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ABSTRACT

We develop a model where a public limit order book (PLB) competes with a Sub-Penny Venue, which allows Sub-Penny Trading (SPT). SPT occurs when a trader undercuts orders in the PLB by less than one penny, a practice we call queue-jumping (QJ). QJ is higher for NASDAQ than for NYSE stocks. We confirm the model's predictions that QJ increases in liquidity and in the tick-to-price ratio. We also find that QJ is associated with improved PLB market quality, especially for large capitalization stocks. Finally, we show that High Frequency Trading is negatively related to QJ.

1 Introduction

On January 15, 2015, a dark pool operator was fined $14.4 million by the U.S. Securities and Exchange Commission (SEC) for violating Rule 612 of Regulation National Market System (Reg NMS).[1] Rule 612 states that no market participant can accept, rank, or display orders priced in sub-pennies. However, between May 2008 and March 2011, the SEC alleges that the operator was accepting and ranking hundreds of millions of orders submitted to his dark pool that were priced in increments smaller than one cent.[2] Select market makers and High Frequency Traders (HFTs) were encouraged by the operator to use sub-penny orders, and were thereby able to undercut orders submitted to exchanges and Alternative Trading Systems (ATSs) without having to offer economically meaningful price improvement. Sub-penny orders helped these market makers and HFTs gain priority over resting orders in the Public Limit Order Book (PLB) queue at the full tick, and enabled HFTs to interact with uninformed order flow in the operator's dark pool. At the same time, the operators' clients using the firm's proprietary algorithms for order routing and execution logic were able to opt out of having their orders executing against market makers and HFTs in the operator's ATS by using restrictions on their orders. The SEC further argues that the operator introduced sub-penny orders without properly disclosing the nature of these order types to the SEC or to other traders using their dark pool. Finally, the SEC argues that the sub-penny order types created an unfair competitive advantage for the operator's dark pool relative to the venues that adhered to Rule 612.

The dark pool sub-penny trades (SPT) discussed above would have been printed off-exchange to one of the Trade Reporting Facilities (TRFs). NYSE's Trade and Quote (TAQ) data shows that such trades represented approximately 10% of the U.S. consolidated equity volume in 2010. Note that the fact that we see significant volume of sub-penny trades printed off-exchange does not necessarily mean that violations of Rule 612 occurred. The Rule allows broker-dealers to execute non-displayed orders (typically retail orders) by matching orders internally at sub-penny prices, against other clients' orders, or against their own inventory, as long as these orders obtain price improvement.[3] Furthermore, the SEC permits two specific

[1] http://www.sec.gov/news/pressrelease/2015-7.html#.VP4D8eFzKZR

[2] The dark pool operator created two specific order types that resulted in sub-penny executions. The Whole Penny Offset (WPO) orders were available from 2008 until June 2010, and allowed a subscriber to submit orders priced at the National Best Bid (NBB) plus $0.01, the National Best Offer (NBO) minus $0.01 or at the midpoint of the NBBO plus or minus $0.01. If the spread at the NBBO was an odd number of cents, the WPO orders would have resulted in sub-penny orders. The PrimaryPegPlus (PPP) orders were introduced in June 2010 and allowed a subscriber to submit orders priced as a percentage of the NBBO plus or minus. For example, if the NBBO was $50.00 to $50.02 a PPP 10% plus buy order would mean an order to buy at $50.002. In addition, technical glitches in the operator's smart order routing and algorithmic execution systems produced sub-penny executions.

[3] Broker-dealers' internalization accounts for a substantial share of the U.S. consolidated equity volume according to the U.S. Securities and Exchange Commission's Concept Release on Equity Market Structure (SEC,

types of sub-penny executions under Rule 612: mid-quote executions and execution at a price determined through a volume-weighted-average-price (VWAP) algorithm.[4] While these types of sub-penny executions are permitted by the Rule, they have the same negative effect on orders at the front of the queue on the PLB, i.e., they are undercut by an economically insignificant amount.

In this paper, we develop a model where a PLB faces competition from a Sub-Penny Venue (SPV). The SPV allows broker-dealers to submit orders on a finer grid to gain priority over resting orders in the PLB queue at the best bid or offer, and we call this phenomenon queue-jumping (QJ). We rely on the model to develop hypotheses about the factors influence the extent of QJ, and about the effects of QJ on market quality. We then test the model's predictions using TAQ data.

Rule 612 was introduced to limit the negative effects of decimalization which took place in 2001 for U.S. stocks.[5] The rationale for decimalization was to lower trading costs, and the empirical evidence shows that quoted and effective spreads indeed fell following decimalization (e.g., Chakravarty, Harris and Wood (2001), Bacidore, Battalio and Jennings (2003), and Bessembinder (2003)). However, decimalization also reduced PLB inside depth and increased market maker participation suggesting that undercutting may have had a detrimental effect on public liquidity provision (SEC, 2012). The SEC realized that if traders could undercut limit orders sitting on the book by an economically insignificant amount, it would potentially reduce the incentive for traders to post limit orders at the top of the PLB, and therefore could have a detrimental effect on inside depth. The SEC also realized that a smaller tick size and hence a lower inside spread would lead to lower profit opportunities for brokers. For these reasons, the SEC adopted Rule 612 which defines the minimum price increment of $0.01 (1 penny) for stocks priced over $1, and a minimum price increment of $0.0001 for stocks priced below $1.

The SEC's allegations in the Rule 612 violation case discussed above suggest that QJ through the use of dark pools is particularly important for traders pursuing algorithmic and HFT strategies. Fast trading includes smart order routers (SORs) which are programs largely used by both the buy and the sell side to search for the best quotes in regular exchanges as well as in dark pools. Therefore, the use of SORs guarantees that orders posted at sub-penny increments are actually executed if they offer the best price. We believe that this is a major reason for why dark trading combined with fast trading facilities has paved the way for the expansion of SPT.

2010), being equal to 17.5% in 2009. According to the SEC Concept Release, in 2009 around 10 exchanges, 5 electronic communication networks, 32 dark pools, and over 200 broker-dealers were active in the U.S.

[4]Further, on July 3, 2012, the SEC issued a new rule that allowed exchanges to provide retail price improvement in sub-penny increments (units of $0.001). NYSE started their Retail Liquidity Program (RLP) and NASDAQ their Retail Price Improvement (RPI) program under this rule. Trades in an approved retail price improvement program are printed on exchanges, and are excluded from our empirical analysis.

[5]In the U.S. from 2001 the minimum price improvement was gradually reduced to 1 cent for stocks above $1 and 0.01 cent for stocks below $1.

For all the reasons discussed above, SPT is one of the main concerns of the SEC and in this paper we aim to study SPT in NASDAQ and NYSE stocks with the objective to understand the frequency of SPT, the factors which induce traders to undertake SPT and the effects that SPT has on market quality. We draw our main empirical hypotheses from a theoretical framework that extends Parlour (1998) to model competition between a PLB and a SPV. We distinguish between cases where the PLB is liquid (there are resting orders at the top of the book, i.e., the stock is tick-constrained) and those for which the PLB is illiquid (there are no orders resting at the top of the book, i.e., the stock is not tick-constrained). The model predicts that QJ is higher for liquid books (high depth and narrow spread) and for stocks with a high tick-to-price ratio, i.e., a low price for a given tick size.

Our empirical analysis is based on a sample of TAQ data for U.S. equities that includes all trades executed in sub-penny increments during 42 trading days in 2010. We include all regular trades printed to a TRF. Note that these trades may come either from broker-dealers' internalization or from trades executed in dark pools. We build a stratified sample of 90 NASDAQ and 90 NYSE listed stocks and use daily Fama-MacBeth regressions to investigate which factors are associated with more SPT, i.e., under which market conditions more sub-penny orders execute. We consider the influence of spread, depth, stock price, share volume, volatility and order imbalance on SPT. To understand the influence of SPT on market quality measured as spread and depth, we use a simultaneous equations model that controls for endogeneity. Finally, the SEC allegation suggests an important link between SPT and HFT, and we therefore investigate the relationship between SPT and HFT based on a commonly used proxy for HFT – the number of quote updates relative to trades.

Before summarizing the main results of the paper, it is important to explain how we classify SPT. We generally classify an execution as SPT when the price improvement associated to the execution price is smaller than $0.01. However, note that there are two main reasons for why we observe trades executed on ATSs at fractions of a penny. First, traders may aim to undercut orders posted at the top of transparent PLBs. Second, the execution system of some dark pools and broker-dealer internalization systems follows a derivative pricing rule according to which all trades execute at the midpoint of the primary market inside spread.[6] Note that undercutting at sub-penny increments smaller than the distance to the midpoint of the PLB spread ostensibly permits traders to provide two-sided liquidity inside the PLB spread. By contrast, midpoint executions do not allow traders to capture a spread by simultaneously posting buy and sell orders. To differentiate between these two categories of SPT, we treat executions at half a cent price improvement as a separate type of SPT and name it Mid-Crossing (MID). [7] We group the remaining SPT with a positive sub-penny price improvement different from half-cent into a

[6] To mention but a few, see ITG Posit, Liquidnet and BATS.

[7] While not included in our dataset, mid-quote executions can also occur within exchange operated dark pools.

category that we label Queue-Jumping (QJ). This way we adopt a parsimonious classification of QJ, and do not mix data which derive from potentially different trading strategies that could have different driving forces and different effects on the quality of the lit markets. However, we acknowledge that by doing so we miss the executions originating from undercutting at exactly half a penny.[8]

Our empirical results regarding the determinants of QJ can be summarized as follows. We find that QJ is on average 7.16% of the consolidated volume for NASDAQ stocks and 6.03% for NYSE stocks. Consistently with the model's predictions, QJ is positively related to the liquidity of the book (depth) and the cent quoted spread, and this is true even after controlling for factors shown in the previous literature (e.g., Buti, Rindi, and Werner (2011)) to affect dark trading. Further, QJ is increasing in depth and in the spread both for NYSE and NASDAQ-listed stocks. However, while QJ is increasing in depth for stocks in all size groups, the quoted cent spread does not significantly influence QJ for large stocks.

We also use the model to study the effects of QJ on market quality. The model predicts that QJ is associated with improved market quality – higher depth and narrower quoted PLB spreads - for liquid books. By contrast, QJ is predicted to be associated with worse market quality for illiquid books. Moreover, the model predicts that when the book is liquid and traders use more market than limit orders, more market orders migrate to the SPV and PLB volume decreases substantially so that consolidated volume also decreases. When instead the book is illiquid, less market orders migrate to the SPV so that consolidated volume increases due to QJ. Finally, the model predicts that the effects of QJ on market quality are dampened if the relative tick size in the PLB is lower.

As QJ and order submissions to the PLB are jointly determined, we need to account for endogeneity when empirically assessing the impact of QJ on market quality. To examine this question, we therefore estimate a simultaneous equation system following Hasbrouck and Saar (2013). As predicted by the model, we find that QJ is generally associated with higher depth and narrower quoted and relative spread. However, closer scrutiny shows that this result is driven by large capitalization stocks. We do not find significant results for the relationship between QJ and market quality for the group of small stocks, presumably because these stocks are less likely to be tick-constrained so that traders can undercut without executing in sub-penny.

We also show that MID executions represent 3.16% of the consolidated volume for NASDAQ stocks and 3.51% of the volume for NYSE stocks. MID executions also increase with depth and the cent quoted spread for the overall sample, and these results are also robust to controlling for the listing exchange, volatility and order imbalance. However, MID executions decrease in depth and cent quoted spread for NASDAQ and small cap stocks, and neither variable

[8] We also miss instances of QJ in stock at the whole penny increment, which may occur in high-priced and wide-spread stocks.

significantly influences MID for large stocks. Moreover, MID executions have no significant effects on market quality, measured either by depth or quoted spread. This is true for the overall sample as well as the subsamples by listing exchange and size.

The incentives for QJ is related to the tick size, and more specifically to the tick-to-price ratio. Two opposite proposals have been put forward from regulators and exchange officials that relate to SPT and the tick size. In 2010, the SEC in its Concept Release on Equity Market Structure stressed that the larger percentage spread that characterizes low-priced stocks may lead to greater internalization by Over-The-Counter (OTC) market makers or more trading volume in dark pools, and proposed a reduction in the tick size for lower priced stocks. While major U.S. exchanges, e.g., NYSE, NASDAQ and BATS, in their comment letters to the SEC Concept Release responded positively to the suggestion of reducing the tick size, there was at the same time a widespread sentiment among practitioners that decimalization curtailed brokers' profits and therefore incentives to supply liquidity for less liquid stocks and for initial public offerings (IPOs). This led Congress to pass the 2012 Jumpstart Our Business Startups Act (Section 106(b) of the "JOBS Act") which instructed the SEC to investigate the possible effects of raising the minimum price increment, i.e., the tick size, for stocks of high growth companies and authorized the SEC to conduct a pilot study to raise the tick size for small and medium capitalization stocks to $0.05.

This tick-size debate motivates an investigation into whether an increase or a decrease in the tick size is the optimal regulatory response to the widespread use of SPT. The SEC concept release and the ensuing comment letters from market participants, as well as the predictions from our model, stress that it is the tick-to-price ratio that matters for policy. It follows that the size of the minimum tick size relative to the price of the stock is the relevant policy instrument in the context of SPT.[9] We therefore study the relationship between relative tick size (i.e., price) and QJ and MID based on daily Fama-MacBeth regressions, controlling for share volume. We find a strong negative empirical relationship between the stock price and QJ after controlling for share volume and listing exchange. By contrast, MID executions are not significantly influenced by the stock price for the overall sample, but are significantly positively related to price for NASDAQ and small cap stocks. Furthermore, both QJ and MID increase significantly in share volume, controlling for listing exchange. Because the sample of NASDAQ and NYSE stocks includes only stocks priced above $1, the tick size is constant at $0.01 and therefore when the price increases the tick-to-price ratio decreases. In other

[9]The SEC (2010) concept release (page 72) reads: "There may be greater incentives for broker-dealer internalization in low-priced stocks than in higher priced stocks. In low-priced stocks, the minimum one cent per share pricing increment of Rule 612 of Regulation NMS is much larger on a percentage basis than it is in higher-priced stocks. For example, a one cent spread in a $20 stock is 5 basis points, while a one cent spread in a $2 stock is 50 basis points – 10 times as wide on a percentage basis. Does the larger percentage spread in low-priced stocks lead to greater internalization by OTC market makers or more trading volume in dark pools? If so, why? Should the Commission consider reducing the minimum pricing increment in Rule 612 for lower priced stocks?"

words, we find a positive empirical relationship between QJ and the tick-to-price ratio as predicted by our model. If the aim of the SEC is to reduce QJ, our empirical results suggest that the right policy action would be to decrease rather than to increase the absolute tick size so that the tick-to-price ratio falls. By contrast, MID executions for NASDAQ and small caps are positively related to the stock price, or negatively related to the tick-to-price ratio. Our empirical results therefore suggest that while a reduction in the tick size will be associated with less QJ, it may result in an increase in MID executions at least for some stocks.

An important caveat is that while we find that a reduction of the tick-to-price ratio reduces QJ, this is not necessarily a good policy objective. First, we find no empirical evidence to suggest that QJ is associated with worse market quality in the sample. Furthermore, empirical studies by Werner, Wen, Rindi, Consonni and Buti (WWRCB, 2015) and O'Hara, Saar and Zhung (2014) find that a reduction in tick-to-price ratio (increase in price for a given tick) is associated with a deterioration of market quality (wider spread and less depth). This suggests that while a lower tick-to-price ratio reduces the incentives for QJ, it also tends to encourage traders to switch from limit orders to market orders. Therefore, while a decrease in the tick size may produce a desirable reduction in QJ using a SPV, it may also be associated with a decrease in PLB liquidity provision, resulting in wider spreads and less depth.

Our model shows that traders with valuations close to the average valuation among traders are more likely to QJ. In other words, traders that QJ tend not to have strong views on the value of the security being either very high or very low. Instead, they trade opportunistically in the SPV to capture small deviations of price from the average valuation. These traders' characteristics fit the description of many HFT traders, and we therefore explore the relationship between HFT and QJ and MID, respectively. We find that our proxy of HFT activity on lit markets is inversely related to both QJ and MID, suggesting a possible substitution effect between transparent and opaque venues.

The paper is organized as follows. In Section 2 we discuss the related literature and in Section 3 we lay out the model and the resulting testable hypotheses. In Section 4 we describe the dataset; in Section 5 we investigate the factors that affect SPT; and in Section 6 we show the effects of SPT on market quality. We conclude in Section 7.

2 Related Literature

As SPT takes place on dark venues, our paper is related to the empirical literature on dark pools. Ready (2013) investigates the determinants of dark trading by considering monthly volume by stock for the period June 2005 to September 2007 in two dark pools, Liquidnet and ITG POSIT, that executed approximately 1% of total market consolidated volume. He finds that dark pools execute most of their volume in stocks with low spread and high share volume. Considering that both Liquidnet and ITG POSIT execute at the mid-quote of the primary market spread, Ready's results are comparable (and consistent) with our findings for mid-crossing which show that the effect of depth on MID is positive and that of spread is negative.

