

Do Commodities Add Economic Value in Asset Allocation?

New Evidence From Time-varying Moments *

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Abstract

Commodities have been increasingly incorporated into traditional Stock-Bond-Cash portfolios by investors. However, their ability to generate sizable economic gains has been questioned recently, especially in an out-of-sample (OOS) context. In this study, we conduct a comprehensive OOS assessment on the economic value of commodities in multi-asset investment strategies for both mean-variance (MV) and non-mean-variance investors who exploit the predictability of time-varying asset return moments. We find that predictability makes the addition of commodities profitable even when short-selling and high leverage are not permitted. For instance, a MV investor with moderate risk aversion would be willing to pay upto 101 bps per year after transaction cost for adding commodities into her stock, bond and cash portfolio.

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1 Introduction

Commodities, as an alternative class of investable assets, have attracted substantial interest from both institutional and individual investors over the course of the past decade. According to a recent report by McKinsey, by the end of 2011, the amount of all forms of alternatives in global assets under management (AUM) has exceeded \$6.5 trillion representing 14 percent weight in the global portfolio, and is expected to reach 17 percent by the end of 2013. Among all alternative strategies, commodities have grown rapidly at an annual rate of 21 percent during 2005-2011, and stand at roughly \$600 billion or approximately 6% in global AUM at the year-end of 2011.¹ Evidently, commodities play an increasingly important role in global capital markets. In the asset management industry the dramatic boom of commodity investing appears to be linked to at least three perceived qualities of commodities: risk diversification, high historical returns and protection against inflation.² Yet, the increasing popularity of commodities recorded in investment practice and the alleged merits pursued by investors do not settle the critical question of how investors should optimally incorporate commodities in their multi-asset portfolios, or even if they should at all. This issue is also reflected in McKinsey's research report: the surveyed traditional asset managers fully agree with the potential of alternatives; on the other hand, they perceive to be constrained by limited knowledge in risk management and product expertise for moving into alternatives.

Academic researchers have conducted numerous analyses of the economic value adding of commodity investment, but the results appear to be inconclusive. On one side, some studies show that investing in commodities indeed improves the risk-return profiles of mean-variance (MV) investors' multi-asset portfolios based on in-sample (IS) assessment. On the other hand, the ability of commodities to generate significant out-of-sample (OOS) economic gains has been questioned in recent studies. As a prominent example, Daskalaki and Skiadopoulos (2011) present the most recent OOS

¹McKinsey's Global Alternative Investment Report is available at http://www.mckinsey.com/client-service/financial-services/knowledge_highlights/recent_reports/mainstreaming%20of%20alternatives.aspx?sc_lang=en.

²According to a survey by Barclays Capital who interviewed over 100 institutional investors and advisors, almost half of respondents choose portfolio diversification as their main reason investing in commodities, a third choose absolute returns, and one tenth inflation protection. The report is available at <http://www.barclays.com/about-barclays-capital/press-office/research-reports.html>.

evidence that investing in commodities adds no value for either mean-variance (MV) or non mean-variance (non-MV) investors. In the present paper we argue that most, if not all, existing studies that examine the OOS economic value of commodities in multi-asset portfolios are based on static and/or backward-looking (BWD) estimates of the return moments that become input to the asset allocation problem. For instance, Daskalaki and Skiadopoulos (2011) rely on rolling sample moments at any given date t as estimates for the moments at time $t + 1$ that, then, determine optimal portfolio weights. But there is by now compelling empirical evidence that asset return moments are time-varying and, to some extent, predictable by variables other than themselves. Indeed, several previous studies show that exploiting the predictability of returns by economic variables or of their volatilities and correlations leads to tangible economic gains for portfolios that include either equities and cash or equities, bonds and cash. A few recent studies point to the importance of including the dynamics of return skewness and/or kurtosis in the asset allocation exercise with equities and/or bonds and cash. No study that we know of, however, investigates the predictable dynamics of return moments in the context of asset allocation strategies that include commodities in addition to equities and bonds. Without careful examination of the return dynamics in a forward-looking (FWD) framework while implementing portfolio strategies, we believe the above conclusion about the lack of value added by commodities to be somewhat in hasty.

