Online Data, Fixed Effects and the Construction of High-Frequency Price Indexes

Jan de Haan[a] **and Rens Hendriks**[b]

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Abstract: Statistics Netherlands has been experimenting with the collection of prices from online stores through web scraping. This paper explores whether the unweighted multilateral time-product dummy, or fixed effects, approach is useful for constructing high-frequency price index numbers from online data. We explain how unmatched (new and disappearing) items are treated and how the time-product dummy index compares to two matched-model price indexes: the chained Jevons index and the multilateral GEKS-Jevons index. We argue that the time-product dummy method is generally preferable to the chained matched-model Jevons method but tends to produce similar, though perhaps slightly less volatile, results as the GEKS-Jevons method. Neither of these methods is suitable for products where quality change is important or where item identifiers, such as web IDs, frequently change. Some examples are provided using data extracted from the website of a Dutch online store.

Key words: fixed-effects models, hedonic quality adjustment, multilateral price index number methods, online prices.

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[a] Division of Process Development, IT and Methodology, Statistics Netherlands; and OTB, Faculty of Architecture and the Built Environment, Delft University of Technology.
Correspondence address: Statistics Netherlands, P.O. Box 24500, 2490 HA The Hague, The Netherlands; email: j.dehaan@cbs.nl.

[b] Division of Economic and Business Statistics and National Accounts, Statistics Netherlands; email: r.hendriks@cbs.nl.

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1. Introduction

Over the past couple of years, Statistics Netherlands has been experimenting with the collection of prices from the Internet through *web scraping*. Online prices could perhaps replace part of the prices observed by price collectors for the compilation of the CPI.[1] Online prices might also replace data that is currently being collected from the Internet in a much less efficient way. Apart from efficiency considerations, web scraping has the advantage that prices can be monitored daily, allowing the estimation of high-frequency price indexes. In the Billion Prices Project, a research initiative at MIT that uses online data to study high-frequency price dynamics and inflation, daily price index numbers have been calculated for several countries around the world, including the Netherlands.[2] For an example on Argentina data, see Cavallo (2012).

Importantly, data on quantities purchased cannot be observed via the Internet. The lack of quantity data is problematic for the construction of price indexes, but the problem is not new to statistical agencies. Weighting information at the item level is generally lacking (unless scanner data is available), and so the agencies are forced to construct unweighted indexes. For each product, the sample of narrowly defined items is typically kept fixed, at least for some time, and the index is based on matched items to compare 'like with like'. When new items are introduced into the sample to replace disappearing items, quality-adjustments should be carried out in order to measure pure price change.

The item samples have traditionally been quite small, particularly to keep things manageable and control costs. A large part of the costs associated with compiling a CPI stems from price collection at the stores. If web scraping turns out to be successful, the costs could be reduced substantially, even when observing all items (displayed on the website) rather than taking small samples. The costs could be further reduced if it were possible to develop a method, including a computer system, where quality adjustments are carried out without manual intervention.

Aizcorbe, Corrado and Doms (2003) claim that it is possible to construct quality-adjusted price indexes without observing any item characteristics. They suggest using a

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[1] Hoekstra, ten Bosch and Harteveld (2012) describe some first experiences with the use of web scraping software, which is part of a broader project at Statistics Netherlands on 'Big Data' (Daas et al., 2011).

[2] The price indexes are currently compiled by PriceStats, a private company; see www. PriceStats.com.

regression model which – instead of including characteristics like in a hedonic model – includes a set of dummy variables indicating the items plus a set of dummy variables indicating the time periods. But their idea sounds too good to be true. Diewert (2004) shows that this method produces a matched-model index in the bilateral case. Silver and Heravi (2005) argue that in the many-period case, the index "will have a tendency to follow the chained matched-model results." De Haan and Krsinich (2012), on the other hand, have found that the *time-product dummy method* did make a difference in the many-period case.

The aim of this paper is threefold: to explain why the multi-period or multilateral time-product dummy index usually differs from its chained matched-model counterpart, to show that the time-product dummy method does not produce quality-adjusted price indexes, and to investigate whether this method is useful for estimating high-frequency price indexes from online data (for goods where quality change is not a major concern). The rest of the paper is structured as follows.

The time-product dummy method can be interpreted as a special case of the time dummy hedonic method, so in section 2 the hedonic method will be discussed in some detail.

Section 3 addresses the relation between the two methods. Essentially, the time-product dummy method is based on a regression model where the hedonic price effects are replaced by item-specific *fixed effects*. This leads to a model where item identifiers are the only 'characteristics' included. An expression for the time-product dummy index in terms of geometric average prices and average fixed effects is derived.

Section 4 discusses the treatment of unmatched (new and disappearing) items in the many-period case. It appears that items with a single price observation in the pooled data set are ignored in the estimation of the time-product dummy index, indicating that this method does not produce a quality-adjusted price index.

In section 5 we argue that the time-product dummy method generates a special type of matched-model price index. We compare the time-product dummy method with an alternative multilateral approach, the GEKS method. The latter method uses all of the matches in the data by taking an average of all possible bilateral price comparisons – in our case using matched-model Jevons indexes – where each period serves as the base. We show that the time-product dummy method and the GEKS-Jevons method basically aim at the same (matched-model) index number formula.

In section 6 we suggest using a rolling window approach to updating the time series and discuss problems that may arise when using daily online price data, including the treatment of regular and sales prices. A related issue is whether the compilation of daily price indexes would be useful.

Section 7 provides some empirical illustrations. Our data set contains daily price observations extracted from the website of a Dutch online retailer for three products: women's T-shirts, men's watches, and kitchen appliances.

Section 8 summarizes our findings and concludes.

2. Time dummy hedonic indexes

A hedonic model explains the price of a product from its (performance) characteristics. Though other functional forms are possible, for convenience we will only consider the log-linear model

$$\ln p_i^t = \delta^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t, \tag{1}$$

where p_i^t denotes the price of item *i* in period *t*; z_{ik} is the (quantity) of characteristic *k* for item *i* and β_k the corresponding parameter; δ^t is the intercept; the random errors ε_i^t have an expected value of zero, constant variance and zero covariance.

The parameters β_k in model (1) are constant across time. Pakes (2003) argues that this is a (too) restrictive assumption,[3] but it allows us to estimate the model on the pooled data of two or more periods, thus increasing efficiency. Suppose we have data for a particular product at our disposal for periods $t = 0,1,...,T$; the samples of items are denoted by $S^0, S^1,..., S^T$ and the corresponding number of items by $N^0, N^1,..., N^T$. The estimating equation for the pooled data becomes

$$\ln p_i^t = \delta^0 + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{k=1}^{K} \beta_k z_{ik} + \varepsilon_i^t, \tag{2}$$

[3] Data permitting, this assumption can be tested. A more flexible method for estimating quality-adjusted price indexes is hedonic imputation where the characteristics parameters are allowed to change over time and the model is estimated separately in each time period. Starting from some preferred index number formula, the 'missing prices' are imputed using the predicted prices from the hedonic regressions. For a comparison of time dummy and imputation approaches, see Silver and Heravi (2007), Diewert, Heravi and Silver (2009), and de Haan (2010).

where the time dummy variable D_i^t has the value 1 if the observation pertains to period t and the value 0 otherwise; the time dummy parameters δ^t shift the hedonic surface upwards or downwards as compared with the intercept term δ^0. The method is usually referred to as the *time dummy method*.

Suppose equation (2) is estimated by Ordinary Least Squares (OLS) regression, yielding parameter estimates $\hat{\delta}^0$, $\hat{\delta}^t$ $(t = 1,...,T)$ and $\hat{\beta}_k$ $(k = 1,...,K)$.[4] Since changes in the item characteristics are controlled for, $\exp(\hat{\delta}^t)$ is an estimator of quality-adjusted aggregate price change going from the base period 0 to each comparison period *t*.[5] An explicit expression for $\exp(\hat{\delta}^t)$ can be derived in the following manner. The predicted prices of item i in the base period 0 and the comparison periods t are

$$\hat{p}_i^0 = \exp(\hat{\delta}^0)\exp\left[\sum_{k=1}^{K}\hat{\beta}_k z_{ik}\right]; \tag{3}$$

$$\hat{p}_i^t = \exp(\hat{\delta}^0)\exp(\hat{\delta}^t)\exp\left[\sum_{k=1}^{K}\hat{\beta}_k z_{ik}\right]; \qquad (t = 1,...,T). \tag{4}$$

Taking the geometric mean of the predicted prices for all items belonging to the samples S^0 and $S^1,...,S^T$, respectively, yields

$$\prod_{i\in S^0}(\hat{p}_i^0)^{\frac{1}{N^0}} = \exp(\hat{\delta}^0)\exp\left[\sum_{k=1}^{K}\hat{\beta}_k \sum_{i\in S^0} z_{ik} / N^0\right]; \tag{5}$$

$$\prod_{i\in S^t}(\hat{p}_i^t)^{\frac{1}{N^t}} = \exp(\hat{\delta}^0)\exp(\hat{\delta}^t)\exp\left[\sum_{k=1}^{K}\hat{\beta}_k \sum_{i\in S^t} z_{ik} / N^t\right]; \qquad (t = 1,...,T). \tag{6}$$

Dividing (6) by (5) and some rearranging gives

$$\exp(\hat{\delta}^t) = \frac{\prod_{i\in S^t}(\hat{p}_i^t)^{\frac{1}{N^t}}}{\prod_{i\in S^0}(\hat{p}_i^0)^{\frac{1}{N^0}}} \frac{\exp\left[\sum_{k=1}^{K}\hat{\beta}_k \sum_{i\in S^0} z_{ik} / N^0\right]}{\exp\left[\sum_{k=1}^{K}\hat{\beta}_k \sum_{i\in S^t} z_{ik} / N^t\right]} = \frac{\prod_{i\in S^t}(\hat{p}_i^t)^{\frac{1}{N^t}}}{\prod_{i\in S^0}(\hat{p}_i^0)^{\frac{1}{N^0}}} \exp\left[\sum_{k=1}^{K}\hat{\beta}_k (\bar{z}_k^0 - \bar{z}_k^t)\right], \tag{7}$$

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[4] Under the classical assumptions, OLS regression will suffice. However, estimating a time dummy model by OLS produces an unweighted price index, which is undesirable from an index number point of view. When quantity or expenditure information at the item level is available, weighted least squares regression is preferable, even if it introduces some heteroskedasticity, because a weighted index results.