Our results are also consistent with Buti, Rindi and Werner (BRW, 2011) who examine a unique dataset for the calendar year 2009 on dark pool activity for a large cross section of U.S. securities. BRW find that liquid stocks are those characterized by more intense dark pool activity. They also find that dark pool volumes increase for stocks with narrow quoted spreads and high inside bid depths, suggesting that a higher degree of competition in the PLB enhances dark pool activity. BRW also investigate the effect of dark trading on market quality and show that increased dark pool activity improves spreads, depth, and short-term volatility. Hatheway, Kwan and Zheng (HKZ, 2013) also study the effects of dark trading on market quality, even though they only focus on effective spread. HKZ look at a sample of NASDAQ and NYSE stocks during the period January-March 2011 and find a positive relation between dark pool market share and effective spread. They explain this result with the effects of SPT that should drive uninformed traders away from the lit markets into dark pools thus increasing adverse selection costs on lit markets. We cannot directly compare HKZ' results with ours because they consider dark pool activity as a whole and do not distinguish between dark trading and SPT when looking at the effects on market quality. However, by controlling for endogeneity we find that SPT does not harm spread and depth at the top of the lit market which does not seem consistent with the conjectured cream skimming effect.

International evidence on dark trading and market quality is presented in Degryse, de Jong and van Kerviel (2015) who consider a sample of 52 Dutch stocks and analyze both internalized trades and trades sent to dark pools. They find that when these two sources of dark liquidity are combined, the overall effect on global liquidity is detrimental. Foley and Putnins (2015) and Comerton-Forde, Malinova, and Park (2015) study the introduction of a trade-at rule in Canada which effectively prohibited SPT. Their evidence suggests that SPT in dark venues using QJ results in improved market quality, but that trading in MID crossing networks are not associated with improvements in market quality. Foley and Putnins (2014) find similar evidence based on a natural experiment in Australia.

Our paper is also related to Kwan, Masulis and McInish (2015) who study the effects of competition for order flow on the fragmentation of U.S. equity markets and point out the relevance of SPT, and in particular of QJ, in the distribution of market shares across lit and dark venues. More precisely, they consider NASDAQ and NYSE stocks and show that when the price of a stock crosses the $1 threshold, volume in dark venues increases while volume in traditional exchanges decreases. Finally, Bartlett and McCrary (2013) find both a significant increase in the fraction of offers submitted as sub-penny orders below the $1.00 cut-off and a positive effect on market quality.

On the theory side, our paper is related to the models that study how limit order books work.[10] As SPT crucially depends on the tick size and the state of the book -measured by depth and spread- to deliver testable empirical hypotheses the model must have discrete prices. Moreover, the model's equilibrium prices cannot be derived under the assumption of steady state which would inevitably imply a constant state of the PLB.[11] To satisfy these requirements and explicitly analyze QJ, we therefore extend Parlour (1998) and draw our main empirical hypotheses from the resulting framework. The framework we build here departs from Werner, Wen, Rindi, Consonni and Buti (WWRCB, 2015) who consider the effects of a reduction in the tick size of a single limit order book on market quality and welfare. Here instead we model competition between two limit order books, a PLB and a SPV, with different tick sizes. Our framework also differs from Buti, Rindi and Werner (2015) in which a public limit order book competes with a dark pool that executes at the mid-quote of the PLB.

3 Model and Empirical Hypotheses

We draw our main empirical hypotheses from a theoretical framework that extends Parlour (1998) to model competition between a PLB and a SPV. In what follows we describe the design of both the PLB and the SPV. We first consider the PLB (our benchmark model) in which traders can only post orders to a standard limit order book. We then compare this model with the extended framework in which a subset of traders - broker-dealers (BDs) - are allowed either to submit orders to the PLB, or to undercut limit orders posted at the top of the PLB by submitting orders in sub-penny increments to the SPV. As discussed in the introduction, Rule 612 prohibits market operators from accepting, ranking, or displaying sub-penny orders. However, as evidenced by the recent SEC allegation, sub-penny orders have been used in dark pools during the 2008-2011 period. Moreover, the Rule makes exceptions for price improvements offered by BDs, and these affect the incentives to submit orders to the PLB in the same way as a formal SPV. Hence, for tractability, we model the ability for BDs to undercut the PLB as a SPV. We derive our empirical hypotheses by investigating how the state of the PLB affects BDs' decisions whether to submit future orders to the SPV or the PLB and how this affects the choice of traders without access to the SPV - regular traders (RTs) - between submitting limit orders to the PLB or opting for market orders. We also study the effects that the SPV trading option has on the market quality of the PLB as reflected in spreads and depth.

[10] To mention but a few, see Buti and Rindi (2013), Goettler, Parlour and Rajan (2005), and Parlour (1998).

[11] See, e.g., Foucault (1999), Foucault, Kadan and Kandel (2005), and Rosu (2009).

3.1 Public Limit Order Book

The PLB works like a double auction market that enforces time and price priority and runs over a trading day divided into 4 periods of time, $t = \{t_1, t_2, t_3, t_4\}$. At each period t a risk neutral trader arrives and can submit an order of size 1 to buy or to sell an asset with average value equal to v. Each trader comes to the market with a private value, $\beta\nu$, where β is drawn from the uniform distribution with support $U(0,2)$. A non aggressive trader has a β close to 1 while an eager one has values of β close to either 0 (seller) or 2 (buyer).

Upon arrival in period t, the trader observes the state of the book, which is characterized by the number of shares available at each level of the price grid (Table 1), and includes the following possible prices: $A_1 = v + \frac{\tau}{2}, A_2 = v + \frac{3}{2}\tau$, and $B_1 = v - \frac{\tau}{2},\ B_2 = v - \frac{3}{2}\tau$. The difference between two adjacent prices -the minimum price increment- is the tick size which we set equal to a constant value $\tau_{PLB} = \tau$, and which also corresponds to the minimum inside spread. We define as $S_t = [Q_t^{A_1}, Q_t^{A_2}, Q_t^{B_1}, Q_t^{B_2}]$ the state of the book that specifies the number of shares Q_t available at each price level. A trading crowd provides liquidity at the highest levels of the PLB, and traders are allowed to submit limit orders queuing in front of it. Upon arrival each trader can choose between limit orders $(+1)$ and market orders (-1), and can also choose not to trade.

Market orders are always executed at the best available ask or bid price and their payoff depends on the traders' personal evaluation of the asset, $\beta\nu$. Limit orders' payoff also depends on their execution probability that we indicate by $p_t(A_k|S_t)$ and $p_t(B_k|S_t)$ for a limit sell and for a limit buy order respectively submitted at the ask price A_k, or at the bid price B_k. The payoff are summarized in Table 2. The trader's strategy space at time t is therefore $H_t = \left\{-1^{A_{k'}}, +1^{A_k}, 0, +1^{B_k}, -1^{B_{k'}}\right\}$, where $+1^{A_k}(-1^{B_{k'}})$ indicates a limit (market) order to sell submitted at the k^{th} level of the book, with $k = 1, 2$, and k' the best available bid price; analogous strategies are available to the trader on the buy side and 0 indicates that the trader decided not to trade.

[Insert Tables 1 and 2 about here]

The state of the PLB evolves according to $S_t = S_{t-1} + h_t$, where S_{t-1} and S_t are respectively the states of the book before and after the trader's order submission and h_t is the change in the PLB induced by the trader's strategy H_t. We assume that when a trader arrives at t_1, the PLB is empty, $S_{t_0} = [0, 0, 0, 0]$. Without loss of generality, in our numerical simulations we assume that the tick size in the PLB is $\tau_{PLB} = 0.1$, and that the price of the asset is $v = 1$.

Figure 1 shows the extensive form of the game. Assume that a trader arriving at the market at t_1 decides to submit a limit sell order at A_2, $H_{t_1} = +1^{A_2}$, so that the state of the book at the end of period t_1 is $S_{t_1} = [1, 0, 0, 0]$. The execution probability of this order depends both on the initial state of the book and on the future orders submitted by the other market participants on both sides of the market. As no shares were standing at A_1 and A_2 before the sell order was submitted, this order is at the top of the queue on the sell side and would be executed if a market buy order arrives in period t_2, and if this does not happen the order could execute in t_3 or t_4 if no subsequent seller submits an order at A_1. Suppose that the trader arriving at t_2 decides among all strategies shown in Figure 1 to opt for a limit buy order at B_1, $H_{t_2} = +1^{B_1}$; in this case the limit sell order posted at t_1 will not be immediately executed but the probability that it is executed at t_3 is higher compared to the scenario in which the incoming trader at t_2 chooses for example a limit order to sell at A_1, for two reasons. First, if a trader submits a limit sell order at A_1, the limit order initially posted at A_2 would be at the back of the queue, with a smaller execution probability; second, a deep book on the bid side at t_2 implies that the potential buyer in the next period t_3 would be more inclined to post a market rather than a limit order, due to the long queue on the bid side. Suppose now that at t_3 the incoming trader after observing $S_{t_2} = [1, 0, 1, 0]$, decides to gain price priority by submitting a limit sell order at A_1, $H_{t_3} = +1^{A_1}$. In this case, at t_4 the book will open with one share both at the first and at the second price level of the ask side , $S_{t_3} = [1, 1, 1, 0]$, and only the order posted at A_1 will have a positive execution probability. Recall that only one trader arrives at each trading round.

[Insert Figure 1 about here]

The model is solved by backward induction. Because at time t_4 the execution probability of limit orders is zero, traders either submit market orders or decide not to trade. Therefore, traders' equilibrium strategies are:

$$H^*_{t_4}(\beta|S_{t_3}) = \begin{array}{ll} -1^{B_{k\prime}} & \text{if } \beta \in [0, \frac{B_{k'}}{v}) \\ 0 & \text{if } \beta \in [\frac{B_{k'}}{v}, \frac{A_{k'}}{v}) \\ -1^{A_{k\prime}} & \text{if } \beta \in [\frac{A_{k'}}{v}, 2] \end{array} \tag{1}$$

By using these equilibrium strategies together with the distribution of β, we calculate the equilibrium execution probabilities of limit orders submitted at t_3:

$$p^*_{t_3}(A_k|S_{t_3}) = \begin{array}{ll} \int_{\beta\in[\beta:H^*_{t_4}=-1^{A_k\prime}]} \frac{1}{2}d\beta = 1 - \frac{A_{k'}}{2v} & \text{if } A_k = A_{k\prime} \text{ and } Q^{A_k}_{t_2} = 0 \\ 0 & \text{otherwise} \end{array} \tag{2}$$

$$p^*_{t_3}(B_k|S_{t_3}) = \begin{array}{ll} \int_{\beta\in[\beta:H^*_{t_4}=-1^{B_k\prime}]} \frac{1}{2}d\beta = 1 - \frac{B_{k'}}{2v} & \text{if } B_k = B_{k'} \text{ and } Q^{B_k}_{t_2} = 0 \\ 0 & \text{otherwise} \end{array} \tag{3}$$

The execution probabilities of limit orders, $p_{t_3}(A_k|S_{t_3})$ and $p_{t_3}(B_k|S_{t_3})$, are the dynamic link between period t_4 and t_3. Because there is only one period left in the trading game, the execution probability of a limit order submitted at t_3 is positive only if the order is posted at the best ask ($A_{k\prime}$) or bid price ($B_{k\prime}$), and if there are no other orders already standing in the book at that price.

A trader arriving at t_3 can choose between a market and a limit order, or decide not to trade. The equilibrium strategies for t_3 depend on the state of the book. As an example, here we discuss the equilibrium strategies for the book opening with room for limit orders on both sides of the market, i.e., $p^*_{t_3}(A_k|S_{t_3}) \neq 0$ and $p^*_{t_3}(B_k|S_{t_3}) \neq 0$.

The trader's optimal strategies are:

$$H^*_{t_3}(\beta|S_{t_2}) = \begin{array}{ll} -1^{B_{k\prime}} & \text{if } \beta \in [0, \beta_{-1^{B_{k\prime}},+1^{A_k},t_3|S_{t_2}}) \\ +1^{A_k} & \text{if } \beta \in [\beta_{-1^{B_{k\prime}},+1^{A_k},t_3|S_{t_2}}, \beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}}) \\ +1^{B_k} & \text{if } \beta \in [\beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}}, \beta_{+1^{B_k},-1^{A_{k\prime}},t_3|S_{t_2}}) \\ -1^{A_{k\prime}} & \text{if } \beta \in [\beta_{+1^{B_k},-1^{A_{k\prime}},t_3|S_{t_2}}, 2] \end{array} \tag{4}$$

where

$$\beta_{-1^{B_{k\prime}},+1^{A_k},t_3|S_{t_2}} = \frac{B_{k'}}{v} - \frac{p^*_{t_3}(A_k|S_{t_3})}{1 - p^*_{t_3}(A_k|S_{t_3})} \frac{A_k - B_{k'}}{v}, \tag{5}$$

$$\beta_{+1^{A_k},+1^{B_k},t_3|S_{t_2}} = \frac{p^*_{t_3}(A_k|S_{t_3})A_k + p^*_{t_3}(B_k|S_{t_3})B_k}{p^*_{t_3}(A_k|S_{t_3}) + p^*_{t_3}(B_k|S_{t_3})} \frac{1}{v}, \tag{6}$$

$$\beta_{+1^{B_k},-1^{A_{k'}},t_3|S_{t_2}} = \frac{A_{k'}}{v} + \frac{p^*_{t_3}(B_k|S_{t_3})}{1 - p^*_{t_3}(B_k|S_{t_3})} \frac{A_{k'} - B_k}{v}. \tag{7}$$

These thresholds are derived by taking into account that the trader arriving at the beginning of period t_3 observes the state of the book S_{t_2}. For instance $\beta_{-1^{B_{k'}},+1^{A_k},t_3|S_{t_2}}$ denotes the threshold between a market sell order hitting the best bid price, $B_{k\prime}$, and a limit sell order posted at the ask price, A_k, and it is derived by equating the payoffs of the two orders. Note that the greater the limit order execution probability, $p^*_{t_3}(A_k|S_{t_3})$, the smaller this threshold and the higher will the probability that traders choose limit rather than market orders be. More generally, if the limit order execution probability at time t is high enough for non-execution costs to be lower than price opportunity costs, the trader will submit a limit order. If instead the execution probability is low, he will choose a market order. This trade-off crucially depends on the value of the tick size and on the price of the stock, v. Specifically, the larger is τ relative to v, the higher is the tick-to-price ratio, and the costlier will the market order be relative to the limit order.

When a trader chooses a limit order, he also has to decide how aggressively to submit this order relative to v. The optimal price at which a trader submits a limit order is also the result of the trade-off between non-execution costs and price opportunity costs: a more aggressive price implies a higher execution probability due to both the lower risk of being undercut by incoming traders and the fact that the order becomes more attractive for traders on the opposite side of the market. However, this is obtained at the cost of lower revenue once the order is executed.

From the equilibrium strategies at t_3 we can derive the execution probabilities for limit orders submitted at t_2 and the corresponding equilibrium strategies. The same procedure is then re-iterated to obtain the equilibrium order submission strategies at t_1. Due to the recursive structure of the game, and because traders are indifferent between orders with a zero execution probability, a unique equilibrium always exists.

3.2 Sub-Penny Venue

In order to include QJ among the trading strategies of market participants, we extend the benchmark model with only a PLB to include competition from a SPV. All else equal, the SPV has a smaller tick size, $\tau_{SPV} = \frac{\tau}{3}$, and therefore a finer price grid with five levels on both the ask and the bid side, a_l and b_l $(l = 1, .., 5)$, which allows market participants to trade at sub-penny increments (Table 1). We assume that at t_1 the SPV opens empty as the PLB, $S_{t_0}^{SPV} = [0]$. Furthermore, since the SPV is trading against the backdrop of the PLB, we assume that the SPV does not have a trading crowd sitting at a_5 and at b_5.[12]

To add further realism to this new framework, which we label PLB&SPV, we assume that the market is populated by two groups of traders, namely broker-dealers (BDs) and regular traders (RTs), who arrive in each period with probability α and $1 - \alpha$, respectively, and that only BDs are allowed to supply liquidity to the SPV.[13] Yet, while only BDs can post limit orders to the SPV, all traders can take advantage of the liquidity offered by both trading platforms and demand liquidity on both the PLB and the SPV. This assumption is consistent with a fast market in which a SOR technology allows all investors to search the best quotes on the consolidated limit order book that includes both the PLB and the SPV. So, as long as the standing orders in the SPV offer better prices than the PLB, all traders, RTs included, can execute against them. Orders posted to the SPV will offer a better price than the PLB when BDs engage in QJ and therefore compete on price against the limit orders posted at the top of the PLB.

As we did for the benchmark framework with only a PLB, we solve the new PLB&SPV framework, in which a SPV competes with the PLB, by backward induction starting from t_4, and denote by $A = A_{k'} \bigvee a_{l'}$ and $B = B_{k'} \bigvee b_{l'}$ the best prices across both PLB and SPV. Note that at t_4 the equilibrium strategies of RTs and BDs are the same: both can

[12] This extension differs from WWRCB (2015) in that here we consider two different trading platform that compete with each other for the provision of liquidity, whereas in WWRCB a PLB opens with a large tick size which is subsequently reduced at the second period of the trading game.

[13] The reason why we do not allow all traders to access the SPV is that in real markets retail traders cannot access SPVs. As for the BDs' trading strategies, we also assume that when the BDs' payoffs from trading across the two markets are the same, they submit orders to both markets with equal probability.

observe the best available price and, because traders submit only market orders, the BDs can't take advantage of their ability to post liquidity on the SPV. Therefore, the orders' execution probabilities at t_3 do not depend on the type of trader arriving at t_4, hence $p^{*RT}_{t_3}(A_k|S^{PLB}_{t_3}, S^{SPV}_{t_3}) = p^{*BD}_{t_3}(A_k|S^{PLB}_{t_3}, S^{SPV}_{t_3}) = p^{*}_{t_3}(A_k|S^{PLB}_{t_3}, S^{SPV}_{t_3})$.