In this paper, we provide a comprehensive analysis of the OOS performance of investment strategies incorporating commodities into traditional asset portfolios for both MV and non-MV investors. A major innovation differentiating our study from the existing literature is that we recursively construct one-period-ahead optimal portfolios by exploiting the predictability of all the first four moments of asset returns. We find that, by exploiting predictability, the inclusion of commodities into traditional asset portfolios does generate significant out-of-sample economic gains. Furthermore, for both traditional and commodity-augmented portfolios, FWD strategies outperform their BWD peers.

2 Literature

The potential benefits of investing in commodities have been suggested in several academic studies. In a seminal paper, Gorton and Rouwenhorst (2006) construct an equally weighted long-only commodity futures index that generates an average annualized geometric return of 9.98% with monthly rebalancing over the 1959-2004 period. They also document the pattern that the monthly (quarterly and annual) returns of the index are insignificantly (negatively) correlated with stocks and bonds but positively correlated with inflation. Consequently, the authors suggest that commodity futures could be an ideal diversifier to traditional investment portfolios. However, Gorton and Rouwenhorst do not attempt to directly assess the empirical benefits of investing in commodities, so it is not clear whether and how the potential benefits could be realized practically. Erb and Harvey (2006) question the "equity-like" return of the Gorton-Rouwenhorst index and argue that such high return is achieved through frequent portfolio rebalancing rather than high returns of individual futures. In a simple asset allocation exercise, they demonstrate that incorporating commodity futures into an equity-bond portfolio improves the risk-return profile significantly if the excess return of the commodity portfolio exceeds 3%.³ Unfortunately, Erb and Harvey do not offer an OOS evaluation based on actual data of asset returns.

Some researchers more directly examine the economic value of commodities in multi-asset allocation exercises, but no consensus on whether investing in commodities adds value to traditional equity-bond investors has been reached thus far. On one hand, a number of studies provide evidence that commodities can be effective alternatives to achieve portfolio diversification. For example, Bodie and Rosansky (1980) find that simply switching from a 100% stock portfolio to 60-40% stocks and commodities makes an investor better off by trimming off one third of the portfolio risk without sacrificing any return performance during the period 1950 to 1976. Similarly, Fortenbery and Hauser (1990) find that adding individual agricultural commodity futures into a well diversified stock portfolio does not enhance the return performance but does reduce risk. After adding

³Erb and Harvey also show that an 18-60-22% commodity-equity-bond portfolio yields much higher return than a traditional 60-40% equity-bond portfolio given the same level of volatility.

commodities into an investment portfolio that consists of stocks, bonds, Treasury bills, and real estate, Jensen, Johnson, and Mercer (2000) report that the Markowitz optimization gives substantial weights towards commodities so that the portfolio's return is enhanced during restrictive monetary policy periods, but little or no weights during expansive monetary periods. Anson (1999) investigates the diversification contribution of commodities with respect to investors' risk tolerance, and finds that more risk averse investors gain higher utility from investing in commodity index funds than less risk averse investors during 1974-1997. More recently, some researchers provide more supportive evidence at the levels of individual commodities and sub-sectors. For instance, Geman and Kharoubi (2008) document that WTI crude oil futures provide significant diversification opportunities to the S&P500 index from 1990 to 2006. You and Daigler (2012) confirm the diversification benefits of 39 commodity futures to traditional equity-bond portfolios in the period 1994-2010. Belousova and Dorfleitner (2012) document the heterogeneous diversification effects across five commodity sectors that energy and precious metals yield the highest value in both return enhancement and risk reduction, and agricultural, livestock and industrial metals only contribute to the risk reduction dimension. It is important to notice that the commodity-augmented portfolios in most of these studies are constructed using historical mean returns and sample variance/covariance (covariance) structures and then evaluated within a MV or IS framework.⁴