[5] The estimator is not unbiased, but the bias is often negligible in practice. For bias correction terms, see Kennedy (1981) or van Garderen and Shah (2002).

where $\bar{z}_k^0 = \sum_{i \in S^0} z_{ik} / N^0$ and $\bar{z}_k^t = \sum_{i \in S^t} z_{ik} / N^t$ are the unweighted sample means of characteristic *k*. Due to the inclusion of time dummies and an intercept into the model, the OLS residuals sum to zero in each period so that $\prod_{i \in S^0} (\hat{p}_i^0)^{1/N^0} = \prod_{i \in S^0} (p_i^0)^{1/N^0}$ and $\prod_{i \in S^t} (\hat{p}_i^t)^{1/N^t} = \prod_{i \in S^t} (p_i^t)^{1/N^t}$. Expression (7) can therefore be written as

$$P_{TD}^{0t} = \exp(\hat{\delta}^t) = \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k (\bar{z}_k^0 - \bar{z}_k^t)\right]; \quad (t = 1,...,T). \tag{8}$$

The exponential factor in equation (8) adjusts the ratio of geometric mean prices for any changes in the average characteristics between period *t* and the base period 0. If $\bar{z}_k^0 = \bar{z}_k^t$, then the quality-adjustment factor equals 1 and the index simplifies to the ratio of geometric mean prices. A specific instance of this condition holds when the samples S^0 and S^t coincide so that all items are matched. In this case the time dummy index is equal to the matched-model Jevons price index, and modelling becomes unnecessary.

In equation (2), period 0 serves as the base and a dummy variable for this period was excluded to identify the model (to prevent perfect multicollinearity). But regression theory tells us that the $\hat{\beta}_k$ are independent of the choice of base period. Hence, the time dummy index is *transitive* and can be written as a period-on-period chained index:

$$P_{TD}^{0t} = \prod_{\tau=1}^{t} \frac{\prod_{i \in S^\tau} (p_i^\tau)^{\frac{1}{N^\tau}}}{\prod_{i \in S^{\tau-1}} (p_i^{\tau-1})^{\frac{1}{N^{\tau-1}}}} \exp\left[\sum_{k=1}^{K} \hat{\beta}_k (\bar{z}_k^{\tau-1} - \bar{z}_k^\tau)\right]; \qquad (t = 1,...,T). \tag{9}$$

It is easily checked that expressions (9) and (8) are indeed equivalent.

A similar expression is obtained if bilateral time dummy indexes were estimated on the pooled data for adjacent periods and subsequently chained. The difference would be that the estimated characteristics parameters are not kept fixed over the entire period $0,...,T$ but will differ across the various links of the chain. An often-heard argument for chaining is that it maximizes the set of matched items and reduces the need for quality adjustments. However, although the set of matched items typically shrinks in the course of time, the argument is not relevant here because the structure of the direct multilateral index is similar to the structure of the chained bilateral index. It is only the difference between the estimated parameters from the two approaches that matters.

3. Time-product dummy indexes

In the time dummy model (1), both the characteristics of an item and the parameters are assumed constant over time. This implies that their combined effect on the log of price is also constant over time. If information on item characteristics is not available, it may be worthwhile to replace the unobservable hedonic effects $\sum_{k=1}^{K} \beta_k z_{ik}$ by item-specific fixed values γ_i. This leads to the *fixed-effects* model

$$\ln p_i^t = \delta^t + \gamma_i + \varepsilon_i^t . \tag{10}$$

Suppose across the entire period $0,...,T$ we observe N different items, many of which may not be available in every time period. The estimating equation for a pooled regression corresponding to (10) is

$$\ln p_i^t = \alpha + \sum_{t=1}^{T} \delta^t D_i^t + \sum_{i=1}^{N-1} \gamma_i D_i + \varepsilon_i^t , \tag{11}$$

where D_i is a dummy variable that has the value of 1 if the observation relates to item i and 0 otherwise. A dummy for an arbitrary item N is not included (so $\alpha = \delta^0 + \gamma_N$) in order to identify the model. The OLS parameter estimates are $\hat{\alpha}$, $\hat{\delta}^t$ $(t = 1,...,T)$ and $\hat{\gamma}_i$ $(i = 1,..., N - 1)$. Note that while items with identical characteristics have identical fixed effects γ_i, the estimates $\hat{\gamma}_i$ will generally differ.[6]

Model (11) is the intertemporal counterpart of the well-known country-product dummy model for estimating price indexes across countries.[7] Following de Haan and Krsinich (2012), we refer to (11) as the *time-product dummy* model. It has been used by various researchers to estimate price indexes across time, including Aizcorbe, Corrado and Doms (2003), Ivancic, Fox and Diewert (2009), Krsinich (2011a,b), de Haan and Krsinich (2012), and Krsinich (2013).[8] From a statistical point of view, the time-product

[6] It is not possible to include both characteristics and product dummies as the model will not be identified; the vector of values for any characteristic can be written as a linear combination of the *N*-1 vectors for the product dummies and the intercept.

[7] The country-product dummy method is due to Summers (1973). Diewert (1999) and Balk (2001) review the different approaches to international price comparisons.

[8] Balk (1980) discusses a weighted version of this model in the context of constructing price indexes for seasonal goods. Aizcorbe, Corrado and Doms (2003) use OLS to estimate equation (11) whereas the other authors listed here use expenditure-share weighted least squares. See the Appendix for details of the latter approach.

dummy method is less efficient than the hedonic time dummy method because more parameters have to be estimated. The time-product dummy method is cost efficient in that there is no need to collect information on item characteristics.

In order to derive an explicit expression for the time-product dummy index, we can follow the same steps as in section 2. For $i = 1,...,N-1$, the predicted prices in the base period 0 and the comparison periods t $(t = 1,...,T)$ are $\hat{p}_i^0 = \exp(\hat{\alpha})\exp(\hat{\gamma}_i)$ and $\hat{p}_i^t = \exp(\hat{\alpha})\exp(\hat{\delta}^t)\exp(\hat{\gamma}_i)$, respectively, while for $i = N$ we have $\hat{p}_N^0 = \exp(\hat{\alpha})$ and $\hat{p}_N^t = \exp(\hat{\alpha})\exp(\hat{\delta}^t)$. By setting $\hat{\gamma}_N = 0$ we can simply write $\hat{p}_i^0 = \exp(\hat{\alpha})\exp(\hat{\gamma}_i)$ and $\hat{p}_i^t = \exp(\hat{\alpha})\exp(\hat{\delta}^t)\exp(\hat{\gamma}_i)$ for all *i*. Taking the geometric mean of the predicted prices across all *i* yields

$$\prod_{i \in S^0} (\hat{p}_i^0)^{\frac{1}{N^0}} = \exp(\hat{\alpha})\exp\left[\sum_{i \in S^0} \hat{\gamma}_i / N^0\right]; \tag{12}$$

$$\prod_{i \in S^t} (\hat{p}_i^t)^{\frac{1}{N^t}} = \exp(\hat{\alpha})\exp(\hat{\delta}^t)\exp\left[\sum_{i \in S^t} \hat{\gamma}_i / N^t\right]; \qquad (t = 1,...,T). \tag{13}$$

Dividing (13) by (12), some rearranging and using $\prod_{i \in S^0} (\hat{p}_i^0)^{1/N^0} = \prod_{i \in S^0} (p_i^0)^{1/N^0}$ and $\prod_{i \in S^t} (\hat{p}_i^t)^{1/N^t} = \prod_{i \in S^t} (p_i^t)^{1/N^t}$ gives for $t = 1,...,T$

$$P_{TPD}^{0t} = \exp(\hat{\delta}^t) = \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \frac{\exp\left[\sum_{i \in S^0} \hat{\gamma}_i / N^0\right]}{\exp\left[\sum_{i \in S^t} \hat{\gamma}_i / N^t\right]} = \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\bar{\hat{\gamma}}^0 - \bar{\hat{\gamma}}^t\right], \tag{14}$$

where $\bar{\hat{\gamma}}^0 = \sum_{i \in S^0} \hat{\gamma}_i / N^0$ and $\bar{\hat{\gamma}}^t = \sum_{i \in S^t} \hat{\gamma}_i / N^t$ are the sample means of the estimated fixed effects, with $\hat{\gamma}_N = 0$.

An important question is whether the time-product dummy method generates a quality-adjusted price index, i.e. whether it properly accounts for new and disappearing items. In other words, apart from random disturbances, does the factor $\exp[\bar{\hat{\gamma}}^0 - \bar{\hat{\gamma}}^t]$ in (14) approximate the quality-adjustment factor $\exp[\sum_{k=1}^{K} \hat{\beta}_k (\bar{z}_k^0 - \bar{z}_k^t)]$ in equation (8) well? In section 5 we argue that this is not the case[9].

[9] This could be empirically shown if information on characteristics was available. Unfortunately, data sets based on web scraping typically do not contain this type of information. Although many websites show some characteristics, it is often difficult to extract and put them into the required format for use in hedonic regressions. Also, different websites tend to show different sets of characteristics for a particular product, making it difficult to attain consistency across online stores.

We will first examine what drives the difference between the unweighted time-product dummy index and the chained matched-model Jevons index. The time-product dummy method is a special case of the time dummy method, and so the time-product dummy index (14) can be expressed as a chain index, similar to equation (9):

$$P_{TPD}^{0t} = \prod_{\tau=1}^{t} \frac{\prod_{i \in S^{\tau}} (p_i^{\tau})^{\frac{1}{N^{\tau}}}}{\prod_{i \in S^{\tau-1}} (p_i^{\tau-1})^{\frac{1}{N^{\tau-1}}}} \exp\left[\bar{\bar{\gamma}}^{\tau-1} - \bar{\bar{\gamma}}^{\tau}\right]; \qquad (t = 1,\ldots,T). \tag{15}$$

In section 4 below, we decompose a single chain link in (15) into the adjacent-period matched-model Jevons index and two factors representing the effects of new items and disappearing items.

4. Unmatched items and the time-product dummy index

To analyse the impact of unmatched items, we need some additional notation. Consider adjacent periods $t-1$ and t. The total set of items in period $t-1$ is $S^{t-1} = S_M^{t-1,t} \cup S_D^{t-1,t}$, where $S_M^{t-1,t}$ denotes the subset of matched items between periods $t-1$ and t and $S_D^{t-1,t}$ the subset of 'disappearing' items (which may appear again later). Similarly, the total set of items in period t is $S^t = S_M^{t-1,t} \cup S_N^{t-1,t}$, where $S_N^{t-1,t}$ denotes the subset of 'new' items (observed in period t but not in $t-1$). We denote the size of the respective sets by N^{t-1}, N^t, $N_M^{t-1,t}$, $N_D^{t-1,t} = N^{t-1} - N_M^{t-1,t}$, and $N_N^{t-1,t} = N^t - N_M^{t-1,t}$. A single chain link in equation (15) for the time-product dummy index can now be decomposed as

$$\frac{P_{TPD}^{0t}}{P_{TPD}^{0,t-1}} = \prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{p_i^{t-1}} \right)^{\frac{1}{N_M^{t-1,t}}} \left[\frac{\prod_{i \in S_N^{t-1,t}} (p_i^t)^{\frac{1}{N_N^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} (p_i^t)^{\frac{1}{N_M^{t-1,t}}}} \right]^{f_N^{t-1,t}} \left[\frac{\prod_{i \in S_D^{t-1,t}} (p_i^{t-1})^{\frac{1}{N_D^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} (p_i^{t-1})^{\frac{1}{N_M^{t-1,t}}}} \right]^{-f_D^{t-1,t}} \exp\left[\bar{\bar{\gamma}}^{t-1} - \bar{\bar{\gamma}}^{t}\right]. \tag{16}$$