Consider as an example the case in which at t_3 the PLB opens as $S^{PLB}_{t_2} = [0110]$ and the SPV is empty, $S^{SPV}_{t_2} = [0]$. This case can originate from a limit buy order at B_1 submitted at t_1, $H_{t_1} = +1^{B_1}$, and a limit sell order at A_1 submitted at t_2, $H_{t_2} = +1^{A_1}$.[14] If a RT arrives at t_3, his possible payoffs are:

$$\begin{array}{rcl} H^{RT}_{t_3} = & -1^{B_1} & : \; B_1 - \beta v \\ H^{RT}_{t_3} = & -1^{A_1} & : \; \beta v - A_1 \\ H^{RT}_{t_3} = & 0 & : \; 0 \end{array} \tag{8}$$

and his equilibrium strategies are:

$$H^{*RT}_{t_3}(\beta|[0110],[0]) = \begin{array}{cl} -1^{B_1} & \text{if } \beta \in [0, \beta^{RT}_{-1^{B_1},0,t_3|[0110],[0]}) \\ 0 & \text{if } \beta \in [\beta^{RT}_{-1^{B_1},0,t_3|[0110],[0]}, \beta^{RT}_{0,-1^{A_1},t_3|[0110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{RT}_{0,-1^{A_1},t_3|[0110],[0]}, 2) \end{array} \tag{9}$$

By using the optimal β-thresholds associated with these strategies, we compute the execution probability of $H_{t_2} = +1^{A_1}$ conditional on a RT arriving at t_3:

$$\begin{aligned} p^{*RT}_{t_2}(A_1|[0110],[0]) \;=\; & \frac{\beta^{RT}_{0,-1^{A_1},t_3|[0110],[0]} - \beta^{RT}_{-1^{B_1},0,t_3|[0110],[0]}}{2} \cdot p^{*}_{t_3}(A_1|[0110],[0]) \\ & + \frac{2 - \beta^{RT}_{0,-1^{A_1},t_3|[0110],[0]}}{2} + \frac{\beta^{RT}_{-1^{B_1},0,t_3|[0110],[0]}}{2} \cdot p^{*}_{t_3}(A_1|[0100],[0]) \end{aligned} \tag{10}$$

If instead a BD arrives at t_3, his possible payoffs are:

$$\begin{array}{rcl} H^{BD}_{t_3} = & -1^{B_1} & : \; B_1 - \beta v \\ H^{BD}_{t_3} = & +1^{a_l} & : \; (a_l - \beta v) \cdot p^{*}_{t_3}(a_l|[0110],[Q^{a_l} = 1]) \\ H^{BD}_{t_3} = & +1^{b_l} & : \; (\beta v - b_l) \cdot p^{*}_{t_3}(b_l|[0110],[Q^{b_l} = 1]) \\ H^{BD}_{t_3} = & -1^{A_1} & : \; \beta v - A_1 \\ H^{BD}_{t_3} = & 0 & : \; 0 \end{array} \tag{11}$$

where for example $[Q^{a_l} = 1]$ indicates a SPV with one unit at a_l and empty at all other price levels. Both limit and market orders are equilibrium strategies because $p^{*}_{t_3}(a_l|[0110],[Q^{a_l} = 1]) \neq 0$ and $p^{*}_{t_3}(b_l|[0110],[Q^{b_l} = 1]) \neq 0$ for $l = 1, 2$. However, traders need to determine the level of aggressiveness of their limit orders. We consider the bid side as an example: for $H_{t_3} = +1^{b_1}$

[14] Another possibility is to have a t_1 a limit sell order at A_1, followed by a limit buy order at B_1.

to be an equilibrium strategy, $\beta_{+1^{b_1},-1^{A_1},t_3|[0110],[0]} > \beta_{+1^{b_2},-1^{A_1},t_3|[0110],[0]}$. Figure 2 shows that this is always the case for $\frac{\tau}{v} \in (0, 0.1]$. Because $\beta_{+1^{b_2},+1^{b_1},t_3|[0110],[0]} > \beta_{+1^{a_2},+1^{b_2},t_3|[0110],[0]}$, also $H_{t_3} = +1^{b_2}$ is an optimal strategy. The equilibrium strategies for the BD are:

$$H^{*BD}_{t_3}(\beta|[0110],[0]) = \begin{cases} -1^{B_1} & \text{if } \beta \in [0, \beta^{BD}_{-1^{B_1},+1^{a_1},t_3|[0110],[0]}) \\ +1^{a_1} & \text{if } \beta \in [\beta^{BD}_{-1^{B_1},+1^{a_1},t_3|[0110],[0]}, \beta^{BD}_{+1^{a_1},+1^{a_2},t_3|[0110],[0]}) \\ +1^{a_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_1},+1^{a_2},t_3|[0110],[0]}, \beta^{BD}_{+1^{a_2},+1^{b_2},t_3|[0110],[0]}) \\ +1^{b_2} & \text{if } \beta \in [\beta^{BD}_{+1^{a_2},+1^{b_2},t_3|[0110],[0]}, \beta^{BD}_{+1^{b_2},+1^{b_1},t_3|[0110],[0]}) \\ +1^{b_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_2},+1^{b_1},t_3|[0110],[0]}, \beta^{BD}_{+1^{b_1},-1^{A_1},t_3|[0110],[0]}) \\ -1^{A_1} & \text{if } \beta \in [\beta^{BD}_{+1^{b_1},-1^{A_1},t_3|[0110],[0]}, 2) \end{cases} \quad (12)$$

It follows that, conditional on a BD arriving at t_3, the execution probability of the limit order posted at A_1 is:

$$\begin{aligned} p^{*BD}_{t_2}(A_1|[0110],[0]) \quad = \quad & \frac{\beta^{BD}_{+1^{a_2},+1^{b_2},t_3|[0110],[0]} - \beta^{BD}_{+1^{a_1},+1^{a_2},t_3|[0110],[0]}}{2} \cdot p^*_{t_3}(A_1|[0110],[a_2]) \quad (13) \\ & + \frac{\beta^{BD}_{+1^{b_2},+1^{b_1},t_3|[0110],[0]} - \beta^{BD}_{+1^{a_2},+1^{b_2},t_3|[0110],[0]}}{2} \cdot p^*_{t_3}(A_1|[0110],[b_2]) \\ & + \frac{\beta^{BD}_{+1^{b_1},-1^{A_1},t_3|[0110],[0]} - \beta^{BD}_{+1^{b_2},+1^{b_1},t_3|[0110],[0]}}{2} \cdot p^*_{t_3}(A_1|[0110],[b_1]) \\ & + \frac{\beta^{BD}_{-1^{B_1},+1^{a_1},t_3|[0110],[0]}}{2} \cdot p^*_{t_3}(A_1|[0100],[0]) + \frac{2 - \beta^{BD}_{+1^{b_1},-1^{A_1},t_2|[1110],[0]}}{2}. \end{aligned}$$

We compute the total execution probability of the limit order posted at A_1 at t_2 as the weighted average of the two conditional probabilities:

$$p^*_{t_2}(A_1|[0110],[0]) = \alpha p^{BD*}_{t_2}(A_1|[0110],[0]) + (1-\alpha) p^{RT*}_{t_2}(A_1|[0110],[0]) \quad (14)$$

Similarly, we compute the equilibrium strategies for all the other possible states of the book at t_3 and obtain the execution probabilities of the different order types available at t_2. This allows us to derive the equilibrium strategies at t_2. We follow an analogous procedure to solve for the equilibrium order submission strategies at t_1.

[Insert Figure 2 about here]

3.3 Empirical Hypotheses

We now use this model to draw our main empirical hypotheses on how liquidity of the PLB affects QJ and on how QJ in turn affects market quality. As the PLB opens empty at t_1, we compare two different states of the book at the opening of the second trading period, t_2, which result from the equilibrium order submission strategies of traders arriving at the first period, t_1. So, we exploit the endogenous liquidity generated at t_1 to compare results for liquid and illiquid books at t_2.

3.3.1 Factors driving QJ

Consider the order submission strategies of traders in period t_2 with two more periods of the game remaining during which a limit order submitted in t_2 may execute. Figure 3 summarizes how a seller's equilibrium order submission strategies at t_2 translate into order submission probabilities for the different price levels under two scenarios: a liquid book (Liquid PLB) which opens with one share at the best ask A_1 (Panel A), and an illiquid book (Illiquid PLB) which opens with no resting orders, just the trading crowd at A_2 (Panel B). In both cases, we assume that the SPV opens empty.[15] The BDs arrival rate is set to $\alpha = 50\%$, and $\tau = 0.1$ in the numerical example. We report the submission probabilities both for the setup with only the PLB and for the PLB&SPV setup in each panel.

Figure 3 shows that BDs engage more intensively in QJ when competition for the provision of liquidity is strong on the PLB (Panel A), i.e., when there are orders at the top of the book and therefore the quoted half-spread $(A_1 - v)$ is small. Resting orders have time priority, which means that new orders at A_1 would go to the back of the queue. BDs therefore have an incentive to gain price priority by undercutting limit orders posted at the top of the PLB by submitting limit orders on the SPV at a_1. Moreover, as the half-spread is constrained by the tick size, there is no room to post PLB limit orders within the inside spread and there is therefore an additional incentive for traders to undercut existing liquidity via QJ on the SPV. All traders understand these incentives, and that future market orders therefore are more likely to be intercepted by the SPV. As a result, they rationally reduce their own PLB liquidity supply at A_2 compared to when no SPV exists.

When the book opens at t_2 without any depth at the inside (Illiquid PLB) and as before the SPV is also empty, traders still decrease their supply of liquidity to the PLB as in the case of the liquid PLB. However, with an illiquid book undercutting at a_1 is smaller in equilibrium because traders can gain priority over the crowd by simply submitting an order at A_2 which generates a larger surplus.

This leads to our first hypothesis on the factors driving QJ.

[Insert Figure 3 about here]

Hypothesis 1. QJ is higher if the book is liquid, i.e., characterized by high inside depth and/or a narrow inside spread, holding the tick size and the stock price constant.

[15] Buyer equilibrium order submission strategies are symmetric.

In our model traders determine whether to submit a market order or a limit order based on the standard trade-off that is related to the tick-to-price ratio. Specifically, the larger is τ relative to v, the higher is the tick-to-price ratio, and the costlier will the market order be relative to the limit orders posted both to the PLB and to the SPV. We solve the model for both $v = 1$ and $v = 1.05$ and therefore compare the results for different relative tick sizes. This leads to our second hypothesis.

Hypothesis 2. QJ is increasing in the tick-to-price ratio: for a given tick size, QJ is decreasing in the stock price.

3.3.2 Effects of QJ on market quality.

We now move to the hypotheses on the effects of QJ on the quality of regular exchanges. When traders post limit orders on the SPV to undercut existing liquidity on the PLB, liquidity provision and hence depth and spread on the PLB may deteriorate. However, liquidity demand on the PLB may also decrease. The reason is that, due to the existence of SORs, the better prices available in the SPV intercept market orders away from the PLB. The reduced liquidity demand mitigates the effect of the reduced liquidity supply on the depth and spread on the PLB. Therefore, the effects of QJ on the quality of regular exchanges depend on the relative proportion of limit and market orders that migrate from the PLB to the SPV. The net effect of the reduced supply and demand of liquidity on the quality of the PLB depends on the characteristics of the stock considered; specifically, it depends on the liquidity of the book as well as on the price of the stock and therefore on the relative tick size.

Consider first a liquid book. As we showed in Figure 3, Panel A, when the SPV platform is introduced, BDs submit limit orders to the SPV at a_1 in period t_2 to undercut the existing depth at A_1.[16] By comparison, Figure 4 illustrates what happens across all remaining periods and shows that liquidity provision in the SPV (LP_{SPV}) increases. These SPV orders intercept future incoming market orders away from the PLB, and SPV executions (VL_{SPV}) also rise. The effect of BDs' undercutting is that both limit and market orders move from the PLB to the SPV thus reducing both liquidity provision (LP_{PLB}) and volume (VL_{PLB}) on the public venue. The result is that inside depth (DP_{BBO}) increases and spread, both quoted (SP) and relative (RSP), narrow. The reason for why market quality improves in the PLB despite the reduction of the liquidity provision is that when the book is liquid, traders use more market than limit orders and consequently the positive effect of the reduced liquidity demand - which helps preserve the low spread and high depth - prevails. In other words, our model shows that for liquid stocks the main effect of sub-penny trading is to foster price competition. Finally, note that as volume in the PLB decreases significantly, the increase in volume in the SPV is not large enough to compensate for the decline, which means that consolidated volume

[16] To economize space we do not present results for the bid side which are symmetric.

($VL_{TOT} = VL_{PLB} + VL_{SPV}$) is smaller in a liquid book with a SPV.

We now turn to the framework with an illiquid opening book (Figure 4). Even for less liquid stocks, when the SPV is introduced both limit and market orders move to the SPV and consequently both liquidity provision (LP_{PLB}) and volume (VL_{PLB}) decline in the PLB. Compared to the liquid case, however, when the book opens illiquid, all else equal, traders tend to use more limit than market orders with the result that more limit than market orders migrate from the PLB to the SPV. As for order flows in the SPV, liquidity provision (LP_{SPV}) and volume (VL_{SPV}) increase but by less than when the book is liquid. Under this scenario with a smaller proportion of market orders migrating to the SPV, the increase in volume in the SPV outweighs the reduction in the PLB and total volume increases.

[Insert Figure 4 about here]

We summarize the results about QJ and market quality in the following hypotheses:

Hypothesis 3. QJ results in a reduction in both the PLB liquidity provision and the PLB volume. SPV volume increases sufficiently for consolidated volume to rise only in the illiquid scenario.

Hypothesis 4. QJ results in improved (worse) depth and spread when the book is liquid (less liquid).

As discussed in the introduction, while several Exchanges have requested that the tick size be lowered to allow sub-penny quoting for all markets, there is at the same time pressure from Congress to raise the tick size for selected stocks following the recommendations in the JOBS Act. This tick-size debate motivates an investigation into how the tick size affects SPT. Because it is the relative tick size that is economically relevant for traders, we compare the model's equilibrium with two different asset values that correspond to different relative tick size. Specifically, we compare the case with $v = 1$ and τ=0.1 to the case with a higher price, $v = 1.05$, and the same tick size which results in a smaller relative tick size. As expected, Figure 4 shows that a smaller relative tick size reduces traders' incentive to post limit orders, thus reducing the incentive to post orders to the SPV. Consequently, a smaller relative tick size results in less QJ activity which clearly attenuates all the QJ effects discussed above. This result motivates the following hypotheses.

Hypothesis 5. With a smaller relative tick size QJ decreases and all the effects of QJ are weaker.

We move to empirically testing the model's hypotheses in the next section.

4 Data Description

4.1 Data and Sample

We construct a sample of stocks stratified by price and market capitalization for both NASDAQ and NYSE. As of December 31, 2009 we identify all common stocks in CRSP which are NYSE or NASDAQ listed. Then we divide all NYSE listed common stocks (i.e., share code 10 or 11) into terciles by market capitalization and price and form nine mutually exclusive groups with the same dimension. From each group we randomly draw 10 stocks for a total of 90 NYSE stocks. We repeat the procedure for NASDAQ stocks using the NYSE breakpoints. This way we create groups of stocks which are comparable across exchanges by market capitalization and price. Our final sample includes 180 stocks and spans from October 1 to November 30, 2010, for a total of 42 trading days.

4.2 Descriptive Statistics and Definitions

The data we use come from the following sources. Data on number of shares outstanding, market capitalization, closing prices, listing exchanges and share codes are from CRSP. Data for S&P 500 Stock Price Index (SP500) and CBOE S&P 500 Volatility Index (VIX) used to capture market-wide activity are from Federal Reserve of Economic Data (FRED). Our main data source is the TAQ database which we describe in more detail below.

TAQ contains intraday transactions data (trades and quotes) for all securities listed on the NYSE and American Stock Exchange (AMEX), as well as NASDAQ. The TAQ database includes a flag indicating the exchange where the trade was executed or the quote displayed. Each exchange is identified by a symbol/capital letter (except for NASDAQ which has two equivalent symbols, T and Q). The letter "D" is used for trades reported to one of the Trade Reporting Facilities (TRFs) and for the NASD Alternative Display Facility (ADF). These facilities record transactions from OTC markets, some transparent non-exchange Electronic Communications Networks (ECNs), broker internalization and dark pools. The crucial information for our analysis is that dark pools trades are reported as "D". The reason is that dark pools do not have to publicly report their quotes and so they do not have to comply with Rule 612 of Reg NMS, which only refers to quoted prices. As a result sub-penny trades executed away from exchanges are reported with the exchange code "D".[17]

We consider only trades and quotes which take place between 9:30:00 AM and 4:00:00 PM. For each day and for each stock we derive the NBBO using the Wharton Research Data Services (WRDS) suggested procedure and we compute the time-weighted bid, ask and total

[17]We can also observe sub-penny trades with other exchange codes. In this case, however, the price improvement is exactly equal to half-cent and is the result of a trade in exchange operated dark pools pricing at the NBBO mid-quote.

depth, the time-weighted quoted spread in cents and in percentage of the mid-quote, and the intraday price range, defined as (high-low)/high, as a measure of intraday volatility.

We remove erroneous and irregular trades; in particular we keep only trades whose correction indicator is either "00" or "01". We then compute the share volume and (buy) order imbalance defined as the absolute value of ((buys-sells)/share volume) where buys are classified using a modified Lee and Ready (1991) algorithm.[18] Table 3 shows descriptive statistics for our sample of stocks divided by exchange and capitalization.