On the other hand, other recent studies have challenged the alleged view by showing that including commodities in investors' portfolios adds little or no value, particularly, in an OOS assessment. The most recent and striking evidence is documented in Daskalaki and Skiadopoulos (2011), in which the authors conduct a comprehensive examination on the diversification value of two most popular commodity indexes (S&P GSCI and DJ-UBSCI) and five individual commodity futures under four different specifications, namely the four cross-combinations of MV or non-MV investors and in- or OOS settings. They find that, after accounting for investors' preferences over higher-order moments, commodities only contribute to non-MV investors in the IS setting, but are

⁴You and Daigler (2012) construct ex ante portfolios without infrequent rebalancing and evaluate their performance based on efficient frontiers and the Sharpe ratios within a MV framework; Belousova and Dorfleitner (2012) consider non-normal returns, but the performance evaluation is based on IS statistical spanning tests.

not beneficial under any of the other specifications. Even though a large body of literature has suggested or directly confirmed the benefits of commodities in an IS setting, the reported results in Daskalaki and Skiadopoulos (2011) inevitably raise some concerns over the validity of such benefits in an OOS context. Moreover, some studies employ regression-based spanning tests to examine whether investing in commodities indexes or individual futures improves the MV efficient frontier of benchmark portfolios within a MV and IS framework, and fail to find any value.⁵ [See Cao, Jayasuriya, and Shambora (2010), Galvani and Plourde (2010) and Nijman and Swinkels (2008).] Another concern on the diversification value of commodities comes from the recent debate on the pattern of increasing correlation between commodity and stock returns since the crash of financial markets in 2007.⁶ Investigating the causes of such phenomena goes beyond the scope of this paper. Nevertheless, the observed rising commodity-equity correlation is certainly of investors' concern, as it could hurt the risk diversification value of commodities, and thus, in turn, would have influence on their optimal portfolio selection. Therefore, it is interesting to re-examine the diversification role using updated data during the post-crisis period.

To summarize, despite the fact that investing in commodities has become popular in recent years and that a large number of academic studies have looked into its potential benefits in improving portfolio performances, there have been thus far mixed answers to the question of whether commodities add value in multi-asset allocation exercises, particularly and more importantly, OOS. To explore the economic value of commodities in portfolio allocation, this paper sets out to better account for the dynamic nature of asset return moments than previously done in the literature.

3 Existing Approaches and Extensions

In this study, we extend the existing literature on the role of commodities in multi-asset allocation along several dimensions. Firstly, the predictability of commodity returns has been largely

⁵It is worth noting that these studies rely on the outcomes of statistical spanning tests. However, there has been a debate on "statistical" vs. "economical" significance for spanning tests. See Glabadanidis (2009) and Kan and Zhou (2012) for detailed discussion.

⁶See Domanski and Heath (2007), Hamilton (2008), Smith (2009), Tang and Xiong (2010), Chong and Miffre (2010), Büyüksahin and Robe (2010) and Kilian and Murphy (2010) for the debate.

neglected by researchers. Instead, all prior studies rely on sample means of historical returns as the estimates for future expected returns.⁷ To the best of our knowledge, this is the first study that explicitly takes into account commodity return predictability in forming optimal portfolios that incorporate multiple asset classes.⁸ Secondly, previous studies exclusively rely on static and/or BWD looking covariance estimators to derive optimal portfolio rules. Instead, we explicitly consider the dynamics of covariance structures of commodities and other asset classes in the portfolio optimization problem. The combined analysis of predictable first and second moments for portfolio allocation is typically absent from previous studies, whether or not considering commodities. Thirdly, the asset allocation exercises involving commodities have been almost always conducted within the classic Markowitz MV framework, which only relies on the first and second moments of commodity returns. As we will detail below, commodity returns exhibit negative skewness and substantial leptokurtosis, making problematic the reliance on first and second moments only when implementing and evaluation portfolio strategies. In addition to the MV case, we also conduct the analysis in a non-MV context by incorporating higher-order moments of asset returns whose significance and time variation are well documented in the literature.⁹ Lastly, the results of existing studies that support the inclusion of commodities into a stock-bond portfolios are exclusively based on the IS assessment. However, there have been legitimate concerns about the OOS validity, which motivates our analysis in an OOS context.¹⁰ In this we exclusively rely on an OOS performance evaluation.