The first bracketed factor in (16) shows the ratio of the geometric average period t prices of the new and matched items, raised to the power of $f_N^{t-1,t} = N_N^{t-1,t} / N^t$ (i.e., the fraction of new items), and the second bracketed factor equals the inverse ratio of the geometric average period $t-1$ prices of the disappearing and matched items, raised to the power of $f_D^{t-1,t} = N_D^{t-1,t} / N^{t-1}$ (the fraction of disappearing items). The factor with the average fixed effects can be written as

$$\exp\left[\bar{\hat{\gamma}}^{t-1} - \bar{\hat{\gamma}}^{t}\right] = \left[\frac{\prod_{i \in S_N^{t-1,t}} [\exp(\hat{\gamma}_i)]^{\frac{1}{N_N^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} [\exp(\hat{\gamma}_i)]^{\frac{1}{N_M^{t-1,t}}}} \right]^{-f_N^{t-1,t}} \left[\frac{\prod_{i \in S_D^{t-1,t}} [\exp(\hat{\gamma}_i)]^{\frac{1}{N_D^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} [\exp(\hat{\gamma}_i)]^{\frac{1}{N_M^{t-1,t}}}} \right]^{f_D^{t-1,t}} . \tag{17}$$

Substituting (17) into (16) yields a decomposition of the kind we are looking for:

$$\frac{P_{TPD}^{0t}}{P_{TPD}^{0,t-1}} = \prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{p_i^{t-1}} \right)^{\frac{1}{N_M^{t-1,t}}} \left[\frac{\prod_{i \in S_N^{t-1,t}} \left(\frac{p_i^t}{\exp(\hat{\gamma}_i)} \right)^{\frac{1}{N_N^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{\exp(\hat{\gamma}_i)} \right)^{\frac{1}{N_M^{t-1,t}}}} \right]^{f_N^{t-1,t}} \left[\frac{\prod_{i \in S_D^{t-1,t}} \left(\frac{p_i^{t-1}}{\exp(\hat{\gamma}_i)} \right)^{\frac{1}{N_D^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^{t-1}}{\exp(\hat{\gamma}_i)} \right)^{\frac{1}{N_M^{t-1,t}}}} \right]^{-f_D^{t-1,t}} . \tag{18}$$

If the fixed effects approximate the quality differences well, then the ratios $p_i^t / \exp(\hat{\gamma}_i)$ and $p_i^{t-1} / \exp(\hat{\gamma}_i)$ are ‘quality-adjusted prices’, normalized with respect to the base item *N*. Expression (18) shows, for example, that new items have an upward effect compared with the adjacent-period matched-model Jevons index when their geometric average of (normalized) quality-adjusted prices is higher than that of the matched items. The larger the fraction $f_N^{t-1,t}$ of new items, the bigger this effect will be.

There is no reason to expect that the effects of the new and disappearing items in (18) are both equal to 1 or cancel each other out. Take for example clothing. The prices of most clothing items decline over time, so a chained matched-model index will have a downward trend. If the time-product dummy method would work well, the unmatched items are likely to counter this trend since we expect the average quality-adjusted price of new (disappearing) items to be above (below) the average quality-adjusted price of the matched items.[10] In that case the bracketed factors in (18) tend to be greater than 1 and the resulting index does not necessarily follow the chained Jevons index.

[10] Using a U.S. scanner data set, Greenlees and McClelland (2010) show that a chained matched-model price index for misses’ tops has a strong downward trend, which is eliminated when hedonic regression is used. A problem with fashion goods such as clothing is that fashion itself may be regarded as a quality-determining feature, so that part of the price decline could be attributed to deterioration in quality. But even if they wanted to, statistical agencies are unable to measure fashion effects. Seasonality in itself is obviously another problem. For different approaches to the treatment of clothing in a CPI, see *Consumer Price Index Manual: Theory and Practice* (ILO et al., 2004).

Now recall that $\hat{p}_i^t = \exp(\hat{\alpha})\exp(\hat{\delta}^t)\exp(\hat{\gamma}_i)$ or $\exp(\hat{\gamma}_i) = \hat{p}_i^t / [\exp(\hat{\alpha})\exp(\hat{\delta}^t)]$, and therefore also $\exp(\hat{\gamma}_i) = \hat{p}_i^{t-1} / [\exp(\hat{\alpha})\exp(\hat{\delta}^{t-1})]$. Substituting these results into the first factor and second factor between square brackets of (18), respectively, gives

$$\frac{P_{TPD}^{0t}}{P_{TPD}^{0,t-1}} = \prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{p_i^{t-1}} \right)^{\frac{1}{N_M^{t-1,t}}} \left[\frac{\prod_{i \in S_N^{t-1,t}} \left(\frac{p_i^t}{\hat{p}_i^t} \right)^{\frac{1}{N_N^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{\hat{p}_i^t} \right)^{\frac{1}{N_M^{t-1,t}}}} \right]^{f_N^{t-1,t}} \left[\frac{\prod_{i \in S_D^{t-1,t}} \left(\frac{p_i^{t-1}}{\hat{p}_i^{t-1}} \right)^{\frac{1}{N_D^{t-1,t}}}}{\prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^{t-1}}{\hat{p}_i^{t-1}} \right)^{\frac{1}{N_M^{t-1,t}}}} \right]^{-f_D^{t-1,t}} . \qquad (19)$$

According to (19), new items will have an upward effect when their average regression residuals are greater than those of the matched items in period *t*, i.e., when their prices are on average unusually high. Decomposition (19) is a well-known result. It holds for any (OLS) multilateral time dummy index and can be directly derived from the fact that the regression residuals sum to zero in each period.

Equation (19) does clarify the role of items which are observed only once during the whole period $0,...,T$. By definition these are unmatched items. When using hedonic regression, they affect measured price change, as they should, but when using the time-product dummy method, they do not. To understand why this is the case, recall that the OLS regression residuals for all $i \in S^{t-1}$ and $i \in S^t$ sum to zero. Because each item has its own dummy variable, the residuals also sum to zero per item. Moreover, for items observed in one period only, there is just a single observation in the data set, implying $\hat{p}_i^{t-1} = p_i^{t-1}$ and $\hat{p}_i^t = p_i^t$ for some $i \in S_D^{t-1,t}$ and $i \in S_N^{t-1,t}$.

When only two time periods, 0 and 1, are considered, equation (19) describes the decomposition of the bilateral time-product dummy index. In this simple case we have $\hat{p}_i^0 = p_i^0$ for all disappearing items and $\hat{p}_i^1 = p_i^1$ for all new items, so the numerators of both bracketed factors in (19) are then equal to 1. The denominators are also equal to 1 because the residuals for the matched items, which are observed in periods 0 and 1, also sum to zero in each time period. Thus, the bilateral time-product dummy index equals the matched-model Jevons price index.

Diewert (2004) has shown this result earlier in the context of two countries.[11] He notes that the method “can deal with situations where say item n* has transactions in one country but not the other” and that “the prices of item n* will be zeroed out”. The

[11] Silver and Heravi (2005) have done the same for two time periods.

fact that, while their fixed effects can be estimated, items with a single observation are zeroed out in the two-period case, carries over to the many-period case. This does not mean that a chained matched-model Jevons index results, as we have seen. Items which are 'new' or 'disappearing' in comparisons of adjacent periods are typically observed multiple times during $0,...,T$ and are not zeroed out. They contain information on price change that is used in a multilateral time-product dummy regression whereas they are ignored in a chained matched-model index.

5. A comparison with the GEKS-Jevons index

The fixed effects in a time-product dummy model can be seen as item-specific hedonic price effects, assuming the parameters of the characteristics in the underlying log-linear hedonic model are constant across time. This leads Aizcorbe, Corrado and Doms (2003) and Krsinich (2013) to believe that the time-product dummy method produces a quality-adjusted price index. But measuring quality-adjusted price indexes without information on item characteristics is just not possible. This is almost trivial from a modelling point of view. In a hedonic model, the exponentiated time dummy coefficients are estimates of quality-adjusted price indexes since we control for changes in the characteristics. In the time-product dummy model, there is nothing to control for as *auxiliary information* on characteristics is not included.

The exponentiated time dummy coefficients in the time-product dummy method do not measure quality-adjusted price change but represent a particular type of matched-model price change. In this section, we will compare the unweighted multilateral time-product dummy method to a competing transitive approach, the unweighted multilateral GEKS method. It will be shown that if a time dummy hedonic model holds true, which is the basic assumption underlying the time-product dummy method, the two methods basically aim at the same matched-model index number formula.

The *GEKS method* was designed for making transitive price comparisons across countries. Ivancic, Diewert and Fox (2011) have adapted the GEKS method to construct transitive comparisons across time.[12] The GEKS index going from period 0 to period t $(t = 0,...,T)$ can be expressed as

[12] For applications on Dutch and New Zealand scanner data, see de Haan and van der Grient (2011) and de Haan and Krsinich (2012), respectively.

$$P_{GEKS}^{0t} = \prod_{l=0}^{T} \left(P^{0l} \times P^{lt} \right)^{\frac{1}{T+1}}, \tag{20}$$

where P^{0l} and P^{lt} are bilateral price indexes between periods 0 and *l* and periods *l* and *t*. Period l $(l = 0,...,T)$ serves as the link period or base period for the various bilateral comparisons. In its standard form, the GEKS method uses bilateral *matched-model* price indexes.

When quantity data is available, as in scanner data, superlative bilateral indexes such as the Fisher or Törnqvist should be used. If quantity data is lacking, as with online data, Jevons indexes can be used instead. Superlative as well as Jevons indexes satisfy the time reversal test, which is a prerequisite for the GEKS method. In this section we focus on bilateral Jevons indexes, so in (20) we have

$$P^{0l} = P_J^{0l} = \prod_{i \in S_M^{0l}} \left(\frac{p_i^l}{p_i^0} \right)^{\frac{1}{N_M^{0l}}}, \tag{21}$$

$$P^{lt} = P_J^{lt} = \prod_{i \in S_M^{lt}} \left(\frac{p_i^t}{p_i^l} \right)^{\frac{1}{N_M^{lt}}}, \tag{22}$$

where S_M^{0l} and S_M^{lt} denote the matched samples between the respective periods with sizes N_M^{0l} and N_M^{lt}. Note that $P_J^{00} = P_J^{tt} = 1$, as required. Thus, the GEKS-Jevons[13] price index (20) can be written as

$$P_{GEKS-J}^{0t} = \prod_{\substack{l=1 \\ l \neq t}}^{T} \left(P_J^{0l} \times P_J^{lt} \times 1 \times P_J^{0t} \times P_J^{0t} \times 1 \right)^{\frac{1}{T+1}} = \prod_{\substack{l=1 \\ l \neq t}}^{T} \left(P_J^{0l} \right)^{\frac{1}{T+1}} \prod_{\substack{l=1 \\ l \neq t}}^{T} \left(P_J^{lt} \right)^{\frac{1}{T+1}} \left(P_J^{0t} \right)^{\frac{2}{T+1}}, \tag{23}$$

showing that P_J^{0t}, the bilateral index going directly from period 0 to period *t*, 'counts twice'.