[Insert Table 3 about here]

4.3 Price Improvement

In order to identify and classify SPT, we construct an auxiliary variable, the price improvement (PI), which is computed as follows. First, prices are rounded up to the closest cent for sell orders and rounded down to the closest cent for buy orders. Second, PI is obtained as the difference between the rounded price and the reported price. It is always restricted to the interval (0.00, 0.01). Therefore PI is equal to:

$$PI = |rounded(price) - price| \tag{15}$$

To better understand what we are actually measuring in the data in Table 4, we report a sample of a 3 seconds interval of trade for IBM. For convenience we number transactions with a progressive identification number. Furthermore we report the rounded price and the price improvement according to our definition. An example of QJ can be seen on line 19, and an example of MID can be found on line 30. We can also see that, in both cases, the transactions have been executed on exchange D. We are now able to properly define SPT as a function of PI. In particular, we have no SPT when PI is equal to zero and SPT when PI is different from zero.

[Insert Table 4 about here]

SPT can be further classified into MID, when PI is exactly equal to half-cent (0.005), and QJ, when PI is strictly positive but different from half-cent.[19] Hence, our rounding procedure is immaterial to the definition of SPT, MID and QJ.

We can use the PI measure to illustrate how large SPT is in the U.S. equity markets.

[18] We first apply a tick-test considering at most two previous trades. We classify a trade as a buy if its price is above the price of the previous trade (or two trades before); otherwise we classify the trade as a sell. If the trade is still unclassified, we classify it as a buy (sell) if the execution price is above (below) the mid-quote at the time of the trade.

[19] Our measure of MID is a lower bound of executions at the mid-quote. The reason is that we are considering as mid-crossing only those executions which result in a sub-penny price. If the spread turns out to be an even multiple of the tick, we will not classify it as MID with our methodology.

Interpreting PI as the gain from SPT and multiplying it by the number of shares traded in SPT, we obtain the dollar volume captured by stepping ahead of the queue. For example, consider Adobe Systems Inc.: in our sample period the daily average of shares traded is 14.8 million. Of these, 1.62 million are traded in SPT divided between QJ (1.25 million) and MID (0.37 million), which correspond respectively to 8.4% and 2.5% of consolidated share volume. Since QJ and MID effectively lower the trading costs for active traders, we can compute the average daily gain from SPT for these traders by multiplying PI by the corresponding number of shares traded. For Adobe Systems Inc., active traders gain on average $2,500 from QJ and $1,850 from MID per day. Note that this gain from SPT for active traders corresponds to a loss for passive traders who are being undercut, but this loss is much more difficult to measure.

Figures 5 and 6 report the plot of the average QJ and MID across the sample period for each stock as a function of price, for NASDAQ and NYSE. Graphically we see that on both markets QJ generally decreases with price, while MID increases. Overall the percentage of volume traded in SPT increases when the stock value decreases and seems to be consistent with the fact that a one cent tick size is a binding constraint for low-priced stocks.

[Insert Figures 5-6 about here]

To study the distribution of PIs, in Figures 7 - 8 we group stocks into 10 bins of size 0.001 for two groups of 30 high-priced NASDAQ stocks and 30 high-priced NYSE stocks, separately. For each bin, we report the associated SPT dollar volume as a percentage of the total traded volume; measuring it in share volume or number of trades yields qualitatively the same results. For example, Figure 7 shows that 3.05% of total NASDAQ traded volume is executed with a PI which lies in the interval (0,0.001], while for NYSE it amounts to 2.44%. The most common type of SPT corresponds to PI equal to 0.005. The distribution of PIs decreases almost monotonically as the PI increases, except for the spike at PI exactly equal to half-cent.

[Insert Figures 7-8 about here]

5 Factors Driving Sub-Penny Trading

To study how SPT varies with market characteristics, and test Hypotheses 1 and 2, we start with daily Fama-MacBeth cross-sectional regressions for both QJ and MID executions. Since MID executions arise both from trading systems using a derivative pricing rule and from undercutting the PLB by exactly half a cent, we expect the results to be weaker for MID executions. We also note that MID executions are more likely to arise from periodic trading systems such as crossing networks, and this may adversely affect traders willingness to submit MID orders when volatility is high. On the other hand, MID executions may be more likely to arise when order imbalances are high as traders using SPVs become more aggressive facing buying/selling pressure.

We are interested in how QJ and MID are affected by the market conditions as captured by quoted spread and depth and the relative tick size as captured by price. However, we also acknowledge that there may be cross-sectional differences that also affect QJ and MID executions but that are not captured by our model. For example, our model does not have fundamental uncertainty and there is no asymmetric information. The model does not incorporates fees, trading is driven entirely by differences in valuation and we assume that the distribution of valuations is symmetric. A skewed distribution of valuations causes either buying or selling pressure depending on the location of the mean, and a smaller range of valuations symmetric around 1 (e.g. [0.5, 1.5]) implies that traders use fewer market orders. We want to control for the β distribution in the cross-section when evaluating the effect of our variables of interest. The β distribution is of course not observable, but we proxy for differences in the β distributions across stocks by the absolute value of the percent order imbalance (to measure buying and selling pressure), and the intraday range and volume (to measure the dispersion in valuations). Finally, we acknowledge that there may be structural differences between stocks predominately traded on the NYSE compared to stocks traded on NASDAQ, and therefore include a dummy to capture the listing exchange.

Motivated by the discussion above, we use two sets of regressors in the analysis. When testing Hypothesis 1 (the positive relationship between QJ and book liquidity), we include a dummy variable which is equal to one when the stock is NYSE listed, time-weighted cent quoted spread and log of time-weighted bid depth to measure market conditions, the absolute order imbalance in percent as a measure of one-sided market, and intraday percent price range as a measure of volatility.[20] When testing Hypothesis 2 (the positive relationship between QJ and tick-to-price ratio), we rely on closing price to examine the effect of the relative tick size, log of share volume as a measure of differences of opinion, the absolute order imbalance in percent as a measure of one-sided market, and intraday percent price range as a measure of volatility. The average daily estimated coefficients and t-statistics are reported in Tables 5 and 6. The t-statistics are based on the Newey-West adjusted standard errors with 5 lags.

[Insert Tables 5-6 here]

We first examine the cross-sectional relationship between QJ and MID and quoted cent spread and time-weighted depth. The results in Table 5, Panel A.1, column (1) show that QJ decreases significantly in quoted spread and increases significantly in depth as predicted by Hypothesis 1. The results indicate that a stock with a quoted spread of 5 cents instead of

[20] We do not include log of share volume in these tests as it is highly correlated with depth which is our main variable of interest.

4 cents (sample mean) has a rate of QJ that is 0.072 percentage points lower, which means QJ goes from 6.59% (sample mean) to 6.52% or a reduction of 1.1%. Similarly, a stock with an average depth that is two times the sample mean of 144.99 round lots has QJ that is 0.3917 percentage points higher than the average firm or an increase of 6.0%. The Table also shows that QJ is significantly higher for NASDAQ than NYSE stocks, and QJ is increasing significantly in volatility, and decreasing significantly in order imbalance. The results are robust to subsampling by listing exchange in columns (2) and (3), and market capitalization in columns (4) and (5). The only exception is that the effect of quoted cent spread is insignificant for the largest stocks. There is simply not enough cross-sectional dispersion of cent spreads within our sample of large stocks to pin down the coefficient.[21] The results are also robust to estimating Fama-MacBeth regressions with quoted cent spread and time-weighted depth separately. [22] Further, we verify that the results hold when estimating a collapsed version of the Fama-MacBeth framework with the time-series average QJ regressed on the time-series average quoted cent spread (coefficient is -0.156 with t=-3.94) and time-weighted depth (coefficient is 0.556 with t=3.31) plus control variables.

Table 5, Panel A.2, reports the same cross-sectional regressions with the frequency of MID executions as the left hand side variable. MID executions decline significantly in the quoted spread. However, while higher depth is associated with more MID executions overall and for NYSE-listed stocks, depth is associated with significantly lower MID executions for NASDAQ stocks and for small caps. Moreover, the amplitude of the estimated coefficients on quoted spread and depth are smaller for the MID regressions. Hence, as expected the effect of market conditions on MID is economically less significant than on QJ. Note also that the significant coefficients on order imbalance and intraday range in the MID regressions are of the opposite sign compared to the QJ regressions. Specifically, MID executions are higher for stocks with lower intraday range and with higher order imbalances. As explained above, this is natural as a significant fraction of MID executions are likely to arise from periodic trading systems using a derivative pricing rule.

We also use the Fama-MacBeth framework to study the effect of the relative tick size on QJ and MID. Since for all stocks in the U.S. with prices above $1.00 the tick size is constant at 1 cent, we use closing price to examine the effect of the relative tick size on QJ. In Table 6, Panel A.1, column (1) we show that QJ decreases significantly in price. In other words, QJ increases significantly in the relative tick size as predicted by Hypothesis 2. This is true after controlling for listing exchange, share volume, order imbalance and volatility. The results imply that a stock with a price $10 below the sample mean of $29.70 has QJ that is 0.35 (6.94%, compared to the sample mean of 6.59%) percentage points higher which represents a 5.3% increase in QJ. The signs on the remaining coefficient are consistent with our earlier

[21]The median cent spread for large stocks is 1.07 and the 75th percentile cent spread is 1.54.

[22]Results are available on request.

results, and the coefficient on share volume has the same sign as the coefficient on intraday range which is consistent with our conjecture that they both proxy the dispersion in valuations. The results are again robust to subsampling by listing exchange in columns (2) and (3), with the exception that the sign on order imbalance switches for NYSE stocks but the estimate is not statistically significant. The results are also robust for small cap stocks in column (5). However, for large cap stocks the effect of price is positive but insignificant and the coefficient on order imbalance is positive and significant. In other words, the relative tick size is not a strong driver for QJ in large cap stocks. This result is not surprising when we consider that there are relatively few large cap stocks for which the relative tick size is large (price is low).

Table 6 (Panel A.2) reports the same cross-sectional regressions with the frequency of MID executions as the left hand side variable. MID executions are not significantly affected by the relative tick size in the overall sample, for NYSE-listed stocks, or for large caps. However, for NASDAQ-listed stocks and small caps, a higher relative tick size (lower price) is associated with fewer MID executions. Moreover, while the signs of the coefficients on share volume remain positive and they are significant across the board, the coefficients on order imbalance and intraday range in the MID regressions are again of the opposite sign compared to the QJ regressions and they are mostly statistically significant. Thus, it again appears that MID executions are affected differently by our control variables, and it might be also due to the fact that MID are a mixture of continuous SPVs and periodic derivative pricing systems.

It is important to remember that the empirical evidence discussed above refers to evaluating a reduction in the tick-to-price ratio that arises because of an increase in the stock price, holding the absolute tick size fixed at \$0.01. The motivation for this approach is that there is no variation in the absolute tick size for the vast majority of stocks in the U.S. The question remains of how QJ would be affected by a reduction in the absolute tick size from say \$0.01 to \$0.0001. This question could in theory be answered by studying stocks that cross the \$1 threshold discussed above to examine whether there is a significant change in QJ at the threshold. However, unfortunately, Financial Industry Regulatory Authority (FINRA) collects prices with a maximum of four decimal digits thus not allowing researchers to study QJ for stocks priced below \$1.[23]

In sum the multivariate Fama-MacBeth regression analysis shows that QJ is significantly higher for NASDAQ than NYSE stocks, that QJ is negatively related to quoted spread and positively related to depth, and that QJ is positively related to the relative tick size (negatively related to price). These results are consistent with both Hypotheses 1 and 2. By contrast, MID executions are higher for NYSE than for NASDAQ stocks, and while they are negatively related to quoted spread and positively related to depth, they are only significantly - and in this case negatively - related to the relative tick size for small caps. Hence, while the results

[23] Informal conversations with U.S. traders informed us that banks are not even aware of the possibility to trade at more than 4 decimals, suggesting that perhaps QJ below \$1 does not even exist.

for MID executions support Hypothesis 1, they do not support Hypothesis 2.

5.1 HFT and QJ

We have documented a significant incidence of QJ in U.S. markets, and have also shown that QJ is more intense when competition among liquidity providers is high, and when the relative tick size is large. However, we have not discussed what type of trader is more likely to engage in QJ. What characteristics would make a trader more prone to QJ? In the model, QJ arises when a trader arrives at the market and would normally have submitted a limit order, but the long queue makes him or her instead QJ. The reason he or she jumps the queue by submitting a sub-penny order is that the execution probability for a limit order submitted to the PLB is low given the long queue and the cost of using a market order too high given the quoted spread. Therefore, a trader that engages in QJ is likely to have a valuation that is not extreme in the support of the β distribution. It follows that if we were to add traders who do not have strong views on the value of the security, but that trade opportunistically, we would see more QJ. These characteristics describe a type of trader that has become increasingly active in securities markets, a High Frequency Trader (Hagstromer and Norden, 2013). This insight begs the question if the incidence of QJ is increasing because of the growth in HFT trading. In other words, is it primarily HFT traders who jump ahead of existing orders on the PLB? To answer this question, we study the relationship between HFT and QJ. Following Angel, Harris and Spatt (2010, 2013) we compute the ratio between quote updates on lit markets and consolidated number of trades as our empirical proxy for HFT. This way we should capture HFTs that affect quotes, in other words only the lit market trading by potential HFTs. Note that quotes are not available for dark trading so we cannot define a similar measure for SPV trading. However, to the extent that HFT on the lit market is strongly correlated with HFT in SPVs, we should be able to use our proxy for lit market HFT to proxy for the relationship between HFT and QJ.

In Table 5, Panel B, we add our proxy for HFT to the specification in Table 5, Panel A, and in Table 6, Panel B, we add our proxy for HFT to the specification we estimated in Table 6, Panel A. We include all control variables from Panel A in each Table, and the estimated coefficients are virtually unchanged. In both cases, we find that our proxy for HFT is negatively related to both QJ and MID for the overall sample and for all the subsamples. The only exception is small caps in Table 6, Panel B.1, where the estimated relationship is positive but insignificant. For the overall sample, a stock with 49 quote updates per trade compared to the sample mean of 24.5 quote updates per trade has QJ that is 0.22-0.29 percentage points lower than the mean, or 6.37%-6.30%, instead of 6.59% (sample mean). The effect is considerably stronger for NASDAQ listed stocks and large stocks. For example, a large stock with 24.5 quote updates per trade more than the average large stock has QJ that is 2.03-2.72 percentage points lower than the mean of 7.68%.

Recall that our HFT proxy only captures trading by HFTs in lit markets. Therefore, if lit market HFT and HFT in SPVs are complements (positively correlated), our results suggest that there is no evidence indicating that HFTs are responsible for the increasing incidence of QJ. On the contrary, our results suggest that HFT trading is actually negatively correlated with QJ. It is of course possible that HFT in SPVs is instead a substitute for HFT trading in the lit market. If this is the case, then HFT in the lit market would be negatively correlated with HFT in SPVs. Under this assumption, our finding of a negative relationship between lit market HFT and QJ implies that QJ is indeed positively correlated with HFT using dark venues. Unfortunately, without a better proxy for HFT in SPVs, we cannot conclusively determine whether HFT in lit markets is a substitute or a complement for HFT in SPVs.[24]

6 Sub-Penny Trading and Market Quality

We now move to test our hypotheses on the effects of SPT on the market quality of regular exchanges. The issue is that PLB market quality and dark trading are jointly determined as pointed out in BRW (2011), so to establish a causal relationship we have to address the endogeneity issue.

To deal with the inherent endogeneity of SPT and market quality, we need to find good instruments for SPT and market quality, respectively. In a recent paper studying the impact of low latency trading on market quality, Hasbrouck and Saar (2013) propose using low latency trading in other stocks during the same time period as an instrument for low latency trading in a particular stock. We follow their suggestion and use SPT for other stocks (not i) on day t as an instrument for SPT in stock i. Because we have observed that there are systematic differences between exchanges and across market capitalization groups in SPT, we refine their instrument slightly. We require that the other stocks (not i) are listed on the same exchange as stock i and that their market capitalization is in the same market capitalization group as stock i. The market quality measures we use are the time-weighted percent quoted spread and the logarithm time-weighted bid-depth. We estimate a two-equation simultaneous model for SPT, which can be either QJ or MID, and market quality measures (MQMs) using both traditional 2SLS and a two step generalized method of moments (GMM) procedure. Specifically, we estimate the following two-equation simultaneous model for each MQM:

$$MQM_{i,t} = a_1 SP_{i,t} + a_2 MQM_{NOTi,t} + \varepsilon_{1,t} \quad (16)$$

[24]Yao and Ye (2015) find that a HFT proxy based on quote updates relative to trades and a version of the strategic run measure proposed by Hasbrouck and Saar (2013) are negatively correlated, highlighting the difficulty of estimating HFT activity based on TAQ data.

$$SP_{i,t} \quad = \quad b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + \varepsilon_{2,t}$$

As instruments for $SP_{i,t}$, we use $SP_{NOTi,t}$, which is the average SPT of other stocks listed on the same exchange, in the same market capitalization group. Note that we exclude stock i. Similarly, as an instrument for $MQM_{i,t}$, we use $MQM_{NOTi,t}$, which is the average market quality measure for other stocks listed on the same exchange, in the same market capitalization group. We again exclude stock i. This estimation method is chosen to address the endogeneity of SPT and MQM and obtain a consistent estimate of the a_1 coefficient that tells us how SPT affects market quality.