⁷Bodie and Rosansky (1980), Fortenbery and Hauser (1990), Jensen, Johnson, and Mercer (2000), Anson (1999) and Belousova and Dorfleitner (2012) all employ constant historical estimators; Daskalaki and Skiadopoulos (2011) use rolling-sample based estimator.

⁸Erb and Harvey (2006) simply assume a sample set of forward-looking commodity futures index returns for demonstrating purpose only.

⁹The studies that document the impact of higher moments of asset returns in asset allocation context include Harvey and Siddique (1999), Ang and Bekaert (2002), Timmermann (2006) and Jondeau and Rockinger (2006, 2012).

¹⁰Daskalaki and Skiadopoulos (2011) perform both in- and out-of-sample tests, and, as noted above, they find commodities add no value to investors OOS. See also Welch and Goyal (2008) for related concerns on IS evaluations of return predictability.

3.1 Predictable commodity expected returns

In principle, expected returns, as one of the crucial inputs to the portfolio optimization problem, should be forward-looking, and thus, forecasted. In practice, as demonstrated in Timmermann and Blake (2005), sophisticated investors indeed forecast time-varying future investment opportunities and then invest accordingly. All previous studies investigating the value of commodities in asset allocation tend to use sample means as the "best" estimates for future expected returns without taking account their time-varying characteristics. The theoretical work by Merton (1971, 1973) implies that return predictability could have significant impact on investors' optimal portfolio choices. Motivated by the theoretical implication and the overwhelming empirical findings on stock return predictability, a number of studies show that such predictability, even statistically weak, could lead to significant economic gains, if exploited by investors who allocate wealth in equity markets. [See Solnik (1993), Pesaran and Timmermann (1995) and Cenesizoglu and Timmermann (2012) for example.] Naturally, two questions arise: are commodity returns predictable? If so, could such predictability pay off economically? The answer to the first question is yes, which we will detail soon. Unfortunately, there has been no study dedicated to the second question, which seems more economically meaningful. Thus, the first goal of this paper is to fill this gap by exploring the economic value of commodity return predictability in multi-asset allocation.

Systematically estimating forward-looking expected returns of commodities could be a challenging task, due to the fact that commodities exhibit great heterogeneity in both return and risk characteristics. Fortunately, recent studies have documented the IS and OOS return predictability by a collection of variables at both individual commodity and broad index levels. Generally, these variables could be categorized into two sets: macro-economic and commodity-specific. The macro-economic set includes the short interest rate, the long term rate of return on bonds, corporate bond yield spreads over government issued bonds, the inflation rate, the growth in Industry Production, the growth in Money Supply, the Baltic Dry Index and the aggregate open interest in commodity futures markets. [Bessembinder and Chan (1992), Bakshi, Panayotov, and Skoulakis

(2011), Gargano and Timmermann (2012) and Hong and Yogo (2012).] The commodity-specific set includes the open interest imbalance between hedgers and speculators (also known as the "hedging pressure"¹¹) and the basis¹².

One possible reason that return predictability has been largely neglected by researchers in the asset allocation literature that involves commodities could be the lack of reliable forecasting models or predictors for future commodity returns. Nevertheless, recent innovation in the literature has updated our knowledge and identified a collection of variables that appear to predict commodity returns. Hence, it seems reasonable and important to investigate the economic value of the predictability in portfolio selection with commodities.