In section 4 we have seen that the price index arising from an unweighted time-product dummy regression on the pooled data of two periods simplifies to the bilateral matched-model Jevons price index. This means we can write the bilateral price indexes (21) and (22) as time-product dummy indexes, which facilitates a comparison with the multilateral time-product dummy index. That is, in equation (23) we set $P_J^{0l} = P_{TPD(0,l)}^{0l}$, $P_J^{lt} = P_{TPD(l,t)}^{lt}$ and $P_J^{0t} = P_{TPD(0,t)}^{0t}$, where (0,*l*), (*l*,*t*) and (0,*t*) refer to bilateral comparisons

[13] Following Balk (2008), we refer to the GEKS method as the procedure to obtain transitivity, no matter what type of bilateral index is used, and write "GEKS-Jevons" when bilateral Jevons price indexes enter the computation.

between periods 0 and l, periods l and t, and periods 0 and t. From section 4 it follows that

$$P_{TPD(0,l)}^{0l} = \frac{\prod_{i \in S^l} (p_i^l)^{\frac{1}{N^l}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\bar{\hat{\gamma}}_{(0,l)}^{0} - \bar{\hat{\gamma}}_{(0,l)}^{l}\right]; \qquad (24)$$

$$P_{TPD(l,t)}^{lt} = \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^l} (p_i^l)^{\frac{1}{N^l}}} \exp\left[\bar{\hat{\gamma}}_{(l,t)}^{l} - \bar{\hat{\gamma}}_{(l,t)}^{t}\right]; \qquad (25)$$

$$P_{TPD(0,t)}^{0t} = \frac{\prod_{i \in S^t} (p_0^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\bar{\hat{\gamma}}_{(0,t)}^{0} - \bar{\hat{\gamma}}_{(0,t)}^{t}\right], \qquad (26)$$

where we have $\bar{\hat{\gamma}}_{(0,l)}^{0} = \sum_{i \in S^0} \hat{\gamma}_{i(0,l)} / N^0$, $\bar{\hat{\gamma}}_{(0,l)}^{l} = \sum_{i \in S^l} \hat{\gamma}_{i(0,l)} / N^l$, $\bar{\hat{\gamma}}_{(l,t)}^{l} = \sum_{i \in S^l} \hat{\gamma}_{i(l,t)} / N^l$, $\bar{\hat{\gamma}}_{(l,t)}^{t} = \sum_{i \in S^t} \hat{\gamma}_{i(l,t)} / N^t$, $\bar{\hat{\gamma}}_{(0,t)}^{0} = \sum_{i \in S^0} \hat{\gamma}_{i(0,t)} / N^0$ and $\bar{\hat{\gamma}}_{(0,t)}^{t} = \sum_{i \in S^t} \hat{\gamma}_{i(0,t)} / N^t$. These are the sample averages of the estimated fixed effects we would find when estimating the bilateral time-product dummy model by OLS regression on the pooled data of periods 0 and l, l and t, and 0 and t, respectively.

After substituting equations (24), (25) and (26) into (23), we find the following expression for the GEKS-Jevons index:

$$P_{GEKS-J}^{0t} = \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \prod_{\substack{l=1 \\ l \neq t}}^{T} \left\{\exp\left[\bar{\hat{\gamma}}_{(0,l)}^{0} - \bar{\hat{\gamma}}_{(l,t)}^{t}\right]\right\}^{\frac{1}{T+1}} \prod_{\substack{l=1 \\ l \neq t}}^{T} \left\{\exp\left[\bar{\hat{\gamma}}_{(l,t)}^{l} - \bar{\hat{\gamma}}_{(0,l)}^{l}\right]\right\}^{\frac{1}{T+1}} \left\{\exp\left[\bar{\hat{\gamma}}_{(0,t)}^{0} - \bar{\hat{\gamma}}_{(0,t)}^{t}\right]\right\}^{\frac{2}{T+1}}$$

$$= \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\sum_{l=1;l \neq t}^{T} \frac{\bar{\hat{\gamma}}_{(0,l)}^{0}}{T+1} - \sum_{l=1;l \neq t}^{T} \frac{\bar{\hat{\gamma}}_{(l,t)}^{t}}{T+1}\right] \exp\left[\sum_{l=1;l \neq t}^{T} \frac{\bar{\hat{\gamma}}_{(l,t)}^{l}}{T+1} - \sum_{l=1;l \neq t}^{T} \frac{\bar{\hat{\gamma}}_{(0,l)}^{l}}{T+1}\right] \left\{\exp\left[\frac{\bar{\hat{\gamma}}_{(0,t)}^{0}}{T+1} - \frac{\bar{\hat{\gamma}}_{(0,t)}^{t}}{T+1}\right]\right\}^{2}$$

$$= \frac{\prod_{i \in S^t} (p_i^t)^{\frac{1}{N^t}}}{\prod_{i \in S^0} (p_i^0)^{\frac{1}{N^0}}} \exp\left[\frac{\sum_{l=1}^{T} \bar{\hat{\gamma}}_{(0,l)}^{0} + \bar{\hat{\gamma}}_{(0,t)}^{0}}{T+1} - \frac{\sum_{l=0;l \neq t}^{T} \bar{\hat{\gamma}}_{(l,t)}^{t} + \bar{\hat{\gamma}}_{(0,t)}^{t}}{T+1}\right] \left\{\exp\left[\sum_{l=1;l \neq t}^{T} \frac{\bar{\hat{\gamma}}_{(l,t)}^{l} - \bar{\hat{\gamma}}_{(0,l)}^{l}}{T-1}\right]\right\}^{\frac{T-1}{T+1}}. \qquad (27)$$

Equation (27) decomposes the GEKS-Jevons price index into three factors. The first factor is the ratio of geometric mean prices in periods t and 0. The second factor is the antilog of the difference between the (arithmetic) averages of $\bar{\hat{\gamma}}^{0}_{(0,l)}$ $(l=1,...,T)$ and $\bar{\hat{\gamma}}^{t}_{(l,t)}$ $(l=0,...,T;l\neq t)$, where $\bar{\hat{\gamma}}^{0}_{(0,t)}$ and $\bar{\hat{\gamma}}^{t}_{(0,t)}$ count twice. The third factor is the antilog of the average of $\bar{\hat{\gamma}}^{l}_{(l,t)}-\bar{\hat{\gamma}}^{l}_{(0,l)}$ $(l=1,...,T;l\neq t)$, raised to the power of $(T-1)/(T+1)$. We expect the third factor to be relatively small and fluctuate around zero over time. The GEKS-Jevons index is therefore most likely driven by the first two factors.

Let us compare decomposition (27) with decomposition (14) for the multilateral time-product dummy index. P^{0t}_{GEKS-J} and P^{0t}_{TPD} are both written as the ratio of geometric mean prices in periods t and 0, adjusted by factors based on differences in average fixed effects. The average fixed effects for period 0 and period t in (27), $\bar{\hat{\gamma}}^{0}_{(0,l)}$ and $\bar{\hat{\gamma}}^{t}_{(l,t)}$, can be viewed as crude approximations of $\bar{\hat{\gamma}}^{0}$ and $\bar{\hat{\gamma}}^{t}$ in (14) because, by assumption, they all measure the same average fixed effects, albeit estimated on different subsets of the data. Thus, the means $(\sum_{l=1}^{T}\bar{\hat{\gamma}}^{0}_{(0,l)}+\bar{\hat{\gamma}}^{0}_{(0,t)})/(T+1)$ and $(\sum_{l=0, l\neq t}^{T}\bar{\hat{\gamma}}^{t}_{(l,t)}+\bar{\hat{\gamma}}^{t}_{(0,t)})/(T+1)$ are also approximations of $\bar{\hat{\gamma}}^{0}$ and $\bar{\hat{\gamma}}^{t}$, but much more stable than the elements $\bar{\hat{\gamma}}^{0}_{(0,l)}$ and $\bar{\hat{\gamma}}^{t}_{(l,t)}$. The third factor in (27), which of course does not appear in (14), adds noise to the first two factors.

This result suggests that the unweighted time-product dummy and GEKS-Jevons indexes essentially aim at the same index number formula and are likely to have similar trends. There can be a difference in smoothness, the time-product dummy index perhaps being a little smoother. Hence, if a time dummy hedonic model describes the data well, then the time-product dummy method can be viewed as a (smoothed) approximation of the matched-model GEKS method.

Our finding is not surprising. The only information entering the estimation of the time-product dummy index is the prices, or price changes, of all items that are observed more than once, i.e., items that are *matched* during some periods of the window $0,...,T$, because the other items are zeroed out. This is the exact same information that is used to construct the GEKS index. The time-product dummy method imputes 'missing prices'[14], but the imputations are unlikely to affect the trend as compared with the GEKS method as long as the underlying assumptions are satisfied.

[14] Summers (1973) proposed the country-product dummy method for estimating transitive price indexes across countries with the aim of obtaining a full set of prices through imputations for products that are purchased in one country but not in all other countries.

When the true characteristics parameters change over time, or if a single model is too restrictive, the basic assumption underlying the time-product dummy model will be violated. As the two methods treat the price changes of the matched items differently, a difference in trend between GEKS and time-product dummy indexes can arise. The time-product dummy method has a potential practical advantage though: the indexes are easier to estimate, assuming the statistical package or computer system can handle large amounts of data and run regressions that include many time and product dummies.

6. Issues with daily online data and daily indexes

A problem with multilateral methods is that when data for the next period ($T+1$ in our case) is added and the model is re-estimated, the results for all previous periods $(1,...,T)$ change. Statistical agencies do not accept continuous revisions of their price statistics. A *rolling window approach* can be used to overcome the revisions problem. For example, a multilateral time-product dummy hedonic model would be estimated on the data of a window with a fixed length, in our case of $T+1$ time periods, which is shifted forward each time period. The most recent period-on-period index movement is then repeatedly spliced onto the existing time series.[15]

A window length of at least one year will be necessary to cope with seasonal goods. On the other hand, it does not seem very helpful choosing a window length that exceeds the average period items – as identified by web IDs or article numbers – are offered for sale. The lifetime will depend on the type of product, the prevailing market circumstances and the stores' or manufacturers' policy of changing item identifiers. The importance of the latter issue has not always been fully recognized.

For measuring price change, items should ideally be identified by a complete set of characteristics; items with identical (quantities of) characteristics are deemed similar. The number of characteristics needed to distinguish between different items can be quite large, so usual practice is to choose a limited set of important characteristics. If there are no item descriptions at all, as we have with web scraping data, or in case it would be too time consuming to identify items uniquely from product descriptions, the only feasible

[15] Ivancic, Diewert and Fox (2011) employ a rolling window approach in the context of GEKS-Fisher price indexes.

solution will be to rely on item identifiers found on websites, such as *article numbers* or *web IDs*.