We estimate the above system of equations for all stocks and days in a panel. To control for stock fixed effects, we de-mean all variables by deducting the in-sample average and divide the de-meaned variables by their in-sample standard deviation. As a result, the estimated coefficients can be interpreted as the response to a one standard deviation shock. We do not include any trend in the system nor do we de-trend our variables since the visual inspection of the data tells us that the variables are stationary in the sample period. We estimate system (16) using a 2SLS procedure; standard errors are double clustered by stock and day.[25] We consider four measures of market quality, time-weighted bid depth to proxy for depth, share volume, time-weighted quoted cent spread, and the time-weighted relative spread.[26]

Tables 7 and 8 report the results from the simultaneous equation model for the relationship between QJ and MID and the four market quality measures (bid depth, share volume, quoted spread and relative spread). The results for share volume will be discussed separately in the next sub-section as the left hand side variables are different. We are primarily interested in the a_1 and b_1 coefficients: a_1 measures the effect of SPT on market quality and b_1 measures the effect of market quality on SPT. The coefficients on our instruments, a_2 and b_2 , are positive and highly significant. In other words, they appear to be good instruments. We present the results for the whole sample in column (1) and for small and large capitalization stocks separately in columns (2) and (3). We rely on market capitalization to proxy for stock liquidity.

Results for the factors driving QJ (b_1) continue to be consistent with Hypothesis 1 even after we attempt to control for endogeneity: QJ is positively related to the liquidity of the book. Specifically, Table 7, Panel A.1, shows that b_1 is positive and statistically significant for depth, while Table 8, Panels A.1 and B.1, shows that it is negative and statistically significant for quoted and relative spread, respectively. The magnitude of the effects are large. A one standard deviation increase in QJ results in a 0.28 standard deviation increase in depth, a 0.19 standard deviation reduction in quoted cent spreads, and a 0.27 standard deviation decrease

[25]We repeat the estimation using a GMM approach allowing for heteroskedasticity of unknown form, still double clustering standard errors. The results are nearly identical.

[26]Results for depth defined as the average of bid depth plus ask depth are virtually identical.

in relative basis point spreads. Similarly, MID is significantly positively related to the liquidity of the book once we take endogeneity into account. Consistent with our previous results, the magnitude of the effects are smaller and it is only significant for the overall sample for depth and quoted spread.

Tables 7 and 8 show that QJ has a positive effect on market quality. The coefficient a_1 is positive and statistically significant in the depth system in Table 7, Panel A.1, and negative and statistically significant in both the quoted cent spread and the relative basis point spread system in Table 8, Panels A.1 and B.1. Recall that variables are standardized, so a one standard deviation increase in QJ results in a 0.34 standard deviation increase in quoted depth. Similarly, a one standard deviation increase in QJ results in a decrease in quoted cent spreads of 0.31 standard deviations and a decrease in relative basis point spreads of 0.49 standard deviations. These are again economically sizable effects.

Interpreting market capitalization as a rough proxy of liquidity, we study the results for large and small capitalization stocks separately in columns (2) and (3) of each table. Consistently with Hypothesis 4, the results show that for stocks with liquid books (large caps) an increase in QJ is associated with an improvement of market quality measured by depth and spreads. By contrast, the relationship is not significant for stocks with less liquid books (small caps).

We report the relationship between MID and our market quality measures taking endogeneity into account in Tables 7 and 8. Recall that the effect of market quality measures on MID is captured by the coefficients b_1. The results for depth (Table 7, Panel A.2), quoted spread, and relative spread (Table 8, Panels A.2 and B.2) are similar, albeit statistically slightly weaker, than they are for QJ. Interestingly, we do not observe any significant effect of MID on depth, quoted or relative spread (a_1). In other words, MID executions do not appear to significantly affect market quality in our sample.

A similar difference between QJ and MID is documented by Comerton-Forde, Malinova, and Park (2015) and Foley and Putnins (2015) for Canadian stocks. They find that when the Canadian securities regulator started enforcing a trade-at-rule (essentially imposing for all platforms one full tick of price improvement or half a tick if the spread is constrained at one tick) in October 2012, dark trading decreased significantly and the result was a deterioration of market quality measures such as quoted and effective spreads and price efficiency measures such as autocorrelation, variance ratio, and delay. The authors pin down the source of the deterioration in market quality to the reduction in dark trading using platforms that offer two-sided liquidity and we label QJ. By contrast, the change in mid-crossings in the Canadian market following the introduction of the trade-at-rule has no statistically significant effect on market quality measures such as quote spread and market efficiency.

In our model, we have no asymmetric information, but we expect informed traders to choose the venue that offers the best liquidity supply, be it a PLB, a SPV, or even a crossing network executing at the midpoint of the PLB. We conjecture that if we added a new competing venue that executes at the mid-quote of the PLB to our model, the crossing network would attract more aggressive traders than the SPV. Hence, we conjecture that the difference in the results for QJ compared to MID can be attributed to other elements possibly associated with HFT access or trading fees.

An important caveat is therefore that we do not control for make-take fees. In practice, traders use dark markets to trade at sub-penny not only to step ahead of existing limit orders at the top of regular exchanges but sometimes also to save on take fees to the extent that fees are lower in SPVs. By internalizing orders a broker-dealer avoids paying the fees imposed by lit venues. Therefore, one should take the effects of different possible make-take fee structures into account both to correctly interpret our results, and to appropriately discuss our policy implications. We cannot take into account make-take fees as for trades marked with the code "D" we cannot distinguish among different trading venues that all report with code "D".

[Insert Tables 7-8 about here]

6.1 Volume

Our model also has predictions about how QJ affects volume. Hypothesis 3 states that consolidated volume should decrease in QJ for liquid books, but increase in QJ for illiquid books. We therefore also study the relationship between consolidated volume and QJ using 2SLS and a two-step GMM procedure. We estimate a two-equation simultaneous model for SPT, which can be either QJ or MID, and volume (VOL) using both traditional 2SLS and a two-step generalized method of moments (GMM) procedure. For this analysis, we measure both QJ and MID in shares (as opposed to relative to consolidated volume) in order to more closely match the empirical tests to the model. Specifically, we estimate the following two-equation simultaneous model for each VOL:

$$\begin{aligned} VOL_{i,t} &= a_1 SP_{i,t} + a_2 VOL_{NOTi,t} + \varepsilon_{1,t} \\ SP_{i,t} &= b_1 VOL_{i,t} + b_2 SP_{NOTi,t} + \varepsilon_{2,t} \end{aligned}$$

As instruments for $SP_{i,t}$, we use $SP_{NOTi,t}$, which is the average SPT of other stocks listed on the same exchange, in the same market capitalization group. Note that we exclude stock i. Similarly, as an instrument for $VOL_{i,t}$, we use $VOL_{NOTi,t}$, which is the average consolidated volume for other stocks listed on the same exchange, in the same market capitalization group. We again exclude stock i. This estimation method is chosen to address the endogeneity of SPT and VOL and obtain a consistent estimate of the a_1 coefficient that tells us how SPT affects market quality.

The results in Table 7, Panel B.1, show that as predicted by Hypothesis 3 there is a positive and significant effect of share volume on QJ (b_1), and a negative but not statistically significant effect of QJ on consolidated volume (a_1) in the overall sample. Columns (2) and (3) show that this conclusion is not affected by dividing the sample by market capitalization.

By contrast, in Table 8, Panel B.2, we do find a significantly negative effect of MID executions on consolidated volume (a_1) for the overall sample. Recall that MID executions have a no significant effect on depth and spreads. Therefore, we conclude once again that while QJ is associated with improved market quality and lower consolidated volume, we find no significant effects of MID on either market quality or consolidated volume.

6.2 Robustness Check

We rerun our simultaneous equation models including additional controls which are exogenous and can affect both market quality and SPT. In particular, following again Hasbrouck and Saar (2013), we include the daily return on SP500 and its volatility (proxied by VIX) to take into account market-wide activity. The same concern has been addressed by Brogaard, Hendershott and Riordan (2014) when studying the relation between HFT and volatility. Specifically, we estimate the following system of equations, which is a modification of system (16):

$$\begin{aligned} MQM_{i,t} &= a_1 SP_{i,t} + a_2 MQM_{NOTi,t} + a_3 SP500_t + a_4 VIX_t + \varepsilon_{1,t} \\ SP_{i,t} &= b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + b_3 SP500_t + b_4 VIX_t + \varepsilon_{2,t} \end{aligned} \tag{17}$$

We present the results in Tables 9 and 10. All our results are robust to the inclusion of the additional controls.

In unreported results, we also find that our results are generally robust to rerunning the analysis of the relationship between SPT and share volume in Section 6.1 using the measures of SPT that are normalized by consolidated volume. The only difference compared to the results reported above is that MID (normalized by consolidated volume) is significantly associated with lower share volume for small cap stocks.

[Insert Tables 9-10 about here]

7 Conclusions and Policy Implications

During the last decade financial markets have been characterized by the growth of dark markets and fast trading. These two elements have fostered the development of sub-penny trading (SPT) which is a particular form of dark trading. SPT takes place when traders take advantage of the Sub-Penny Rule (Rule 612) and its exceptions by executing orders in the dark market or internalizing customers' orders at fractions of the tick size. In this way they gain price priority over the orders sitting at the inside quotes of regulated markets. Sub-penny orders are then executed against traders who demand liquidity by using smart order routing programs that allow them to hit the best quotes available both in lit and in dark markets.

In this paper we show that in the U.S. approximately 10% of share volume executes at sub-penny increments. Approximately 3% executes exactly at the mid-quote, and could be the result either of queue-jumping (QJ) or of a midpoint cross (MID) in an opaque venue. These volumes are rapidly increasing and SPT is a concern to regulators as it can reduce the incentive for liquidity providers to post limit orders on regular exchanges and hence worsen market depth. For this reason in 2010 the SEC proposed the Trade-At Rule to curtail dark trading, and more recently such rule has been introduced in Canada (October 2012) and in Australia (May 2013).[27] In the 2010 concept release on market microstructure the SEC also opened the debate on SPT by asking questions on the factors that drive SPT, the effects of SPT on the quality of lit markets and more precisely on the adequate policy instrument to use to influence SPT. In this paper we answer most of these questions and to draw our empirical hypotheses we build a model of competition between a Public Limit Order Book (PLB) and a Sub-Penny Venue (SPV).

We find that SPT varies significantly by listing exchange, as QJ is significantly higher for NASDAQ than for NYSE listed stocks. Consistently with our model's predictions, we show that QJ is positively related to depth and negatively related to quoted cent and relative basis point spreads, and to stock price. This means that broker-dealers use dark markets for QJ especially for stocks where competition for the provision of liquidity is high and hence it is difficult to gain price priority on lit platforms. The use of QJ is also intense in low-priced stocks for which the profit from price improvement is higher relative to the asset value.

As predicted by our model, we find that QJ improves both depth and quoted cent and relative basis point spreads, especially for large capitalization stocks. Also as predicted by our model, QJ is associated with lower share volume, but this relationship is not statistically significant. By contrast, we find no relationship between MID and market quality and MID is significantly negatively related to share volume. In sum, we do not find any evidence supporting that SPT harms liquidity. We also build a proxy of HFT as the ratio between quotes updates

[27] On October 12, 2012 the Investment Industry Regulatory Organization of Canada introduced a Trade-At Rule that now gives lit orders priority over dark orders in the same venue. In particular small dark orders under 5,000 shares or C$100,000 dollars in value must offer at least half a tick in price improvement for stocks that have a one tick spread, and a full tick of price improvement for stocks with higher spreads. Following the implementation of this rule, the Canadian dark share of volume dropped by more than 50% (Rosenblatt Securities Inc., February 2013). Even the Australian Securities and Investments Commission on May 26, 2013, adopted a new regulation for dark venues aimed at containing dark trading for transactions of size smaller than blocks. The key component of the new regulatory regime is the adoption of a minimum size threshold for dark orders.

on lit markets and consolidated number of trades and find that QJ is negatively related to lit market HFT, suggesting that they could be strategic substitutes in traders' order submission decisions. If an increase in QJ is associated with both a reduction in HFT and an improvement in PLB market quality, it might well be that the improvement in market quality is also due to HFTs migrating from the PLB to the SPV. Therefore it would be interesting to investigate whether all market participants (HFTs and non-HFTs), both in the PLB and in the SPV, benefit from the existence of QJ. We leave this interesting topic for future research.

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Figure 1: **Public Limit Order Book**
Example from the extensive form of the game in the framework with a Public Limit Order Book only. The book opens empty at t_1: $S_{t_0} = [0,0,0,0]$. H_t refers to the strategy of the trader who arrives at the market in period t. All available H_t are defined in Table 2.

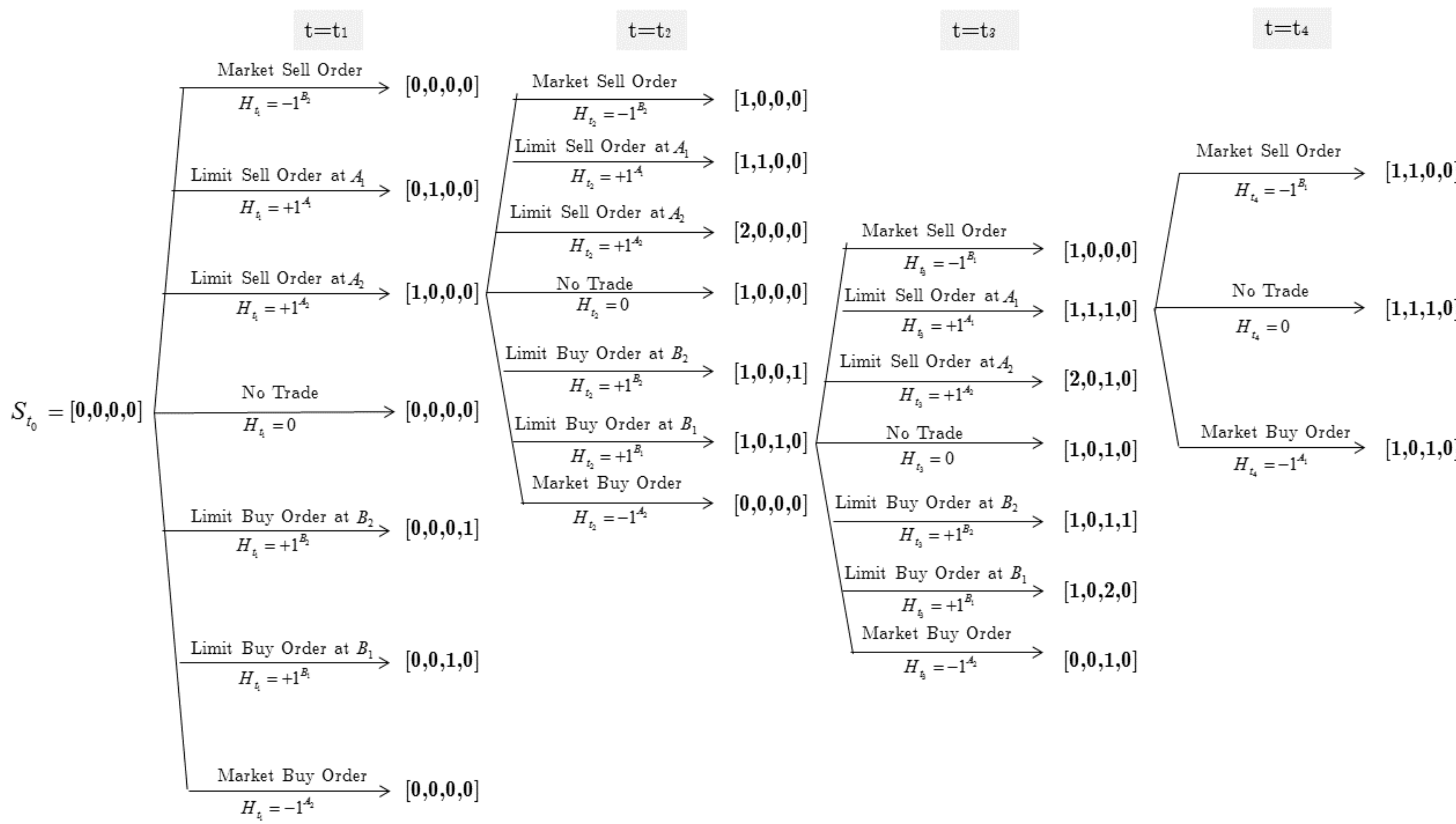

Figure 2: **Broker-dealers' β-thresholds.** This figure shows the BDs' β-thresholds at t_3, for $S_{t_1} = [0110]$ and $SPV_{t_1} = [0]$. The thresholds are computed for $\tau = 0.1$ and $\alpha = 50\%$.