3.2 Predictable volatility and correlations

It is now widely agreed that covariance of financial asset returns vary substantially across assets, asset classes, countries, time periods, market conditions and business cycles.¹³ Particularly, Chong and Miffre (2010) and Büyüksahin, Haigh, and Robe (2010) document the large variation in commodity-equity correlations at both index and individual futures levels over time, which, in turn, implies time-varying diversification benefits of commodities for traditional equity-bond investors. Some researchers have examined the impact of covariance dynamics on portfolio strategies without considering commodities, and find that it yields substantial economic gains (loss) for investors who account for (ignore) the dynamics of covariance structures. For example, Fleming, Kirby, and Ostdiek (2001, 2003) demonstrate that volatility predictability in stock markets would be worth 50-200 bps per year for an investor who allocates her wealth in the S&P500 index, Treasury bonds and gold futures. Della Corte, Sarno, and Tsiakas (2012) find substantial economic value in timing correlations in addition to the gains from volatility timing in FX markets. Engle and Colacito

¹¹See Hirshleifer (1989), Bessembinder and Chan (1992) and De Roon, Nijman, and Veld (2000).

¹²The basis is defined as the difference between the current spot price and the contemporaneous futures price, which is closely related to the convenience yield that serves as an important determinant pricing futures contracts in the Theory of Storage. See Fama and French (1987) and Gorton, Hayashi, and Rouwenhorst (2012).

¹³See Erb, Harvey, and Viskanta (1994), Longin and Solnik (1995), Ball and Torous (2000), Ang and Bekaert (2002), Moskowitz (2003), Goetzmann, Li, and Rouwenhorst (2005), and Cappiello, Engle, and Sheppard (2006) for international and U.S. equities and bonds; Büyüksahin, Haigh, and Robe (2010), Chong and Miffre (2010) for commodities and stocks; Huang and Zhong (2010) for commodities, real estate and TIPS.

(2006) theoretically and empirically show that, keeping other conditions constant, the economic loss of stocks-bonds portfolio performance could be as high as 40% of return if a static correlation specification is assumed but the true structure is dynamic.

Given the empirical evidence on time-varying covariance highlighted in the data and the economic significance of dynamic covariance modeling, it is surprising that all previous studies evaluating the value of commodities in asset allocation fail to take into account the dynamic nature in the OOS portfolio analyses. Hence, another question this paper aims to address is whether capturing such covariance dynamics between commodities and other assets adds OOS economic value for investors.

3.3 Predictable Skewness and Kurtosis

One of the inadequacies of the classic Markowitz portfolio theory and its empirical implementations is its inability to handle higher-order moments of the return distribution. However, it seems reasonable to assume that risk averse investors favor positive skewness and low kurtosis in asset returns.¹⁴ Moreover, there exists overwhelming empirical evidence suggesting that financial asset returns exhibit excess skewness and kurtosis rather than normality.¹⁵ Gorton and Rouwenhorst (2006), Erb and Harvey (2006) and Gorton, Hayashi, and Rouwenhorst (2012) all report that monthly return distributions of commodity futures indexes and individual futures have positive skewness and excess kurtosis during the periods of 1959-2004, 1982-2004 and 1971-2010, respectively. At weekly frequency, the results are mixed. You and Daigler (2010) report 55% of 20 commodity futures have positive skewness during the 1992-2006 period. At daily frequency, Eastman and Lucey (2008) show that the returns of 14 futures are all negatively skewed except the 10-year notes. In summary, the skewness and kurtosis of commodity returns are well pronounced in the data although sensitive to data frequencies.

The inconsistency between the presence of skewness and kurtosis in the data and the normality

¹⁴Scott and Horvath (1980) establish the fact that investors prefer odd moments and are averse to the even ones. Harvey and Siddique (2000) and Dittmar (2002) offer the empirical evidence.

¹⁵E.g., see Ang and Bekaert (2002) for the evidence in equity markets.

assumption in the classic portfolio theory has drawn attention by researchers in asset pricing and asset allocation. Several studies find that risk averse investors do adjust optimal portfolio weights accordingly to time-varying skewness and kurtosis in stock returns, and that portfolio strategies accounting for the dynamics of higher-order moments leads to economically significant gains.¹⁶ However, the literature has been extremely skewed towards the impact of higher moments in optimal allocations within equity markets. There has been very limited exploration on whether these findings in equity markets are preserved in other asset classes, in particular, commodities.¹⁷ Given the presence of excess skewness and kurtosis in commodity return data, another goal of this study is to analyze the economic value of predicting higher-order moments and co-moments of commodity returns in an OOS asset allocation context.