However, these item identifiers may be too detailed for our purpose as different article numbers or web IDs can relate to items which are similar from the consumers' point of view.[16] Item churn will then be overestimated and matched-model indexes will be based on fewer matches than desirable. Price changes of items whose article numbers or web IDs have changed but otherwise remained unchanged are captured by hedonic regression methods, although the results become increasingly model dependent. But such *hidden price changes* will be missed by matched-model methods,[17] including the time-product dummy method.

There are a number of issues that are specific to web scraping data. Online prices for supermarkets in the Netherlands are usually 5-6% above shelf prices due to delivery costs. In turn, shelf prices may differ from average transaction prices (i.e. unit values) as a result of promotional sales and the like. Representativity of the online data is another issue. The range of products shown on websites need not be the same as offered in the corresponding (physical) outlets and may change very frequently, even on a daily basis. Changes made to the website are a potential problem associated with web scraping since it could lead to missing price observations. Also, for clothing in particular, online stores sometimes classify items that are on sale in a separate (clearing) sales category, and this category should not be overlooked.

The last point raises an important issue. It is obvious that both regular and sales prices should be taken into account for measuring price change, but it is not so obvious how they should be treated. Suppose first, following the Billion Prices Project at MIT, that we observe prices on a daily basis and follow the price change over time of each individual item as long as it is available. While this price trajectory shows the change in offer prices, the observed price trend is not necessarily the right one from the average consumer's perspective as a result of promotional sales. For example, it might happen

[16] This issue was also mentioned by Bradley et al. (1997), who investigated the potential uses of scanner data in the U.S. CPI, and by de Haan (2002).

[17] The first step traditionally followed by statistical agencies when an item is replaced by a newly sampled item is to find out whether or not the items are comparable. If they are, the prices are directly compared (and if they are not, a quality adjustment should be made); see for example Chapter 17 in the U.S. Bureau of Labor Statistics *Handbook of Methods*, available at www.bls.gov. This procedure ensures that hidden price changes will not be missed.

that regular prices stay constant over time but sales prices show an upward trend. Since promotional sales occur infrequently relative to the number of days with regular prices, the overall trend seems to be almost flat. However, if consumers mainly buy the item at times of sales,[18] then the change in sales prices would be a better indicator of the change in prices actually paid.

Partly due to promotional sales, daily price indexes may be quite volatile, at least at the product level. It is questionable whether users benefit from volatile price indexes, and they may want to smooth out volatility. Alternatively, the statistical agency could do 'internal smoothing' by constructing price indexes at a lower frequency, for example weekly or monthly. There is another reason why the construction of *daily* indexes from online data may not be very meaningful. Prices of many products, particularly services, cannot be observed online. For these products, official price indexes are available from most statistical agencies on a monthly basis only, so it seems that a month is the shortest period possible to combine official figures with online data to calculate an overall CPI.[19] Ideally, monthly unit value indexes are computed at the item level, which would resolve the sales problem mentioned above. But without information on quantities purchased, calculating unit values from daily online data is not possible.

In many cases, scanner data is an ideal source for computing unit values, but that might be different for online purchases, in particular on clothing. Statistics Netherlands has received a research scanner database from a major Dutch online store. An issue is the way in which goods that were returned by the customers are treated. The quantities registered in a particular month refer to quantities delivered minus quantities returned. This means that quantities can be negative, something we observed in the database for a number of goods. More generally, it is not immediately clear how quantities purchased in each month and the corresponding unit values should be determined. This issue needs more attention.

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[18] We saw this for a number of products in supermarket scanner data. An illustrative example for the most popular make of detergents in the Netherlands is shown in de Haan (2008). Quantities sold at the regular price were negligible.

[19] PriceStats uses officially published monthly indexes for unobserved products to calculate daily overall inflation measures, apparently by assuming that these indexes are constant across the whole month: "Most of the categories that we are not able to cover are services. This, however, is not a problem for our goal to detect the main changes in inflation trends. Services are usually quite stable and not the main source of volatility." (www.PriceStats.com/faqs)

7. Empirical results

In this section we present some empirical results. The main goal is to illustrate that the different types of price indexes discussed in the paper, i.e. TPD, chained matched-model Jevons, and GEKS-Jevons indexes, can have quite different trends and are often highly volatile when constructed at a daily frequency. Our data set contains daily offer prices extracted from the website of a Dutch online store for three products: women's T-shirts, men's watches, and kitchen appliances. Actually, we exploit two data sets. The first data set covers the period of 6 October 2012 to 8 April 2013; the sample period is extended to 12 August 2013 in the second data set.[20] Note that this online retailer has no physical store, and so the data relate to (potential) online purchases only.

Figures 1-3 compare daily time-product dummy (TPD) and chained Jevons price indexes for the three products (with 6 October 2012 as the base period), based on the initial small data set. The change in unweighted arithmetic and geometric average prices is also plotted. As shown by equation (14), the difference between the TPD index and the ratio of geometric average prices results from differences in the sample means of the estimated fixed effects. A couple of things are worth mentioning.

For women's T-shirts (Figure 1), we observe a noticeable difference between the TPD and chained Jevons indexes, the TPD sitting above the chained Jevons. This is in accordance with our expectations, as discussed in section 4. Both price indexes appear to have substantial downward bias, which also meets our expectations because both are matched-model indexes based on 'too detailed' item identifiers and/or a lack of quality adjustment. The two indexes are very volatile. Due to compositional changes, average prices are even more volatile. Yet the trend in average prices seems a lot more plausible as an indicator of aggregate price change than the trend of the TPD index. Although the sample period is too short to draw any definitive conclusions, a seasonal pattern appears to emerge with average prices declining in autumn and winter, and then rising again in spring.

Heterogeneity probably is greater for men's watches and kitchen appliances than for women's T-shirts, which at least partly explains the erratic behaviour of the average prices (Figures 2 and 3a). The trends of the TPD and chained Jevons indexes for these

[20] The price indexes based on the extended data set were kindly estimated by Frances Krsinich (Statistics New Zealand).

products look reasonable. In Figure 3b the left scale has been adjusted in order to show that the TPD and chained Jevons indexes for kitchen appliances are also volatile, though much less so than average prices. The differences in volatility as well as in index levels between the two indexes are minor.

Figure 1: Daily price indexes of women's T-shirts (small data set)

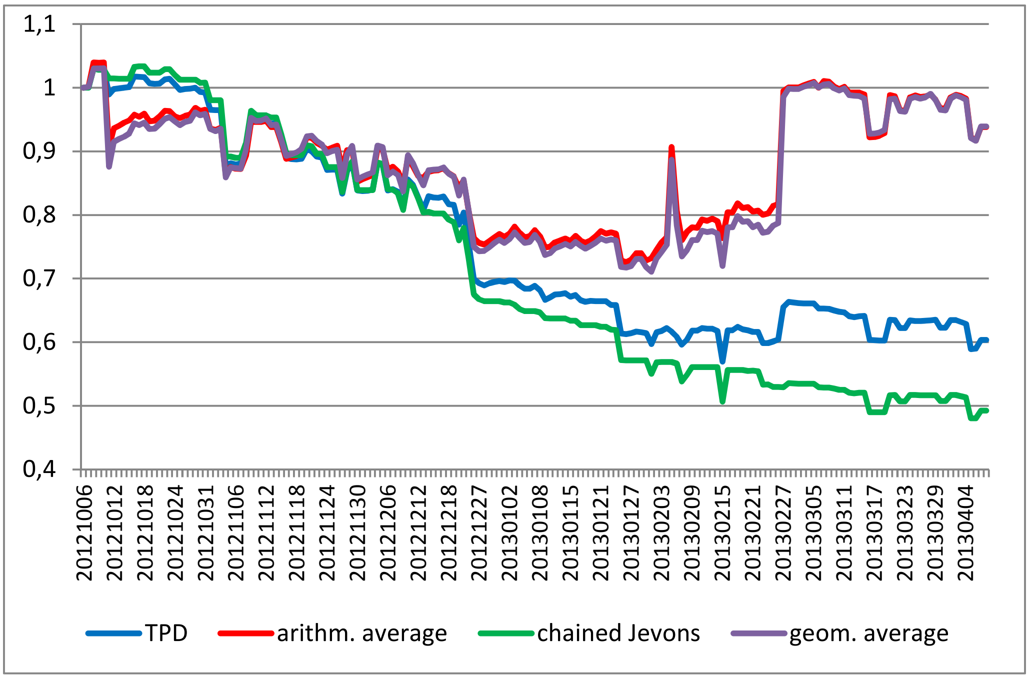


Figure 2: Daily price indexes of men's watches (small data set)

Figure 3a: Daily price indexes of kitchen appliances (small data set)

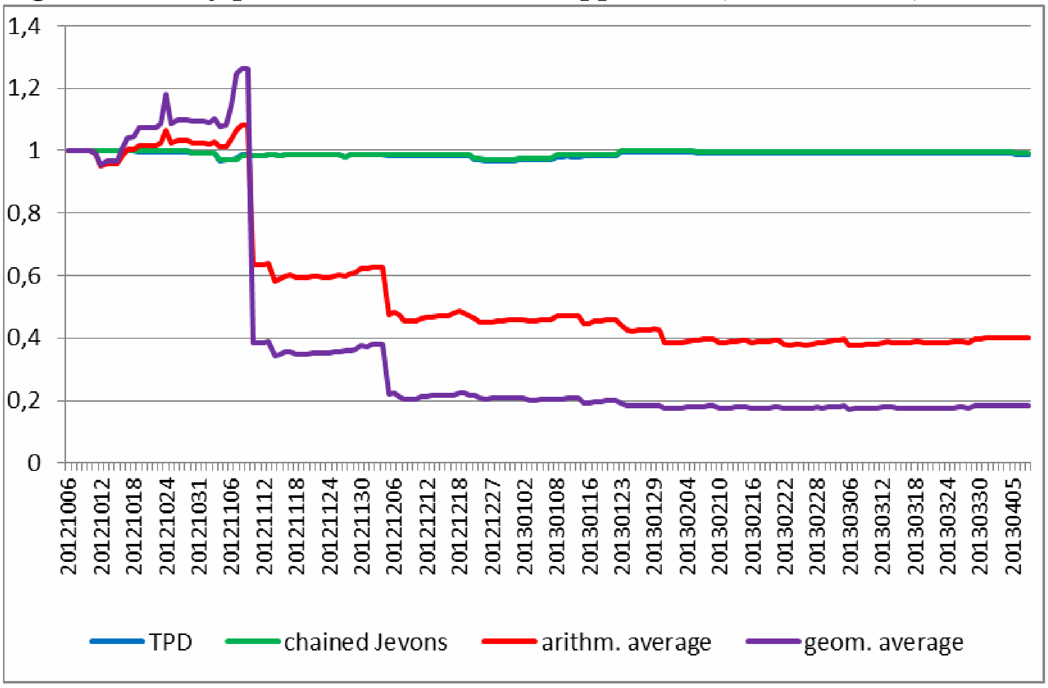
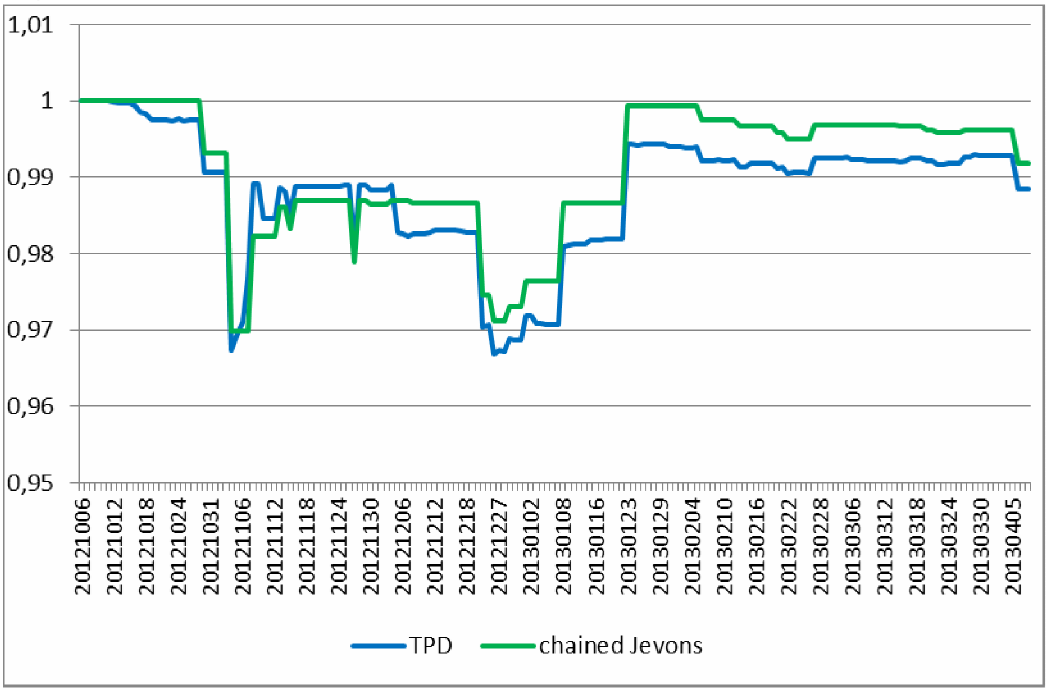