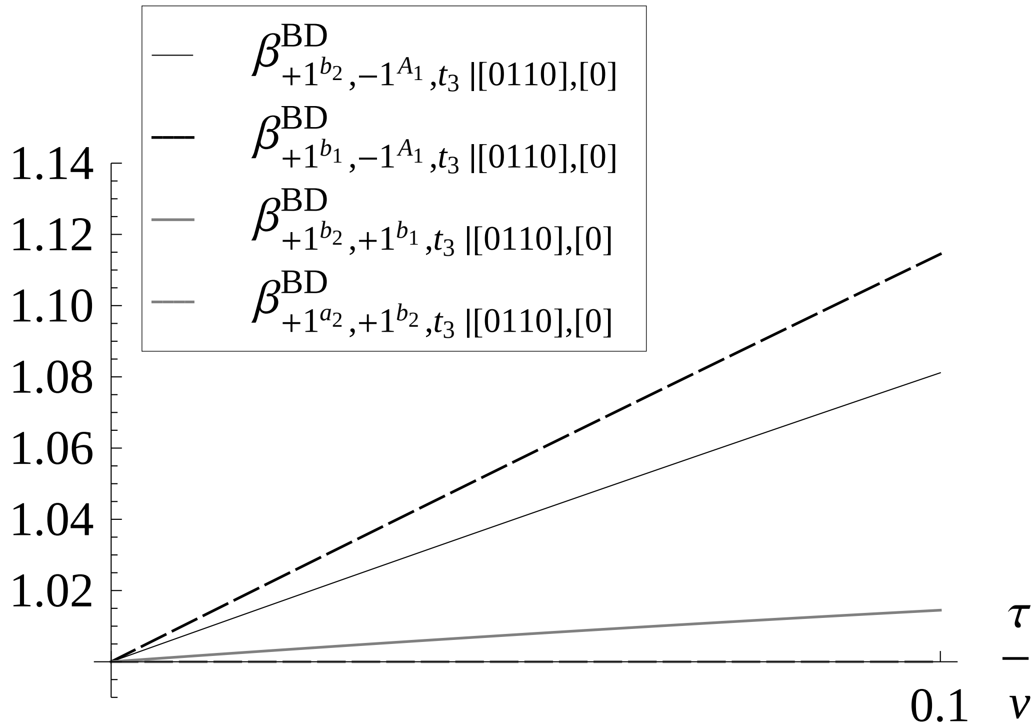

Figure 3: **Trading Strategies at t_2**
This figure reports the equilibrium submission probabilities at t_2 of limit orders posted on the ask side under two equilibrium opening states of the Public Limit Order Book (PLB) at t_2, "Liquid PLB" (Panel A) and "Illiquid PLB" (Panel B), that differ for the number of shares at the first level of the ask side of the book, $S_{t_1} = [0, 1, 0, 0]$, and $S_{t_1} = [0, 0, 0, 0]$, respectively. We denote by "PLB ONLY" the framework with only a PLB with tick size $\tau_{PLB} = \tau$, and price grid $\{A_1, A_2\}$, and with "PLB&SPV" the framework with both a PLB and a Sub-Penny Venue (SPV) with tick size $\tau_{SPV} = \frac{\tau}{3}$, and price grid $\{a_1, a_2, a_3, a_4, a_5\}$. Results are computed assuming broker-dealers' arrival rate $\alpha = 50\%$, and $\tau = 0.1$.

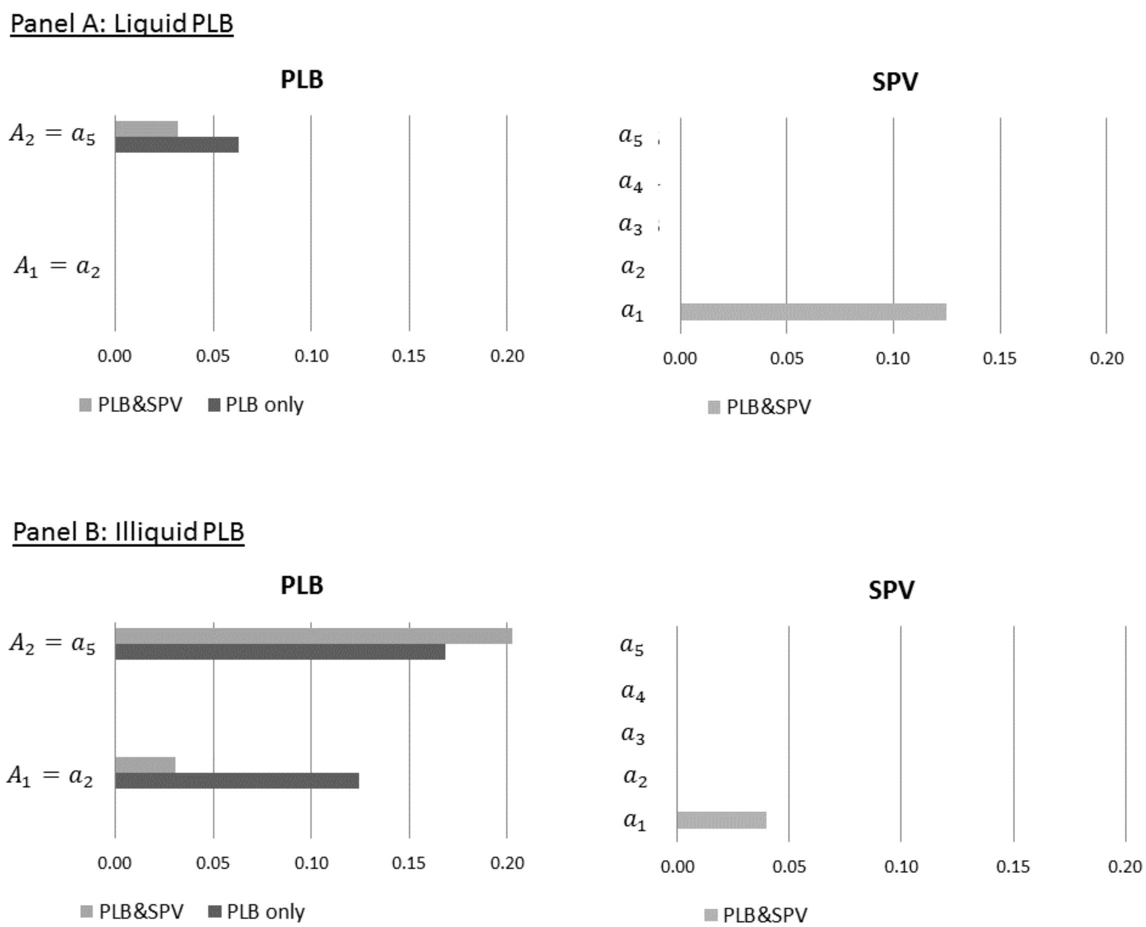

Figure 4: **Order Flows and Market Quality**

This figure reports statistics for order flows and market quality that result from the comparison between the benchmark framework with only a Public Limit Order Book (PLB) with tick size $\tau_{PLB} = \tau$, and the extended model (PLB&SPV) in which a PLB with the same tick size as the benchmark model competes with a Sub-Penny Venue (SPV) characterized by a smaller tick size, $\tau_{SPV} = \frac{\tau}{3}$. We consider two equilibrium opening states of the PLB at t_2, "Liquid" and "Illiquid", that differ for the number of shares at the first level of the ask side of the book, $S_{t_1} = [0, 1, 0, 0]$, and $S_{t_1} = [0, 0, 0, 0]$, respectively. For order flows, this panel reports statistics on liquidity provision and trading volume both for the PLB (LP_{PLB} and VL_{PLB}) and for the SPV (LP_{SPV} and VL_{SPV}). We also provide total trading volume (VL_{TOT}=VL_{PLB}+VL_{SPV}). For market quality, we report statistics on depth at the best bid-offer (DP_{BBO}), absolute and relative spread (SP and RSP). The statistics are computed as the average difference between the value of the measure in the framework in which the PLB competes with the SPV, and the value of the same statistic in the benchmark framework with only a PLB: $\Delta y = \frac{1}{k}\sum_{t=t_2}^{t_k}[y_t^{PLB\&SPV} - y_t^{PLB\ only}]$, where $y = \{DP_{BBO}, SP, RSP, VL_{PLB}, LP_{PLB}, VL_{SPV}, LP_{SPV}, VL_{TOT}\}$, $k = 2$ for $\{DP_{BBO}, SP, RSP, LP_{PLB}, LP_{SPV}\}$, and $k = 3$ for $\{VL_{PLB}, VL_{SPV}, VL_{TOT}\}$. We present results for a broker-dealers' arrival rate $\alpha = 50\%$, $\tau = 0.1$, and $v = \{1, 1.05\}$.

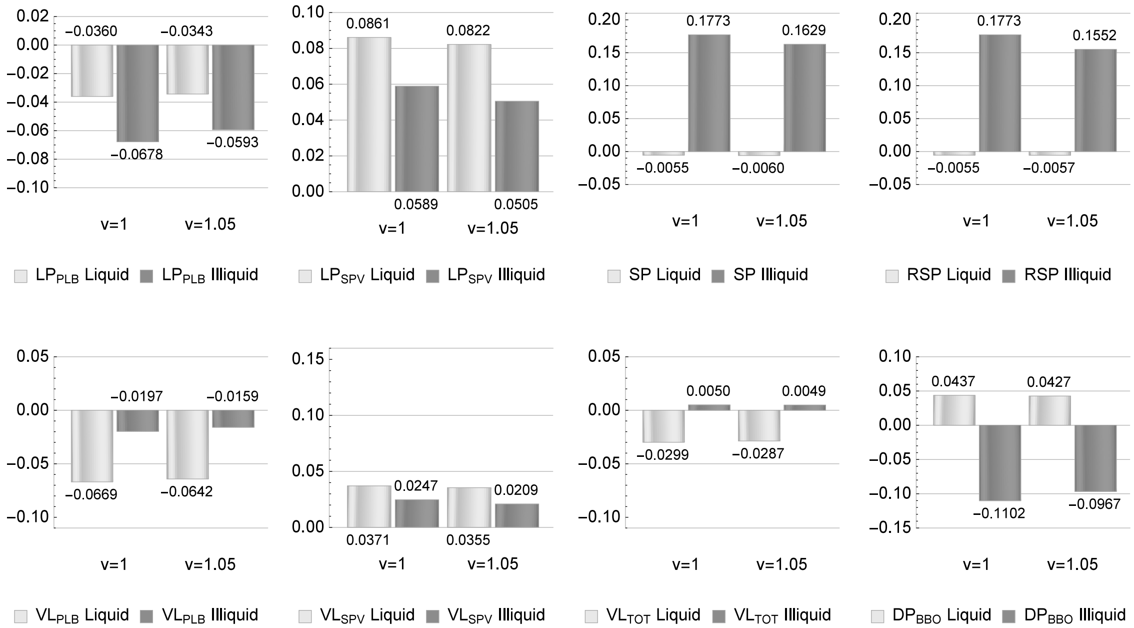

Figure 5: **NASDAQ queue-jumping and mid-crossing versus stock price**
This figure reports the queue-jumping and mid-crossing percentage over the sample period for each NASDAQ stock against the price of the stock.

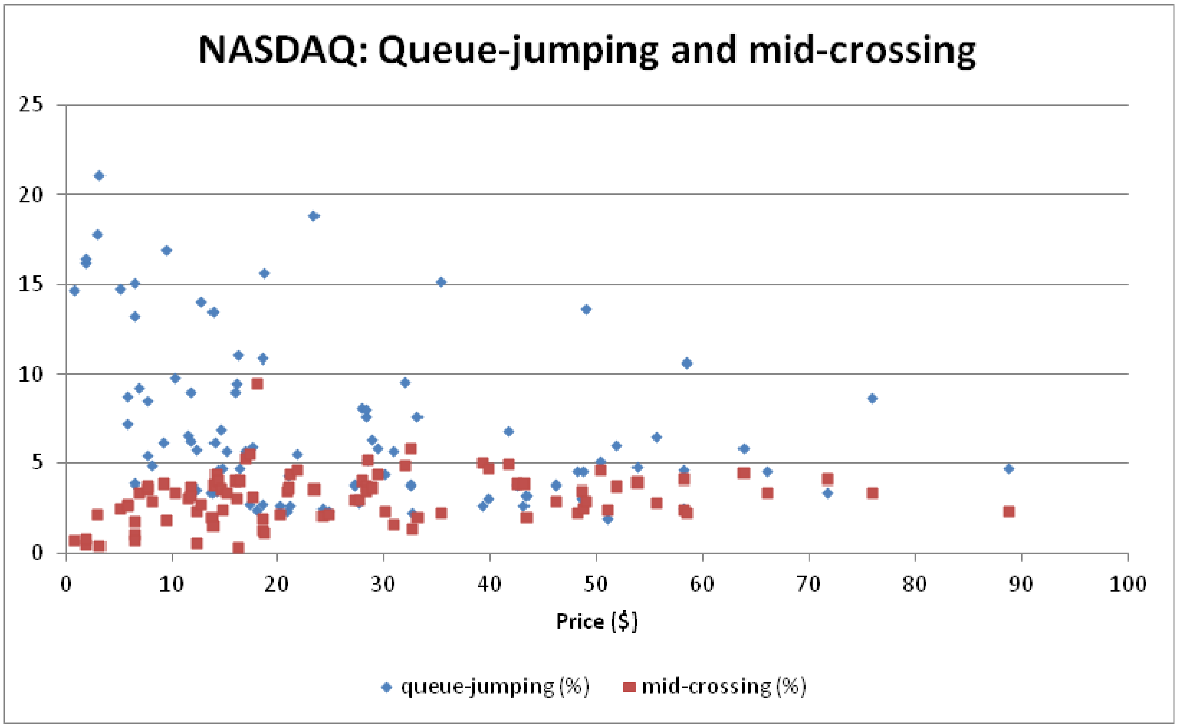

Figure 6: **NYSE queue-jumping and mid-crossing versus stock price**
This figure reports the queue-jumping and mid-crossing percentage over the sample period for each NYSE stock against the price of the stock.

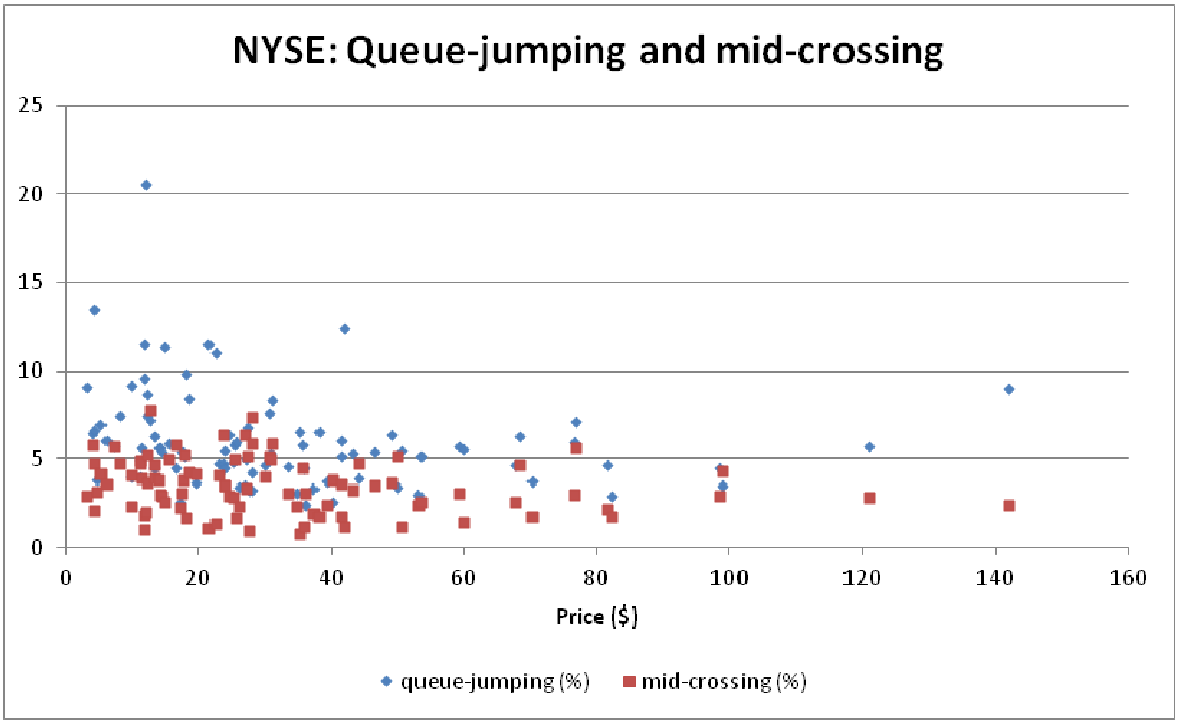

Figure 7: **NASDAQ distribution of Sub-Penny (including mid-crossing)**
This figure reports the sub-penny in percentage of consolidated volume for each bin over the sample period and for the group of 30 high-priced NASDAQ stocks.

Figure 8: **NYSE distribution of Sub-Penny (including mid-crossing)**
This figure reports the sub-penny in percentage of consolidated volume for each bin over the sample period and for the group of 30 high-priced NYSE stocks.

