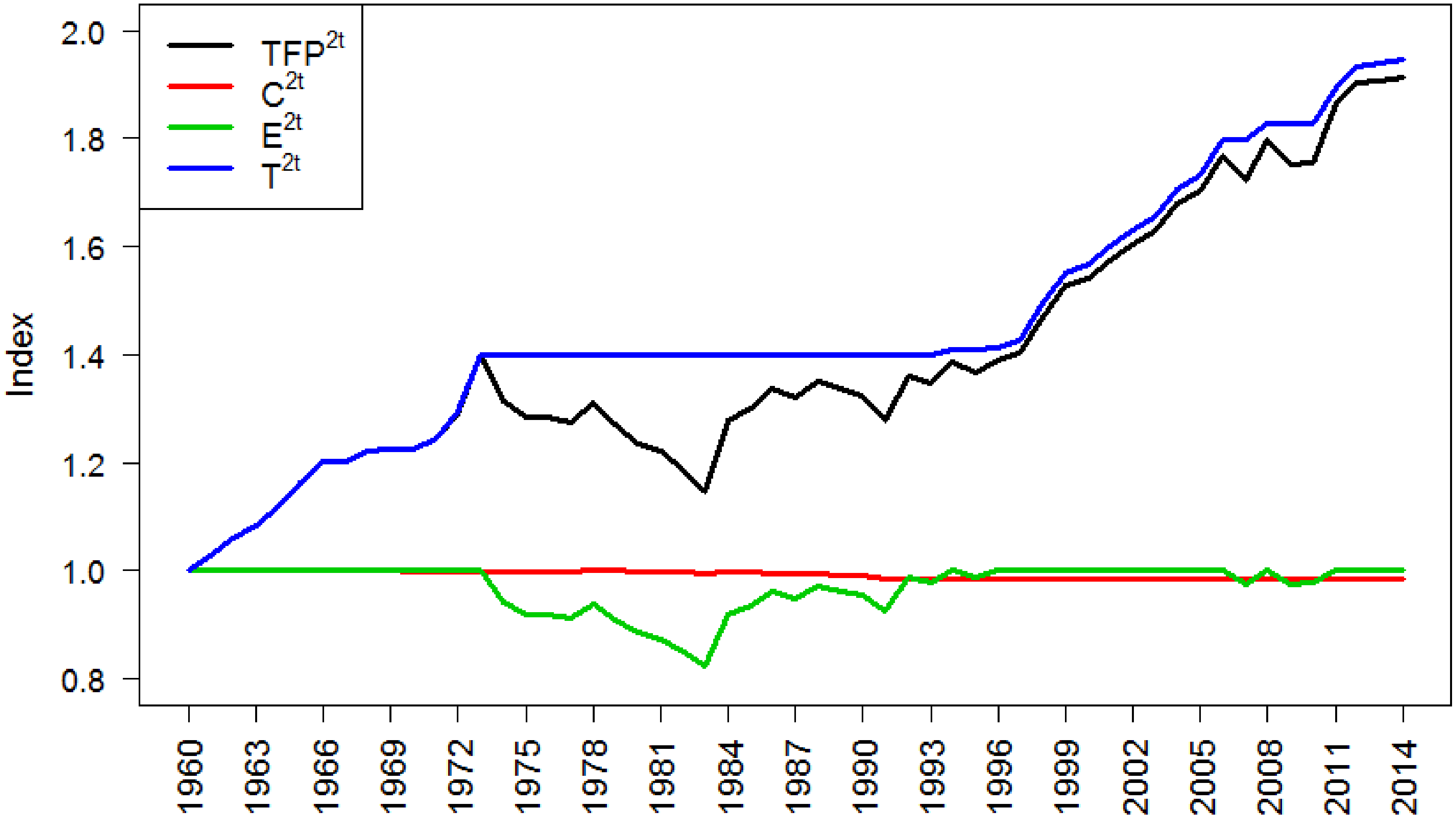
Figure 1: Sector 1 Level of TFP, Input Mix, Value Added Efficiency and Technology



TFP Growth for the U.S. Corporate Nonfinancial Sector, 1960-2014

- There was a substantial decline in value added efficiency over the years 2006-2009
- TFP has grown at a slower than average rate since 2006. The level of TFP also fell in the 1974, 1979, 1982, 1989 and 2001 recessions when efficiency growth dipped below one.
- On the whole, TFP growth in the U.S. Corporate Nonfinancial Sector has been satisfactory.

Figure 2: Sector 2 Level of TFP, Input Mix, Value Added Efficiency and Technology



TFP Growth for the U.S. Noncorporate Nonfinancial Sector, 1960-2014

- The loss of value added efficiency in Sector 2 was massive over the 20 years 1974-1993. This loss of efficiency dragged down the level of Sector 2 TFP over these years. TFP growth resumed in 1994 and was excellent until 2006 when TFP growth again stalled with the exception of two good years of growth in 2011 and 2012.