Figure 3b: Daily price indexes of kitchen appliances (small data set)



Figures 4-6 display daily TPD indexes for the three products, estimated from the extended data set. Figure 4 confirms that the TPD index of women's T-shirts is severely biased downwards: nobody believes an aggregate price decline of almost 60% within 10 months. A comparison of Figures 4-6 with the TPD indexes shown in Figures 1-3 tells us that the revisions of index numbers previously estimated from the small data set are negligible in relation to the volatility of the indexes.

Figure 4: Daily TPD price indexes of women's T-shirts (large data set)

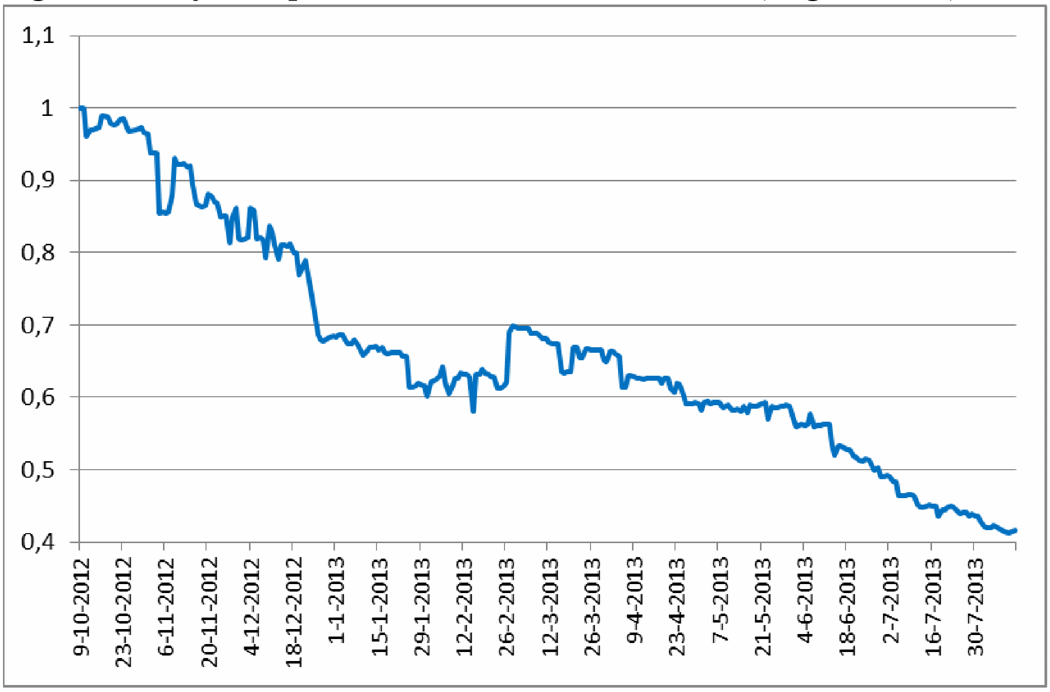


Figure 5: Daily TPD price indexes of men's watches (large data set)

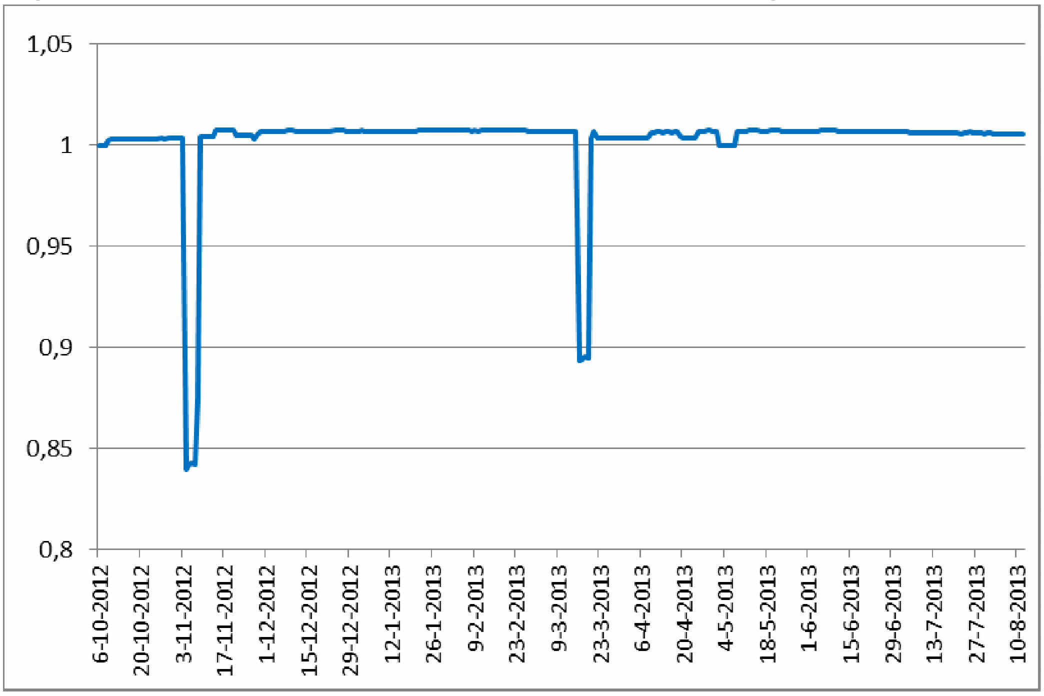
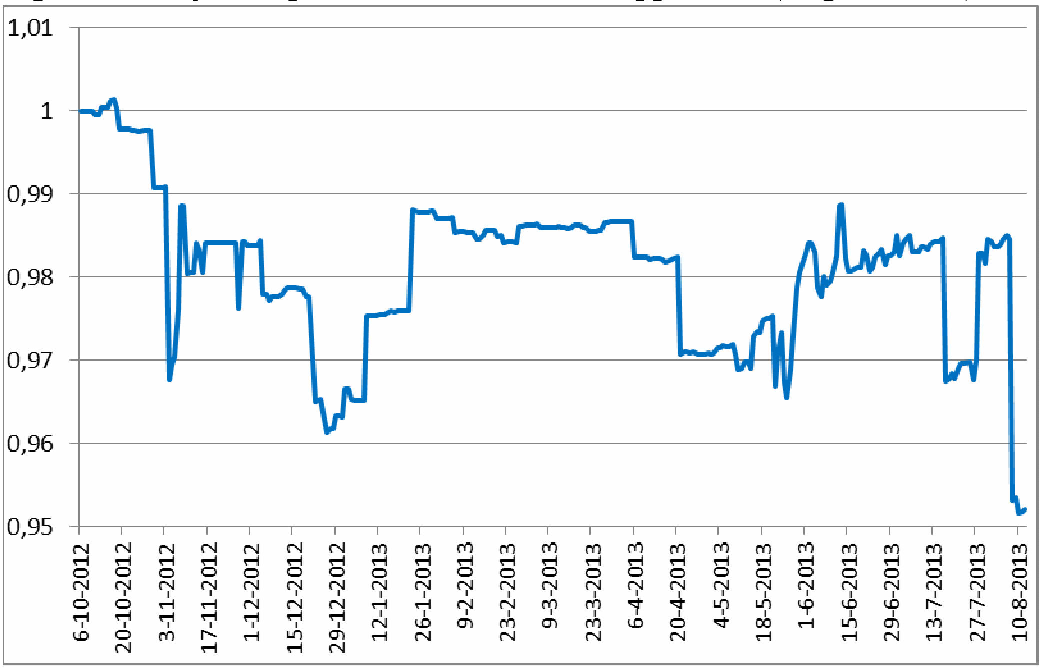


Figure 6: Daily TPD price indexes of kitchen appliances (large data set)



When trying to estimate GEKS-Jevons indexes on the big data set, it turned out that the SAS program was unable to handle such a large amount of data. We decided to randomly sample one observation out of seven daily observations per item in each of the 43 weeks. Two independent samples were drawn to get a better understanding of the potential effects of sampling in time. Figures 7-9 compare the resulting weekly GEKS-Jevons price indexes with weekly TPD indexes estimated from the same samples.

For women's T-shirts (Figure 7), the GEKS-Jevons index does not fall as fast as the TPD index, which is a bit surprising, but for men's watches (Figure 8) and kitchen appliances (Figure 9), the two indexes are very similar. Note that the difference between the indexes estimated from sample 1 and sample 2 is small for each product. Comparing Figures 7-9 with Figures 1-2 reveals that drawing samples does not change the picture much (during 6 October 2012 to 8 April 2013), both in terms of trends and volatility. Apparently, there is a lot of redundancy in the daily data set. This again raises doubts about the usefulness of observing prices on a daily basis. For the three products, weekly web scraping would suffice, unless of course the aim is to explicitly compile daily price indexes.