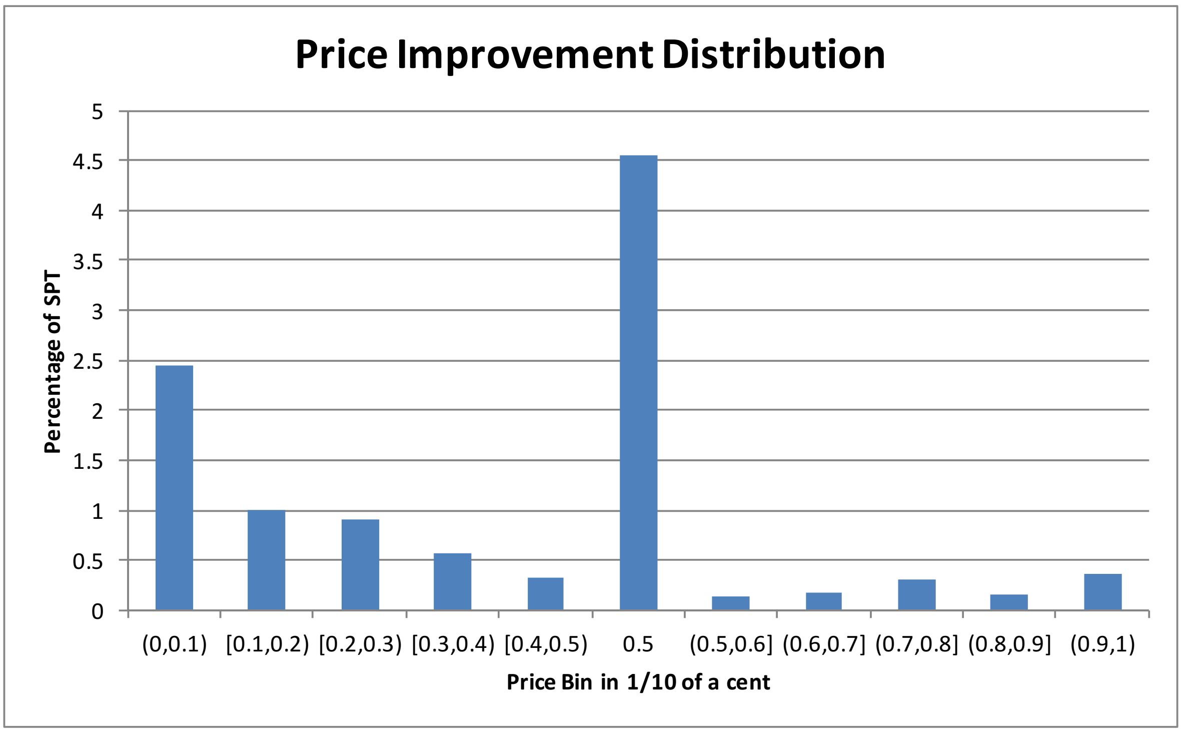

Table 1: **Price Grid**
This Table shows the price grid for both the Public Limit Order Book (PLB) and the Sub-Penny Venue (SPV). v is the asset value and τ is the tick size.

PLB	Price	SPV
A_2	$v + \frac{9}{6}\tau$	a_5
	$v + \frac{7}{6}\tau$	a_4
	$v + \frac{5}{6}\tau$	a_3
A_1	$v + \frac{3}{6}\tau$	a_2
	$v + \frac{1}{6}\tau$	a_1
	$v - \frac{1}{6}\tau$	b_1
B_1	$v - \frac{3}{6}\tau$	b_2
	$v - \frac{5}{6}\tau$	b_3
	$v - \frac{7}{6}\tau$	b_4
B_2	$v - \frac{9}{6}\tau$	b_5

Table 2: **Order Submission Strategy Space.**
This Table reports in column 3 the payoffs, $U(\cdot)$, of the order strategies H_t listed in column 1. In the case of market orders, $A_{k'}$ and $B_{k'}$ always refer to the best ask and bid prices.

Strategy	H_t	$U(\cdot)$
Market Sell Order	$-1^{B_{k'}}$	$B_{k'} - \beta v$
Limit Sell Order	1^{A_k}	$p_t(A_k \vert S_t)(A_k - \beta v)$
No Trade	0	0
Limit Buy Order	1^{B_k}	$p_t(B_k \vert S_t)(\beta v - B_k)$
Market Buy Order	$-1^{A_{k'}}$	$\beta v - A_{k'}$

Table 3: **Stocks descriptive statistics**
The sample period is October 1, 2010 - November 30, 2010. All variables are daily (time-weighted) averages. Market cap is the stock's market capitalization in millions from CRSP. Closing price is in dollar from CRSP. Relative spread is the difference between the bid and ask price over the mid-quote. Quoted spread is the difference between the bid and ask price (cents $). Price range is the difference between the maximum and the minimum price over the maximum price. Share volume is the number of shares traded in millions. Bid depth is the bid depth at NBBO in unit of trade (100 shares). Queue-jumping is the QJ on D computed as the ratio of volume executed in QJ over the total consolidated volume. Mid-crossing is the MID on D computed as the ratio of volume executed in MID over the total consolidated volume. Sub-Penny is the SPT on D computed as the ratio of volume executed in SPT over the total consolidated volume. Statistics are reported for the full sample (NASDAQ and NYSE) and for the NASDAQ and the NYSE separately. The flag "D" indicates in TAQ data transactions from over the counter markets, some transparent non-exchange Electronic Communications Networks, broker internalization and dark pools. Furthermore, for each panel, we report the summary statistics for the three market capitalization groups (SMALL, MEDIUM and LARGE). The first five variables are in Panel A of the table, while the other five are in the Panel B of the table.

	Panel A					
	Market Cap (Million $)	Closing Price ($)	Time-weighted relative spread ($^{0}/_{000}$)	Time-weighted quoted spread (cents $)	Price range (%)	HFT Proxy
NASDAQ+NYSE	**7,324.30**	**29.70**	**0.149**	**3.96**	**0.04**	**24.5**
Small	534.48	26.78	0.272	6.53	0.05	40.5
Medium	1740.48	29.10	0.112	3.92	0.03	25.0
Large	19,697.94	33.22	0.062	1.43	0.02	7.89
NASDAQ	**5,807.27**	**26.58**	**0.153**	**3.18**	**0.03**	**17.2**
Small	462.41	22.75	0.287	4.64	0.04	25.4
Medium	1,648.04	26.92	0.111	3.30	0.03	18.8
Large	15,311.36	30.05	0.061	1.41	0.02	6.4
NYSE	**8,841.33**	**32.83**	**0.145**	**4.74**	**0.04**	**31.7**
Small	606.55	30.81	0.257	8.85	0.05	59.0
Medium	1,832.93	31.28	0.114	4.46	0.03	30.5
Large	24,084.53	36.40	0.064	1.46	0.02	9.2

	Panel B				
	Share Volume (M)	Time-weighted bid depth (UoT)	Queue-jumping on D (%)	Mid-crossing on D (%)	Sub-Penny on D (%)
NASDAQ+NYSE	**5.92**	**144.99**	**6.59**	**3.34**	**9.93**
Small	0.41	9.58	7.68	2.51	10.19
Medium	1.77	23.22	5.49	3.40	8.89
Large	15.61	399.17	6.61	4.10	10.71
NASDAQ	**3.69**	**37.67**	**7.16**	**3.16**	**10.32**
Small	0.55	14.39	9.11	2.11	11.23
Medium	2.32	27.01	5.79	3.51	9.30
Large	8.61	71.59	6.57	3.86	10.43
NYSE	**8.15**	**250.31**	**6.03**	**3.51**	**9.54**
Small	0.23	4.76	6.25	2.90	9.15
Medium	1.28	19,43	5.18	3.29	8.48
Large	22.16	726.73	6.65	4.35	11.00

Table 4: **Price improvement calculation**
In this table we report an example of price improvement calculation and sub-penny identification according to our definition. We report a 3 seconds interval of transactions for IBM on date 10/01/2010.

ID	Symbol	Date	Time	Exchange	Price	Size	Rounded Price	Price Improvement	Type of Sub-Penny
1	IBM	20101001	10:00:00	K	135,76	100	135,76	0	None
2	IBM	20101001	10:00:00	K	135,77	100	135,77	0	None
3	IBM	20101001	10:00:00	K	135,84	100	135,84	0	None
4	IBM	20101001	10:00:00	N	135,75	100	135,75	0	None
5	IBM	20101001	10:00:00	N	135,77	100	135,77	0	None
6	IBM	20101001	10:00:00	N	135,78	400	135,78	0	None
7	IBM	20101001	10:00:00	N	135,8	600	135,8	0	None
8	IBM	20101001	10:00:00	N	135,84	779	135,84	0	None
9	IBM	20101001	10:00:00	P	135,73	100	135,73	0	None
10	IBM	20101001	10:00:00	P	135,75	100	135,75	0	None
11	IBM	20101001	10:00:00	P	135,84	400	135,84	0	None
12	IBM	20101001	10:00:00	T	135,73	200	135,73	0	None
13	IBM	20101001	10:00:00	T	135,75	100	135,75	0	None
14	IBM	20101001	10:00:00	T	135,77	300	135,77	0	None
15	IBM	20101001	10:00:00	T	135,8	2039	135,8	0	None
16	IBM	20101001	10:00:00	T	135,81	100	135,81	0	None
17	IBM	20101001	10:00:00	T	135,84	200	135,84	0	None
18	IBM	20101001	10:00:01	D	135,64	196	135,64	0	None
19	**IBM**	**20101001**	**10:00:01**	**D**	**135,7375**	**200**	**135,74**	**0,0025**	**Queue-Jumping**
20	IBM	20101001	10:00:01	N	135,67	100	135,67	0	None
21	IBM	20101001	10:00:01	N	135,69	100	135,69	0	None
22	IBM	20101001	10:00:01	N	135,72	100	135,72	0	None
23	IBM	20101001	10:00:01	N	135,73	600	135,73	0	None
24	IBM	20101001	10:00:01	N	135,74	200	135,74	0	None
25	IBM	20101001	10:00:01	P	135,71	100	135,71	0	None
26	IBM	20101001	10:00:01	Z	135,74	200	135,74	0	None
27	IBM	20101001	10:00:02	P	135,69	100	135,69	0	None
28	IBM	20101001	10:00:03	B	135,76	100	135,76	0	None
29	IBM	20101001	10:00:03	D	135,76	100	135,76	0	None
30	**IBM**	**20101001**	**10:00:03**	**D**	**135,825**	**300**	**135,83**	**0,005**	**Mid-Crossing**
31	IBM	20101001	10:00:03	J	135,76	100	135,76	0	None
32	IBM	20101001	10:00:03	K	135,75	200	135,75	0	None
33	IBM	20101001	10:00:03	K	135,76	200	135,76	0	None
34	IBM	20101001	10:00:03	N	135,71	100	135,71	0	None
35	IBM	20101001	10:00:03	N	135,73	100	135,73	0	None
36	IBM	20101001	10:00:03	N	135,74	300	135,74	0	None
37	IBM	20101001	10:00:03	N	135,75	200	135,75	0	None
38	IBM	20101001	10:00:03	N	135,76	500	135,76	0	None
39	IBM	20101001	10:00:03	N	135,77	100	135,77	0	None
40	IBM	20101001	10:00:03	N	135,78	200	135,78	0	None
41	IBM	20101001	10:00:03	N	135,79	200	135,79	0	None
42	IBM	20101001	10:00:03	N	135,8	300	135,8	0	None
43	IBM	20101001	10:00:03	N	135,81	100	135,81	0	None
44	IBM	20101001	10:00:03	N	135,82	100	135,82	0	None
45	IBM	20101001	10:00:03	N	135,83	400	135,83	0	None
46	IBM	20101001	10:00:03	P	135,76	200	135,76	0	None
47	IBM	20101001	10:00:03	P	135,77	200	135,77	0	None
48	IBM	20101001	10:00:03	T	135,75	400	135,75	0	None
49	IBM	20101001	10:00:03	T	135,76	800	135,76	0	None
50	IBM	20101001	10:00:03	T	135,77	200	135,77	0	None
51	IBM	20101001	10:00:03	T	135,81	600	135,81	0	None
52	IBM	20101001	10:00:03	T	135,83	300	135,83	0	None
53	IBM	20101001	10:00:03	Z	135,76	200	135,76	0	None
54	IBM	20101001	10:00:03	Z	135,77	100	135,77	0	None
55	IBM	20101001	10:00:03	Z	135,81	400	135,81	0	None

Table 5: **Queue-jumping and Mid-crossing: Spread and Depth**

This table reports the results of regressions of queue-jumping and mid-crossing, separately, on contemporaneous market characteristics based on daily cross-sectional Fama-MacBeth regressions. Queue-jumping is 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). Mid-crossing is 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We take the log of depth. Dollar spreads are multiplied by 100. The relative order imbalance is the absolute value of (buys-sells)/share volume. The intraday price range is defined as (high-low)/high. NYSE is a dummy variable which takes the value of 1 for stocks whose primary listing exchange is NYSE. HFT proxy is the ratio between quotes updates on lit markets and consolidated number of trades. Panel A reports the results for the baseline specification, while Panel B reports the results including the HFT proxy along with the same controls. We report the average daily coefficients on top and t-statistics below, computed according to Newey-West with 5 lags. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively.

Panel A: Queue-jumping and Mid-crossing: Spread and Depth

	A.1: Queue-jumping					A.2: Mid-crossing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
	All	NYSE	NASDAQ	Large Cap	Small Cap	All	NYSE	NASDAQ	Large Cap	Small Cap
NYSE	-0.933***			0.171**	-1.894***	0.440***			0.489***	0.571**
	(-15.466)			(3.005)	(-15.973)	(5.045)			(7.235)	(3.018)
quoted spread cents (TAQ)	-0.072***	-0.032***	-0.107***	0.111	-0.060***	-0.024***	-0.014**	-0.086***	-0.174	-0.017*
	(-8.743)	(-5.183)	(-6.467)	(1.474)	(-4.438)	(-4.649)	(-2.703)	(-6.235)	(-1.421)	(-2.389)
time-weighted depth (TAQ)	0.573***	0.376***	0.758***	0.328***	2.012***	0.126***	0.323***	-0.128***	0.035	-0.209***
	(23.354)	(16.856)	(16.067)	(12.119)	(19.916)	(4.500)	(7.798)	(-4.100)	(0.672)	(-4.427)
order imbalance (TAQ)	-2.269***	0.694	-4.503***	3.955***	-5.160***	2.132***	1.912**	2.490***	-0.345	2.405**
	(-4.008)	(1.193)	(-6.077)	(5.618)	(-4.717)	(4.290)	(2.915)	(4.066)	(-0.840)	(2.722)
(high-low)/high (TAQ)	45.530***	7.480*	103.564***	18.574*	34.903***	-7.711***	-3.905	-16.776***	-0.229	1.089
	(5.588)	(2.134)	(13.915)	(2.409)	(4.737)	(-4.376)	(-1.462)	(-9.438)	(-0.049)	(0.397)
Constant	2.488***	3.369***	-0.244	2.488***	-3.617***	2.239***	1.232***	4.448***	3.879***	3.350***
	(8.156)	(18.300)	(-0.583)	(8.692)	(-7.337)	(11.713)	(3.730)	(16.497)	(6.562)	(9.539)
Observations	7,560	3,780	3,780	2,520	2,520	7,560	3,780	3,780	2,520	2,520
R^2	0.121	0.075	0.227	0.113	0.266	0.050	0.081	0.075	0.086	0.077

Panel B: Queue-jumping and Mid-crossing: Spread, Depth and HFT

	B.1: Queue-jumping					B.2: Mid-crossing				
HFT proxy	-0.009***	-0.003	-0.042***	-0.111***	-0.010**	-0.019***	-0.014***	-0.048***	-0.132***	-0.017***
	(-3.921)	(-1.213)	(-6.288)	(-3.969)	(-3.401)	(-7.112)	(-7.331)	(-6.044)	(-6.510)	(-5.121)
Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	7,560	3,780	3,780	2,520	2,520	7,560	3,780	3,780	2,520	2,520
R^2	0.131	0.104	0.239	0.138	0.281	0.073	0.102	0.109	0.131	0.122

Table 6: **Queue-jumping and Mid-crossing: Price**

This table reports the results of regressions of queue-jumping and mid-crossing, separately, on contemporaneous market characteristics based on daily cross-sectional Fama-MacBeth regressions. Queue-jumping is 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). Mid-crossing is 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We take the log of share volume. Closing price is the price at the end of the day. The relative order imbalance is the absolute value of (buys-sells)/share volume. The intraday price range is defined as (high-low)/high. NYSE is a dummy variable which takes the value of 1 for stocks whose primary listing exchange is NYSE. HFT proxy is the ratio between quotes updates on lit markets and consolidated number of trades. Panel A reports the results for the baseline specification, while Panel B reports the results including the HFT proxy along with the same controls. We report the average daily coefficients on top and t-statistics below, computed according to Newey-West with 5 lags. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively.