3.4 Summary and potential challenges

In the present study, we assess the OOS performance of investment strategies incorporating commodities into traditional asset portfolios by more carefully modeling forward-looking expected commodity returns, dynamics of volatility and correlation structures and higher-order moments. Our analysis builds on the recursive construction of optimal OOS portfolios consisting of commodities, stocks, bond and cash.

This is not a challenge-free exercise. One of the main challenges lies in the well-known estimation risk, which has been discussed in a number of asset allocation studies.¹⁸ Admittedly, any sophisticated asset allocation models that either account for time-varying and state-dependent properties or introduce higher moments raise the concerns of estimation error. Specifically, the true model parameters are unknown and thus need to be estimated from the data. Therefore, the estimated optimal portfolio rule is subject to parameter uncertainty that makes the estimated rule significantly different from the true optimal rule. Increasing the richness and, thus, the dimensionality

¹⁶See Bekaert, Erb, Harvey, and Viskanta (1998), Patton (2004), Jondeau and Rockinger (2006), Guidolin and Timmermann (2008) and Harvey, Liechty, Liechty, and Müller (2010), among others.

¹⁷Daskalaki and Skiadopoulos (2011) and You and Daigler (2010) are exceptions.

¹⁸For example, see Kandel and Stambaugh (1996) and Jagannathan and Ma (2003).

of the model exacerbates the problem.¹⁹

4 The asset allocation strategies

In this section, we discuss the asset allocation strategies under different specifications and the assessment of their OOS performance. In general, we consider the case of a risk averse investor with preferences characterized by a continuous, increasing and concave utility function facing an asset allocation problem with N risky assets and a risk-free asset. The investor recursively chooses portfolio weight vectors $x_t \in \mathbb{R}^{N+1}$ to maximize her one-period-ahead expected utility $E_t[U(W_{t+1})]$.²⁰ Let $x_t = [\theta_t, \omega_t]'$ denote the portfolio weight vector at time t . In particular, θ_t is a scalar denoting the weight on the risk-free asset, $\omega_t \in \mathbb{R}^N$ is a vector denoting the weights on N risky assets, and i^{th} element of $\omega_{i,t}$ is the fraction of wealth to the i^{th} risky asset. The investor's initial wealth is normalized to one. At each time t , the investor's portfolio optimization problem with respect to x_t is given by:

$$\begin{aligned} \max_{x_t} \quad & E_t[U(W_{t+1})] \\ \text{s.t.} \quad & i'x_t = 1 \end{aligned} \tag{1}$$

where i is a $N \times 1$ vector of 1.

For comparison purposes, we evaluate the performance of the following investment strategies.

1. Traditional asset / backward-looking strategy (*S-B/BWD*): the investor employs backward-looking sample moments of asset returns as the inputs to the optimization problem defined in Eq.(1) to allocate her wealth in Equity, Bond and Cash only.

¹⁹This is reminiscent of the issue raised by the provocative work by DeMiguel, Garlappi, and Uppal (2009). The authors question the value of 14 static MV portfolio optimization models and show that none of the 14 models can beat the naive 1/N rule (i.e. an asset allocation strategy that invests 1/N of wealth on each of the N available assets at each rebalancing date.) in terms of OOS Sharpe ratios. In the present study, the issue appears to be of a less concern as we are dealing with a substantially lower dimensional problem.

²⁰As most academic studies in asset allocation only focus on solving recursive myopic portfolio optimization problems [See, for example, DeMiguel, Garlappi, and Uppal (2009), Daskalaki and Skiadopoulos (2011) and Cenesizoglu and Timmermann (2012).], and given also that industry practice is mostly about one-period problems as illustrated by Brandt (2009), we leave the dynamic portfolio choice analysis for future study.