The above results are preliminary. In future work we should take a closer look at the microdata. Previous analysis of web scraping data from another Dutch online store indicated that many items were missing as a result of day-to-day changes in the website, even though these items were most likely available for purchase. It may be worthwhile to impute temporarily 'missing prices', for example by carrying forward the latest price observations. In particular, it would be interesting to investigate how imputations affect the volatility of the daily and weekly time series.

Figure 7: Weekly price indexes of women's T-shirts (large data set)

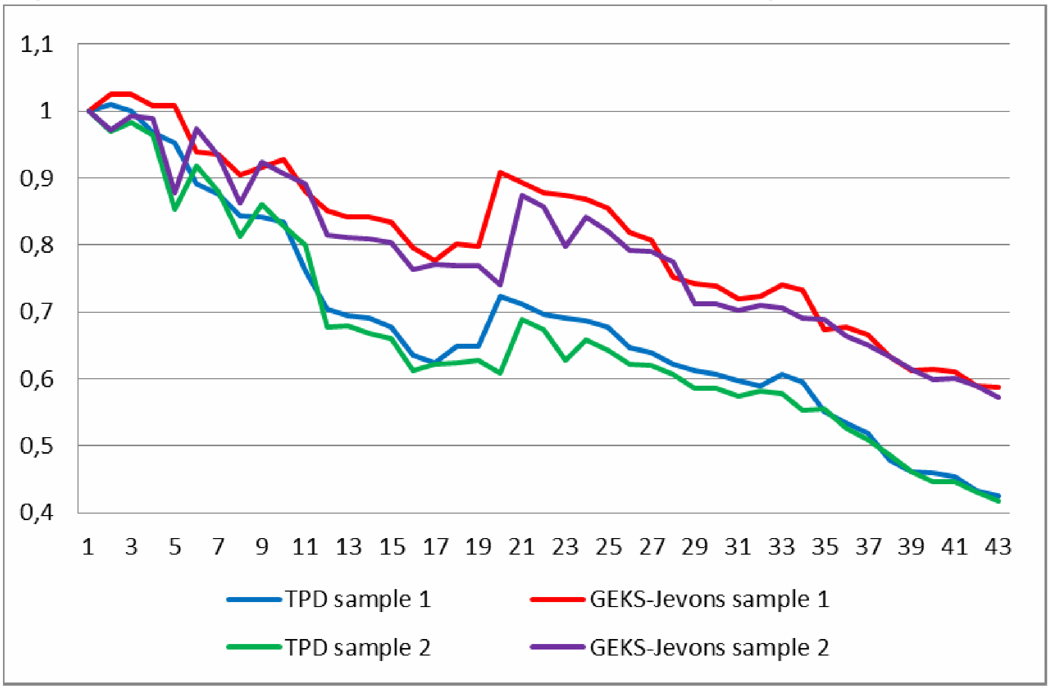


Figure 8: Weekly price indexes of men's watches (large data set)

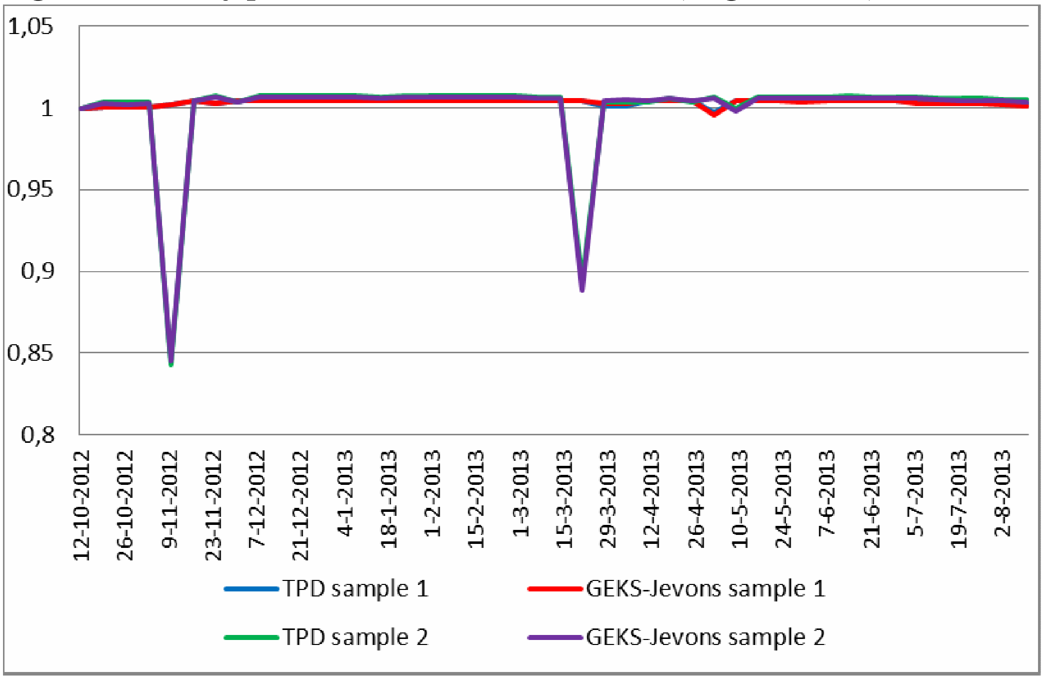
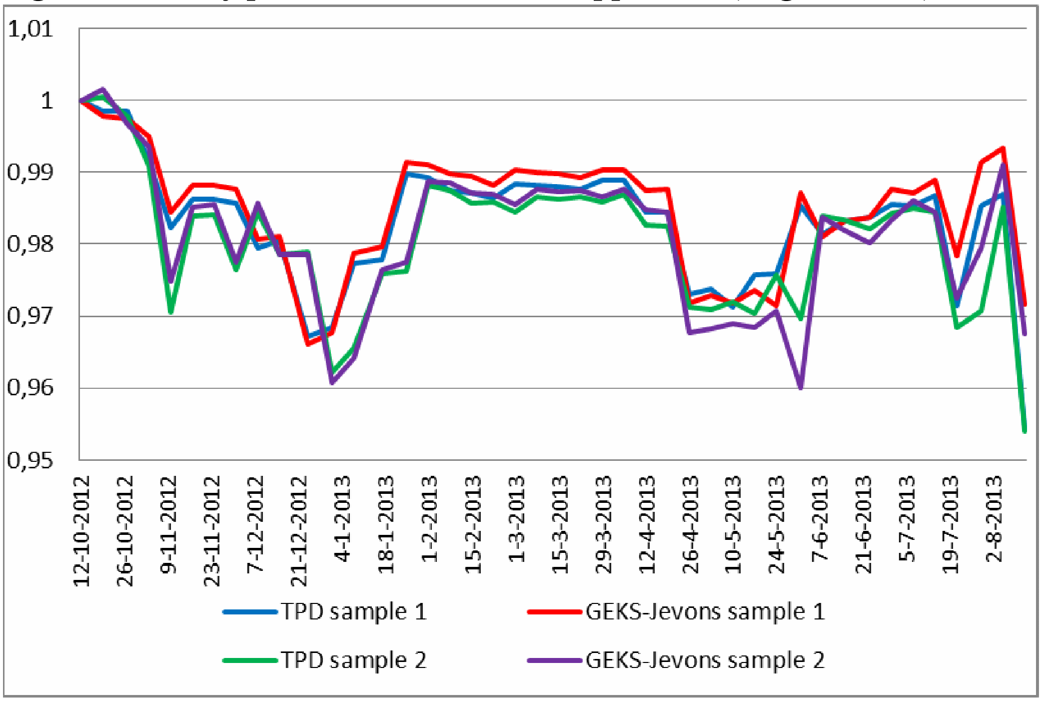


Figure 9: Weekly price indexes of kitchen appliances (large data set)



8. Conclusions

Some authors, e.g., Aizcorbe, Corrado and Doms (2003) and Krsinich (2013), refer to the time-product dummy method as a hedonic regression approach. In our opinion, this is not appropriate. Hedonic analysis is about decomposing heterogeneous products into their price-determining characteristics, measuring the characteristics' marginal prices or price elasticities, and often estimating quality-adjusted price indexes. As characteristics are not included, the time-product dummy model is not a hedonic model.

The confusion arises from the fact that 'fixed effects' in a time-product dummy model can be viewed as item-specific hedonic effects if the characteristics parameters of the underlying log-linear hedonic model are constant across time. This does not imply, however, that the time-product dummy method produces quality-adjusted price indexes. The only data effectively entering the estimation of time-product dummy indexes is the price changes of items that are matched in one or more bilateral comparisons across the whole sample period, which is the exact same data that enters the estimation of GEKS-Jevons price indexes. To clarify our point, we have shown that if a time dummy hedonic model holds true, the unweighted time-product dummy method and the matched-model GEKS-Jevons method essentially aim at the same index number formula.

Measuring quality-adjusted price change without data on item characteristics is just not possible. The two multilateral methods should therefore not be applied to goods where quality change is important.[21] De Haan and Krsinich (2012) show how the GEKS method can be modified to account for quality change by using hedonic rather than matched-model price indexes as input in the GEKS system.[22] For goods where quality change is of minor importance, the two methods have much to offer as compared to a period-on-period chained matched-model price index since they use all of the matches across the whole sample period. We would prefer the GEKS method because it is the most straightforward way to obtain transitive indexes and because it is a nonparametric approach whereas the time-product dummy method is model-based. Minimising model dependence seems like good advice for producing official statistics. The identification of items remains an issue. Any matched-model method breaks down when changes in item identifiers and price changes occur simultaneously.

The time-product dummy method has a practical advantage though, in particular when the aim is to construct high-frequency price index numbers using online data. If the production system can deal with very large data sets, time-product dummy indexes may be easier to estimate than GEKS indexes. Also, our equations (18) and (19) provide practitioners with the opportunity to decompose the latest period-on-period price change into a matched-model index and the effects of items that are new or disappearing with respect to the previous period. The latter effects are implicitly based on the data of many earlier periods. Staff involved in production of the CPI may not like this aspect, but it is unavoidable with multilateral methods.

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[21] This is also true for the chained matched-model Jevons method, which is how PriceStats compiles daily indexes for each product category. On their website (www.PriceStats.com/faqs) it is mentioned that "We treat all individual products [what we call items] as separate series, without making product substitutions or hedonic quality adjustments. Only consecutive price observations for exactly the same product are used to calculate price changes. So, for example, if a TV is replaced with a new, more expensive model, we do not have a price change in that category. Only when the new model starts changing its price will the index start to be affected by that product. Similarly, when a product disappears from the sample, we assume it is temporarily out of stock for a set amount of time. After that period, the product is discontinued from the index." We think their approach can give rise to upward bias for high-technology goods (due to a lack of quality adjustment) and to downward bias for clothing (due to a combination of high-frequency chaining and the use of too-detailed item identifiers).

[22] As mentioned in footnote 6, it is not possible to incorporate characteristics into a time-product dummy model; the product dummies must be left out to identify the model, turning it into a time dummy hedonic model.