- Illustrates the adverse influence of recessions when output falls but inputs cannot be adjusted optimally due to the fixity of many capital stock (and labour) components of aggregate input.

- Under these circumstances, production takes place in the interior of the production possibilities set and for Sector 2, the resulting waste of resources was substantial.

Figure 3: Approximate National Level of TFP, Input Mix, Value Added Efficiency and Technology

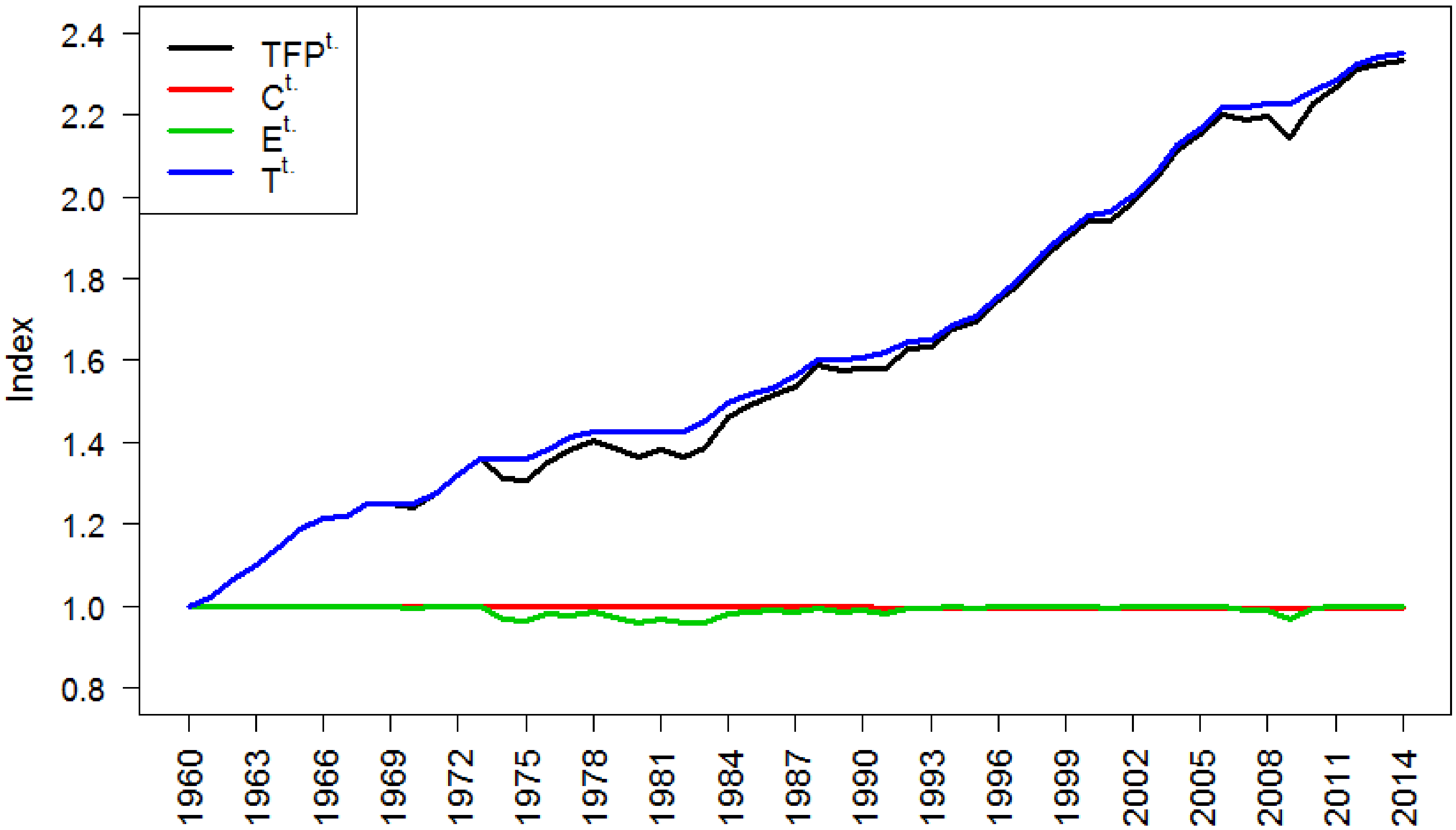


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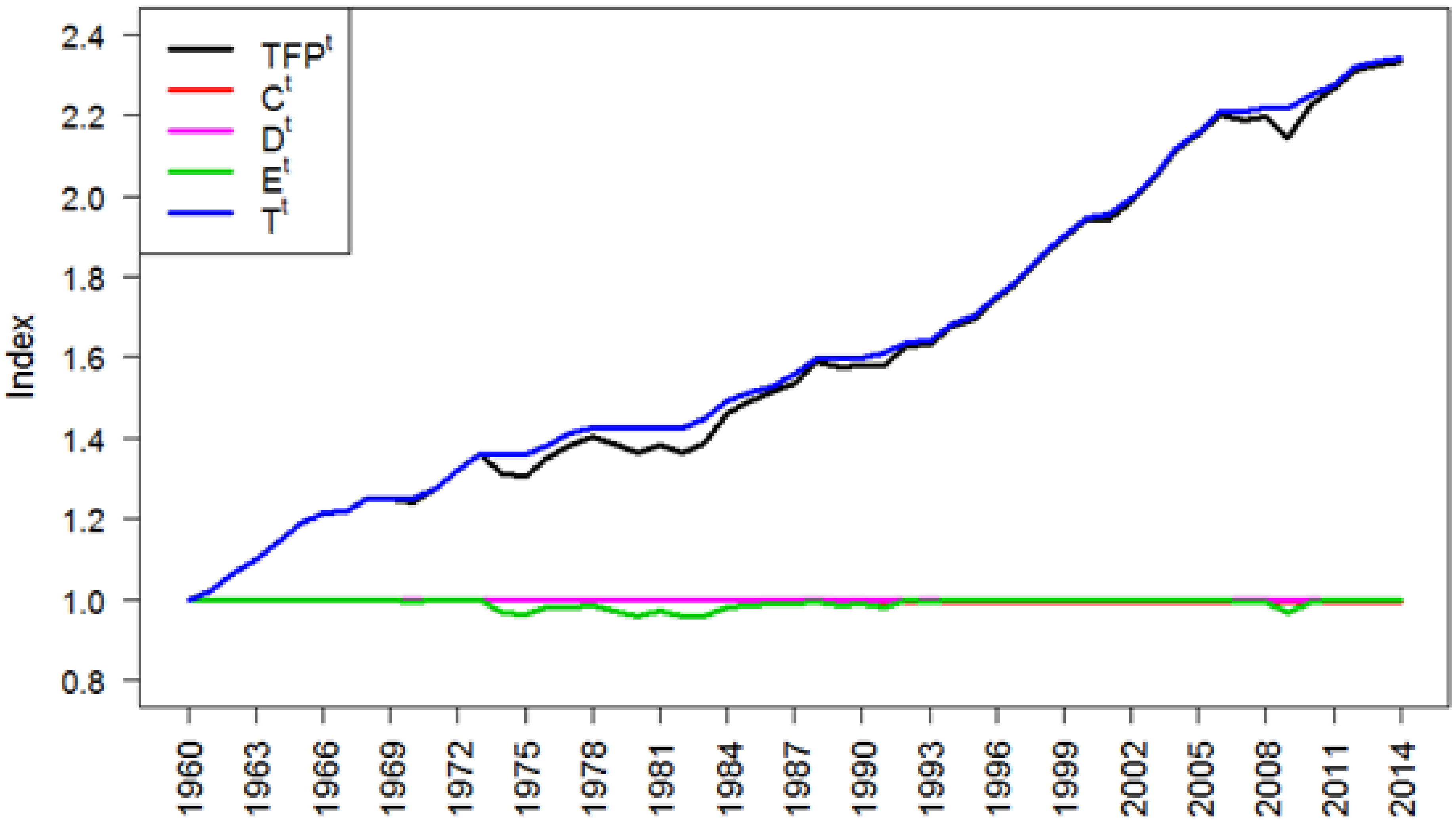
Figure 4: National Level of TFP, Input Mix, Returns to Scale, Value Added Efficiency and Technology



Summary

- Derived decompositions of nominal value added growth (and TFP growth) for a single sector into explanatory factors.

- We also used two alternative approaches to relating the sectoral decompositions to a national growth decomposition:
  - a weighted average sectoral approach and
  - a national value added function approach.

- A main advantage of our new approach is that our new nonparametric measure of technical progress never indicates technical regress.

- During recessions, value added efficiency drops below unity and depresses TFP growth.

Summary

- For our U.S. data set, TFP growth is well explained as the product of value added efficiency growth times the rate of technical progress.
- For the U.S. Noncorporate Nonfinancial Sector, we found that the cost of recessions was particularly high.
- Implementation of the decompositions can provide key insights into the drivers of economic growth at a detailed sectoral level.
- Hence, we believe that they will provide new insights into the sources of economic growth.
- Our decompositions may also indicate data mismeasurement problems that can then be addressed by statistical agencies.