Panel A: Queue-jumping and Mid-crossing: Price										
	A.1: Queue-jumping					A.2: Mid-crossing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
	All	NYSE	NASDAQ	Large Cap	Small Cap	All	NYSE	NASDAQ	Large Cap	Small Cap
NYSE	-0.890***			0.149*	-2.101***	0.437***			0.533***	0.836***
	(-15.821)			(2.321)	(-12.548)	(5.220)			(8.327)	(4.092)
share volume (TAQ)	0.276***	0.245***	0.241***	0.446***	1.065***	0.374***	0.418***	0.361***	0.225***	0.534***
	(9.264)	(6.290)	(8.787)	(9.604)	(16.166)	(11.162)	(9.228)	(14.104)	(4.753)	(8.595)
closing price (TAQ)	-0.035***	-0.016***	-0.064***	0.002	-0.078***	0.003	-0.005	0.020***	-0.002	0.014***
	(-19.189)	(-8.022)	(-21.818)	(0.764)	(-15.197)	(1.845)	(-1.966)	(5.849)	(-0.765)	(4.857)
order imbalance (TAQ)	-1.424*	0.747	-3.146***	2.177*	-5.744***	0.442	-0.160	0.795	-1.340**	2.076*
	(4.707)	(0.995)	(13.421)	(2.247)	(6.888)	(0.739)	(-0.164)	(1.356)	(-2.849)	(2.229)
(high-low)/high (TAQ)	33.050***	3.864	84.015***	18.129*	30.855***	-4.772**	-0.978	-9.393***	-4.236	-3.049
	(3.577)	(2.803)	(3.498)	(-3.630)	(-6.017)	(-3.166)	(-0.420)	(-3.962)	(-0.952)	(-1.193)
Constant	2.262***	1.953**	2.281**	-3.270***	-7.251***	-3.644***	-3.761***	-3.776***	-0.194	-7.089***
	(3.578)	(2.801)	(3.502)	(-3.628)	(-6.021)	(-6.055)	(-4.816)	(-7.329)	(-0.198)	(-6.569)
Observations	7,560	3,780	3,780	2,520	2,520	7,560	3,780	3,780	2,520	2,520
R^2	0.112	0.076	0.219	0.118	0.246	0.083	0.108	0.117	0.099	0.111

Panel B: Queue-jumping and Mid-crossing: Price and HFT										
	B.1: Queue-jumping					B.2: Mid-crossing				
HFT proxy	-0.012***	-0.001	-0.058***	-0.083***	0.002	-0.009***	-0.006**	-0.022***	-0.104***	-0.008**
	(-4.053)	(-0.414)	(-8.947)	(-4.011)	(0.594)	(-3.851)	(-3.364)	(-5.068)	(-5.009)	(-2.938)
Controls	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	7,560	3,780	3,780	2,520	2,520	7,560	3,780	3,780	2,520	2,520
R^2	0.121	0.092	0.238	0.141	0.258	0.092	0.118	0.134	0.133	0.132

Table 7: **Queue-jumping and Mid-crossing: Depth and Share Volume**

This table reports the results from the analysis of the relation between queue-jumping and mid-crossing activity and time-weighted bid depth (Panel A) and share volume (Panel B). We measure queue-jumping activity as QJ, which is defined as 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We measure mid-crossing activity as MID, which is defined as 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). When analyzing the relation between QJ (or MID) and share volume, we use gross measure (i.e., not normalized to total consolidated volume). Due to the potential simultaneity between time-weighted bid depth (log of) and share volume (log of), respectively, and queue-jumping and mid-crossing activity, we estimate the following two-equation simultaneous model for SP, which can be either QJ or MID, and MQM, which can be either time-weighted bid depth or share volume :

$MQM_{i,t} = a_1 QJ_{i,t} + a_2 MQM_{NOTi,t} + \varepsilon_{1,t}$

$SP_{i,t} = b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + \varepsilon_{2,t}$

As an instrument for $SP_{i,t}$ (which can be either queue-jumping or mid-crossing), we use $SP_{NOTi,j}$, which is the daily average SP activity of other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). Similarly, as an instrument for $MQM_{i,t}$ we use $MQM_{NOTi,t}$, **which is the average time-weighted bid depth (log of), quoted spread and relative spread,** respectively, for other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). We estimate the simultaneous equation model by pooling observations across all stocks and days in the sample. To make the pooling meaningful, we de-mean all variables by deducting the stock-specific average and scale all variables by dividing by the stock-specific standard deviation to control for stock fixed effects. We report the estimated coefficients on top and t-statistics below. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively. Estimation is done with 2SLS with two-way clustered standard errors (i.e., stock and day).

	Panel A: Time-weighted depth						Panel B: Share volume					
	A.1: Queue-jumping			A.2: Mid-crossing			B.1: Queue-jumping			B.2: Mid-crossing		
	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large
a_1	0.342***	0.044	0.281*	0.202	-0.041	0.253	-0.294	-0.434	-0.216	-1.108*	-0.510	-2.583
	(3.358)	(0.240)	(2.338)	(0.940)	(-0.198)	(0.790)	(-1.678)	(-1.559)	(-1.521)	(-1.972)	(-1.633)	(-0.641)
a_2	0.523***	0.371***	0.645***	0.602***	0.368***	0.719***	1.028***	1.029***	1.023***	1.231***	0.981***	1.887
	(10.266)	(4.294)	(8.665)	(13.768)	(4.279)	(9.500)	(9.603)	(7.522)	(11.148)	(6.132)	(8.021)	(1.249)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520
b_1	0.284***	0.058	0.251**	0.110	-0.055	0.153*	0.219***	0.222***	0.160***	0.239***	0.140*	0.323***
	(3.800)	(0.236)	(2.694)	(1.775)	(-0.219)	(1.963)	(6.521)	(4.658)	(3.371)	(6.439)	(2.573)	(6.542)
b_2	0.421***	0.421***	0.548***	0.251***	0.276**	0.279**	0.490***	0.391***	0.623***	0.246***	0.344***	0.186
	(8.155)	(5.057)	(6.938)	(4.746)	(3.556)	(2.944)	(6.121)	(3.932)	(6.903)	(3.951)	(3.483)	(1.814)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520

Table 8: **Queue-jumping and Mid-crossing: Quoted and Relative Spread**

This table reports the results from the analysis of the relationship between queue-jumping and mid-crossing activity and quoted spread (Panel A) and relative spread (Panel B)(both time-weighted). We measure queue-jumping activity as QJ, which is defined as 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We measure mid-crossing activity as MID, which is defined as 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). Due to the potential simultaneity between quoted spread and relative spread, respectively, and queue-jumping and mid-crossing activity, we estimate the following two-equation simultaneous model for SP, which can be either QJ or MID, and and MQM, which can be either quoted spread and relative spread:

$MQM_{i,t} = a_1 QJ_{i,t} + a_2 MQM_{NOTi,t} + \varepsilon_{1,t}$

$SP_{i,t} = b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + \varepsilon_{2,t}$

As an instrument for $SP_{i,t}$ (which can be either queue-jumping or mid-crossing), we use $SP_{NOTi,j}$, which is the daily average SP activity of other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). Similarly, as an instrument for $MQM_{i,t}$ we use $MQM_{NOTi,t}$, which is the average time-weighted bid depth (log of), quoted spread and relative spread, respectively, for other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). We estimate the simultaneous equation model by pooling observations across all stocks and days in the sample. To make the pooling meaningful, we de-mean all variables by deducting the stock-specific average and scale all variables by dividing by the stock-specific standard deviation to control for stock fixed effects. We report the estimated coefficients on top and t-statistics below. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively. Estimation is done with 2SLS with two-way clustered standard errors (i.e., stock and day).

	Panel A: Quoted spread						Panel B: Relative spread					
	A.1: Queue-jumping			A.2: Mid-crossing			B.1: Queue-jumping			B.2: Mid-crossing		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
	Full sample	Small	Large	Full sample	Small	Large	Full sample	Small	Large	Full sample	Small	Large
a_1	-0.312**	-0.426	-0.148	-0.436	-0.133	-0.462	-0.493***	-0.551	-0.288*	-0.321	-0.076	-0.581
	(-2.601)	(-1.943)	(-1.372)	(-1.819)	(-0.768)	(-1.121)	(-3.749)	(-1.837)	(-2.339)	(-1.360)	(-0.209)	(-1.283)
a_2	0.683***	0.604***	0.732***	0.690***	0.682***	0.670***	0.631***	0.582***	0.684***	0.742***	0.719***	0.669***
	(8.407)	(6.378)	(8.671)	(11.321)	(9.029)	(5.348)	(11.720)	(5.614)	(9.592)	(17.162)	(12.300)	(5.874)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520
b_1	-0.187*	-0.213	-0.120	-0.160**	-0.0795	-0.197	-0.268***	-0.259**	-0.231**	-0.129**	-0.045	-0.223**
	(-2.522)	(-1.901)	(-1.534)	(-2.693)	(-1.058)	(-1.901)	(-5.630)	(-2.778)	(-2.866)	(-3.210)	(-0.603)	(-2.933)
b_2	0.448***	0.339***	0.647***	0.208**	0.304**	0.239	0.351***	0.282**	0.544***	0.221**	0.282**	0.203*
	(6.634)	(3.601)	(9.648)	(2.802)	(2.791)	(1.682)	(6.392)	(2.847)	(6.881)	(4.068)	(3.499)	(2.017)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520

Table 9: **Queue-jumping and Mid-Crossing: Depth and Share Volume (Robustness)**
This table reports the results from the analysis of the relation between queue-jumping and mid-crossing activity and time-weighted bid depth (Panel A) and share volume (Panel B). We measure queue-jumping activity as QJ, which is defined as 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We measure mid-crossing activity as MID, which is defined as 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). When analyzing the relation between QJ (or MID) and share volume, we use gross measure (i.e., not normalized to total consolidated volume). Due to the potential simultaneity between time-weighted bid depth (log of) and share volume (log of), respectively, and queue-jumping and mid-crossing activity, we estimate the following two-equation simultaneous model for SP, which can be either QJ or MID, and MQM, which can be either time-weighted bid depth or share volume. We include as controls the return on SP500 and the VIX:

$MQM_{i,t} = a_1 SP_{i,t} + a_2 MQM_{NOTi,t} + a_3 SP500_t + a_4 VIX_t + \varepsilon_{1,t}$
$SP_{i,t} = b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + b_3 SP500_t + b_4 VIX_t + \varepsilon_{2,t}$
As an instrument for $SP_{i,t}$ (which can be either queue-jumping or mid-crossing), we use $SP_{NOTi,j}$, which is the daily average SP activity of other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). Similarly, as an instrument for $MQM_{i,t}$ we use $MQM_{NOTi,t}$, which is the average time-weighted bid depth (log of), quoted spread and relative spread, respectively, for other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). We estimate the simultaneous equation model by pooling observations across all stocks and days in the sample. To make the pooling meaningful, we de-mean all variables by deducting the stock-specific average and scale all variables by dividing by the stock-specific standard deviation to control for stock fixed effects. We report the estimated coefficients on top and t-statistics below. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively. Estimation is done with 2SLS with two-way clustered standard errors (i.e., stock and day).

	Panel A: Depth						Panel B: Share volume					
	A.1: Queue-jumping			A.2: Mid-crossing			B.1: Queue-jumping			B.2: Mid-crossing		
	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large	(1) Full sample	(2) Small	(3) Large
a_1	0.321***	0.055	0.252*	0.161	-0.046	0.163	-0.379	-0.491	-0.347	-1.167	-0.506	-5.510
	(3.134)	(0.323)	(2.055)	(0.769)	(-0.220)	(0.566)	(-1.841)	(-1.112)	(-1.463)	(-1.723)	(-1.222)	(-0.392)
a_2	0.521***	0.358***	0.621***	0.594***	0.359***	0.691***	1.056***	1.036***	1.075***	1.238***	0.973***	2.913
	(7.520)	(4.693)	(4.988)	(10.375)	(4.714)	(6.601)	(11.349)	(6.021)	(9.091)	(5.988)	(8.269)	(0.558)
a_3	0.015	-0.017	0.035	0.030	-0.016	0.044	0.0289	0.0381	0.0314	0.0277	0.0189	0.195
	(1.312)	(-0.449)	(1.914)	(1.890)	(-0.441)	(1.939)	(1.301)	(0.838)	(1.049)	(0.810)	(0.513)	(0.369)
a_4	0.000025	0.000090	-0.000042	-0.000094	0.000080	-0.00014	-9.294	1.765	-17.15	-10.11	3.175	-86.03
	(1.898)	(0.337)	(1.377)	(0.559)	(0.244)	(0.412)	(-0.734)	(0.072)	(-0.888)	(-0.551)	(0.133)	(-0.412)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520
b_1	0.261***	0.071	0.234*	0.096	-0.060	0.121	0.231***	0.218***	0.195***	0.236***	0.139**	0.330***
	(3.233)	(0.331)	(2.100)	(1.444)	(-0.266)	(1.265)	(6.95)	(4.49)	(3.30)	(9.00)	(3.07)	(8.29)
b_2	0.408***	0.409***	0.541***	0.242***	0.283***	0.264***	0.451***	0.376***	0.540***	0.241***	0.344***	0.146
	(6.043)	(4.573)	(4.499)	(4.499)	(3.532)	(2.868)	(7.69)	(3.88)	(5.73)	(4.41)	(3.82)	(1.75)
b_3	0.026	0.016	0.018	0.016	-0.0060	0.032	0.0280*	0.0273	0.0345	0.0104	0.00321	0.0278
	(0.616)	(0.287)	(-0.107)	(0.634)	(-0.133)	(0.761)	(2.08)	(1.16)	(1.65)	(0.70)	(0.11)	(1.32)
b_4	-0.00024	-0.00015	-0.00023	-0.000081	0.00010	-0.00020	-11.09	0.532	-19.37	-5.283	2.393	-12.15
	(-1.142)	(-0.961)	(-1.482)	(-1.129)	(0.229)	(-1.207)	(-1.31)	(0.03)	(-1.38)	(-0.67)	(0.15)	(-0.97)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520

Table 10: **Queue-jumping and Mid-Crossing: Quoted and Relative Spread (Robustness)**
This table reports the results from the analysis of the relationship between queue-jumping and mid-crossing activity and quoted spread (Panel A) and relative spread (Panel B)(both time-weighted). We measure queue-jumping activity as QJ, which is defined as 100 times daily queue-jumping dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). We measure mid-crossing activity as MID, which is defined as 100 times daily mid-crossing dollar volume on Alternative Display Facility (ADF) divided by daily consolidated dollar volume (volume computed during trading hours 9:30 AM - 4:00 PM ET). Due to the potential simultaneity between quoted spread and relative spread, respectively, and queue-jumping and mid-crossing activity, we estimate the following two-equation simultaneous model for SP, which can be either QJ or MID, and and MQM, which can be either quoted spread and relative spread. We include as controls the return on SP500 and the VIX:

$MQM_{i,t} = a_1 QJ_{i,t} + a_2 MQM_{NOTi,t} + a_3 SP500_t + a_4 VIX_t + \varepsilon_{1,t}$
$SP_{i,t} = b_1 MQM_{i,t} + b_2 SP_{NOTi,t} + b_3 SP500_t + b_4 VIX_t + \varepsilon_{2,t}$
As an instrument for $SP_{i,t}$ (which can be either queue-jumping or mid-crossing), we use $SP_{NOTi,j}$, which is the daily average SP activity of other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). Similarly, as an instrument for $MQM_{i,t}$ we use $MQM_{NOTi,t}$, which is the average time-weighted bid depth (log of), quoted spread and relative spread, respectively, for other stocks listed on the same exchange, in the same market capitalization grouping (LARGE, MEDIUM, SMALL; excluding stock i). We estimate the simultaneous equation model by pooling observations across all stocks and days in the sample. To make the pooling meaningful, we de-mean all variables by deducting the stock-specific average and scale all variables by dividing by the stock-specific standard deviation to control for stock fixed effects. We report the estimated coefficients on top and t-statistics below. The symbols ***, ** and * indicate statistical significance at the 0.1%, 1%, and 5% levels, respectively. Estimation is done with 2SLS with two-way clustered standard errors (i.e., stock and day).

	Panel A: Quoted spread						Panel B: Relative spread					
	A.1: Queue-jumping			A.2: Mid-crossing			B.1: Queue-jumping			B.2: Mid-crossing		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
	Full sample	Small	Large	Full sample	Small	Large	Full sample	Small	Large	Full sample	Small	Large
a_1	-0.329*	-0.547*	-0.135	-0.450	-0.223	-0.442	-0.461**	-0.602	-0.271*	-0.303	-0.132	-0.538
	(-2.301)	(-2.012)	(-0.861)	(-1.601)	(-0.739)	(-0.948)	(-2.600)	(-1.214)	(-2.376)	(-1.085)	(-0.389)	(-1.036)
a_2	0.641***	0.463***	0.709***	0.639***	0.576***	0.652***	0.611***	0.500***	0.669***	0.714***	0.661***	0.659***
	(12.344)	(4.501)	(8.029)	(10.472)	(6.322)	(5.773)	(7.240)	(4.069)	(4.199)	(9.405)	(6.484)	(3.082)
a_3	-0.048**	-0.102**	-0.036	-0.055**	-0.085**	-0.031	-0.028	-0.068*	-0.017	-0.039***	-0.057*	-0.015
	(-2.612)	(-2.861)	(-1.443)	(-3.067)	(-2.666)	(-1.021)	(-1.090)	(-1.906)	(-0.293)	(-3.414)	(-1.928)	(-0.054)
a_4	-17.685	-30.339	-9.413	-8.718	-11.666	-9.595	0.00014	0.00027	0.00017	0.00023	0.00027	0.00020
	(-1.448)	(-1.459)	(-0.488)	(-0.810)	(-0.594)	(-0.501)	(0.569)	(-0.069)	(1.337)	(1.503)	(0.557)	(1.651)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520
b_1	-0.187**	-0.319*	-0.0924	-0.171***	-0.129	-0.174*	-0.262***	-0.322**	-0.214*	-0.121**	-0.067	-0.201*
	(-2.991)	(-2.102)	(-0.811)	(-3.504)	(-1.189)	(-1.968)	(-3.620)	(-2.548)	(-2.134)	(-2.591)	(-0.871)	(-2.117)
b_2	0.403***	0.284**	0.583***	0.201***	0.292**	0.211*	0.348***	0.259*	0.528***	0.219***	0.268***	0.194
	(6.988)	(2.758)	(7.351)	(3.501)	(3.262)	(2.278)	(5.666)	(2.281)	(5.734)	(4.055)	(3.426)	(1.917)
b_3	-0.000973	-0.0573	0.0221	-0.0113	-0.0282	0.0182	0.0086	-0.035	0.026	0.0040	-0.016	0.027
	(-0.049)	(-1.428)	(0.779)	(-0.678)	(-0.809)	(0.722)	(0.256)	(-1.210)	(0.881)	(0.224)	(-0.438)	(1.036)
b_4	-27.22**	-28.25	-27.73	-10.539	-0.796	-16.412	-0.000084	0.000084	-0.00013	0.0000055	0.00015	-0.000072
	(-2.941)	(-1.582)	(-1.843)	(-1.302)	(-0.062)	(-1.131)	(-1.164)	(-0.612)	(-0.451)	(-0.020)	(0.346)	(0.558)
Observations	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520	7,560	2,520	2,520