A major drawback of web scraping is that quantities purchased/sold cannot be observed. If quantities or expenditures at the item level are available, as in scanner data, then this information can be used in the estimation of the time-product dummy model in order to obtain weighted price indexes. This has been done by Ivancic, Diewert and Fox (2009), de Haan and Krsinich (2012), and Krsinich (2013), using the items' expenditure shares as regression weights. In the Appendix, a decomposition of the expenditure-share weighted time-product dummy index is derived along the lines for the unweighted case in section 4. The treatment in scanner data sets of products that have been returned by customers deserves more attention. Previous analysis by Statistics Netherlands indicated that this was a problem in scanner data from an online store, particularly for clothing, and in scanner data from a Do-It-Yourself store.

Our empirical results for three products confirm that daily price indexes can be highly volatile. For kitchen appliances and men's watches, the TPD and GEKS-Jevons indexes are similar, as expected, but the chained Jevons index performs just as well. For women's T-shirts the situation is different: the chained Jevons index sits below the TPD index, as expected, but the TPD and GEKS-Jevons indexes differ. The latter indexes are heavily biased downwards due to changing identifiers for comparable items.

Appendix: The weighted time-product dummy method

In this Appendix we will show what drives the difference between the expenditure-share weighted time-product dummy index and the period-on-period chained matched-model Törnqvist price index. We assume that the time-product dummy model (11) is estimated by WLS regression, where the items' expenditure shares s_i^0 and s_i^t in the base period 0 and the comparison periods t $(t = 1,...,T)$ serve as weights. Note that the shares sum to unity in each period, i.e., $\sum_{i \in S^0} s_i^0 = 1$ and $\sum_{i \in S^t} s_i^t = 1$. The predicted prices are given by $\tilde{p}_i^0 = \exp(\tilde{\alpha})\exp(\tilde{\gamma}_i)$ and $\tilde{p}_i^t = \exp(\tilde{\alpha})\exp(\tilde{\delta})\exp(\tilde{\gamma}_i)$, where $\tilde{\alpha}$, $\tilde{\delta}$ and $\tilde{\gamma}_i$ denote the WLS parameter estimates. Taking geometric means of the predicted values yields

$$\prod_{i \in S^0} (\tilde{p}_i^0)^{s_i^0} = \exp(\tilde{\alpha}) \exp\left[\sum_{i \in S^0} s_i^0 \tilde{\gamma}_i\right]; \qquad (A.1)$$

$$\prod_{i \in S^t} (\tilde{p}_i^t)^{s_i^t} = \exp(\tilde{\alpha}) \exp(\tilde{\delta}^t) \exp\left[\sum_{i \in S^t} s_i^t \tilde{\gamma}_i\right]; \qquad (t = 1,...,T). \qquad (A.2)$$

By dividing (A.2) by (A.1), rearranging and using $\prod_{i \in S^0} (\tilde{p}_i^0)^{s_i^0} = \prod_{i \in S^0} (p_i^0)^{s_i^0}$ and $\prod_{i \in S^t} (\tilde{p}_i^t)^{s_i^t} = \prod_{i \in S^t} (p_i^t)^{s_i^t}$ (which holds because the weighted regression residuals sum to zero in each time period), an explicit expression for the weighted time-product dummy index is found:

$$P_{WTPD}^{0t} = \exp(\tilde{\delta}^t) = \frac{\prod_{i \in S^t} (p_i^t)^{s_i^t}}{\prod_{i \in S^0} (p_i^0)^{s_i^0}} \exp\left[\overline{\tilde{\gamma}}^0 - \overline{\tilde{\gamma}}^t\right]; \qquad (t = 1,...,T), \tag{A.3}$$

where $\overline{\tilde{\gamma}}^0 = \sum_{i \in S^0} s_i^0 \tilde{\gamma}_i$ and $\overline{\tilde{\gamma}}^t = \sum_{i \in S^t} s_i^t \tilde{\gamma}_i$ are the expenditure-share weighted sample means of the estimated fixed effects, with the effect for the base item item N set to zero ($\hat{\gamma}_N = 0$).

Just like its unweighted counterpart (15), the weighted index (A.3) is transitive and can be written as a chain index:

$$P_{WTPD}^{0t} = \prod_{\tau=1}^{t} \frac{\prod_{i \in S^\tau} (p_i^\tau)^{s_i^\tau}}{\prod_{i \in S^{\tau-1}} (p_i^{\tau-1})^{s_i^{\tau-1}}} \exp\left[\overline{\tilde{\gamma}}^{\tau-1} - \overline{\tilde{\gamma}}^\tau\right]; \qquad (t = 1,...,T). \tag{A.4}$$

A single chain link in equation (A.4) can be written as

$$\frac{P_{WTPD}^{0t}}{P_{WTPD}^{0,t-1}} = \frac{\prod_{i \in S^t} (p_i^t)^{s_i^t}}{\prod_{i \in S^{t-1}} (p_i^{t-1})^{s_i^{t-1}}} \left[\frac{\prod_{i \in S^t} [\exp(\tilde{\gamma}_i)^{s_i^t}]}{\prod_{i \in S^{t-1}} [\exp(\tilde{\gamma}_i)^{s_i^{t-1}}]} \right]^{-1} = \frac{\prod_{i \in S^t} \left(\frac{p_i^t}{\exp(\tilde{\gamma}_i)} \right)^{s_i^t}}{\prod_{i \in S^{t-1}} \left(\frac{p_i^{t-1}}{\exp(\tilde{\gamma}_i)} \right)^{s_i^{t-1}}} = \frac{\prod_{i \in S^t} (\tilde{p}_i^t)^{s_i^t}}{\prod_{i \in S^{t-1}} (\tilde{p}_i^{t-1})^{s_i^{t-1}}}, \tag{A.5}$$

with $\tilde{p}_i^{t-1} = p_i^{t-1} / \exp(\tilde{\gamma}_i)$ and $\tilde{p}_i^t = p_i^t / \exp(\tilde{\gamma}_i)$. Chain link (A.5) can alternatively be written as

$$\frac{P_{WTPD}^{0t}}{P_{WTPD}^{0,t-1}} = \frac{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{s_i^t}}{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{s_i^{t-1}}} \frac{\prod_{i \in S_N^{t-1,t}} (\tilde{p}_i^t)^{s_i^t}}{\prod_{i \in S_D^{t-1,t}} (\tilde{p}_i^{t-1})^{s_i^{t-1}}}. \tag{A.6}$$

We introduce some additional notation. The aggregate expenditure shares of the items that are matched in periods $t-1$ and t are $s_M^{t-1} = \sum_{i \in S_M^{t-1,t}} s_i^{t-1}$ and $s_M^t = \sum_{i \in S_M^{t-1,t}} s_i^t$. Thus, $s_{iM}^{t-1} = s_i^{t-1} / s_M^{t-1,t}$ and $s_{iM}^t = s_i^t / s_M^t$ represent the matched items' normalized expenditure shares, such that $\sum_{i \in S_M^{t-1,t}} s_{iM}^{t-1} = \sum_{i \in S_M^{t-1,t}} s_{iM}^t = 1$. Multiplying (A.6) by the adjacent-period matched-model Törnqvist price index $\prod_{i \in S_M^{t-1,t}} (p_i^t / p_i^{t-1})^{(s_{iM}^{t-1}+s_{Mi}^t)/2}$ and dividing again by the same index, but now written as $\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{(s_{iM}^{t-1}+s_{Mi}^t)/2} / \prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{(s_{iM}^{t-1}+s_{iM}^t)/2}$, gives

$$\frac{P_{WTPD}^{0t}}{P_{WTPD}^{0,t-1}} = \prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{p_i^{t-1}} \right)^{\frac{s_{iM}^{t-1}+s_{iM}^t}{2}} \frac{\left[\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{s_{iM}^t} \right]^{s_M^t}}{\left[\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{s_{iM}^{t-1}} \right]^{s_M^{t-1}}} \frac{\prod_{i \in S_N^{t-1,t}} (\tilde{p}_i^t)^{s_i^t}}{\prod_{i \in S_D^{t-1,t}} (\tilde{p}_i^{t-1})^{s_i^{t-1}}} \left[\frac{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{\frac{s_{iM}^{t-1}+s_{iM}^t}{2}}}{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{\frac{s_{iM}^{t-1}+s_{iM}^t}{2}}} \right]^{-1} . \qquad (A.7)$$

Using $s_D^{t-1} = \sum_{i \in S_D^{t-1,t}} s_i^{t-1} = 1 - s_M^{t-1}$, $s_N^t = \sum_{i \in S_N^{t-1,t}} s_i^t = 1 - s_M^t$ and the normalized shares $s_{iD}^{t-1} = s_i^{t-1} / s_D^{t-1,t}$ and $s_{iN}^t = s_i^t / s_N^{t-1,t}$ for unmatched items, equation (A.7) can be written as:

$$\frac{P_{WTPD}^{0t}}{P_{WTPD}^{0,t-1}} = \prod_{i \in S_M^{t-1,t}} \left(\frac{p_i^t}{p_i^{t-1}} \right)^{\frac{s_{iM}^{t-1}+s_{iM}^t}{2}} \left[\frac{\prod_{i \in S_N^{t-1,t}} (\tilde{p}_i^t)^{s_{iN}^t}}{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{s_{iM}^t}} \right]^{s_N^t} \left[\frac{\prod_{i \in S_D^{t-1,t}} (\tilde{p}_i^{t-1})^{s_{iD}^{t-1}}}{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{s_{iM}^{t-1}}} \right]^{-s_D^{t-1}} \frac{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^t)^{\frac{s_{iM}^t - s_{iM}^{t-1}}{2}}}{\prod_{i \in S_M^{t-1,t}} (\tilde{p}_i^{t-1})^{\frac{s_{iM}^{t-1} - s_{iM}^t}{2}}} . \qquad (A.8)$$

Three points are worth noting about equation (A.8). First, although this is trivial, if in both periods the expenditure shares are the same for all items, i.e. if $s_i^{t-1} = 1/N^{t-1}$ and $s_i^t = 1/N^t$ for all *i*, then (A.8) simplifies to decomposition (18) for the unweighted case.

Second, if there are no new or disappearing items between periods $t-1$ and *t*, then $s_N^t = s_D^{t-1} = 0$ and the chain link equals the product of the adjacent-period matched-model Törnqvist price index and the last factor in equation (A.8). Because the Törnqvist index is not transitive, high-frequency chaining can lead to a drifting time series.[23] So we could say that the last factor in (A.8) eliminates chain drift in the Törnqvist index.

Third, unlike the unweighted index, (a chain link of) the weighted time-product dummy index depends on the model specification if all items are matched. This type of model dependency holds for any weighted multilateral time dummy method, including the time-product dummy method.[24]

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[23] It is empirically well established that high-frequency chaining of superlative indexes, such as Törnqvist and Fisher price indexes, can lead to substantial drift; for evidence, see Ivancic (2007), Ivancic, Diewert and Fox (2011), de Haan and van der Grient (2011), and de Haan and Krsinich (2012).

[24] For the two-period case, de Haan (2004) proposes a set of regression weights such that the time dummy method implicitly generates an imputation Törnqvist price index; when there are no new and disappearing items, a matched-model Törnqvist index results and modelling has no influence.

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