2. Traditional asset / forward-looking strategy (*S-B/FWD*): the investor employs forward-looking moments as the inputs to the optimization problem defined in Eq.(1) to allocate her wealth in Equity, Bond and Cash only.
3. Commodity-augmented / backward-looking strategy (*S-B-C/BWD*): the investor employs backward-looking sample moments of asset returns as the inputs to the optimization problem defined in Eq.(1) to allocate her wealth in Equity, Bond, Commodity and Cash.
4. Commodity-augmented / forward-looking strategy (*S-B-C/FWD*): the investor employs forward-looking moments as the inputs to the optimization problem defined in Eq.(1) to allocate her wealth in Equity, Bond, Commodity and Cash.

For all four strategies, we evaluate and compare their OOS performance based on economic criteria for both MV and non-MV cases.

4.1 The MV case

We first consider the case for a MV investor whose preferences are characterized by a quadratic utility function. So her expected utility maximization problem is given by:

$$\max_{\omega_t} E_t[r_{p,t+1}] - \frac{\gamma}{2} var_t[r_{p,t+1}] \quad (2)$$

where γ measures the investor's tolerance for risk, $r_{p,t+1} = \omega_t' \mu_{t+1}$ is the portfolio return at the time $t + 1$, and $var_t[r_{p,t+1}] = \omega_t' \Sigma_{t+1} \omega_t$ is the $t + 1$ portfolio return variance. The solution to the optimization problem defined in Eq.(2) is given by:

$$\omega_t^* = \frac{1}{\gamma} \Sigma_{t+1}^{-1} \mu_{t+1} \quad (3)$$

where $\mu_{t+1} \equiv E_t[r_{t+1}] \in \mathbb{R}^N$ the vector of expected excess returns, and $\Sigma_{t+1} \equiv E_t[(r_{t+1} - \mu_{t+1})(r_{t+1} - \mu_{t+1})'] \in \mathbb{R}^{N \times N}$ is the covariance matrix of N risky assets. The optimal weight

for the risk-free asset is given by:

$$\theta_t^* = 1 - i' \omega_t^* \quad (4)$$

Thus, to obtain the optimal weights $x_t^* = [\theta_t^*, \omega_t^*]'$, one needs to input the estimates for μ_{t+1} and Σ_{t+1} . In this study, we consider two sets of estimators: (i) the BWD estimators $\bar{\mu}_{t+1}$ and $\bar{\Sigma}_{t+1}$ which denote the sample mean and sample covariance matrix up to time t ; (ii) the FWD estimators $\hat{\mu}_{t+1|t}$ and $\hat{\Sigma}_{t+1|t}$ which denote the conditional OOS forecasts for the expected excess returns and covariance matrix using conditional information up to time t . We detail the OOS forecasting in the following sections.

4.1.1 Forecasting excess returns

In this section, we discuss the predictive variables and forecasting methods for each asset.

Commodities. A number of related studies motivate the set of predictive variables used in our study. [See Bessembinder and Chan (1992); Bakshi, Panayotov, and Skoulakis (2011); Gargano and Timmermann (2012); Hong and Yogo (2012).] Most of the predictors are macro economic variables that have been identified as reliable signal of higher economic activity and, consequently, future movements in asset prices. [See, for example, Hong and Yogo (2012).] Specifically, the short rate (*TB3*) is measured by the 90-day Treasury bill yield; the long-term rate of returns (*LTR*) is calculated as the equally-weighted average of 10-, 20- and 30-year U.S. Treasury bond yields; the default yield spread (*DFS*) is computed as the difference between Baa- and Aaa-rated corporate bond yields; the inflation (*INFL*) is the year-on-year (log) growth rate of Consumer Price Index; the monthly (quarterly) growth rate of Industrial Production Index (ΔIP) is calculated as the log difference of month-end (quarter-end) Industrial Production index and the growth rate of Baltic Dry Index (*BDI*) is the log changes in the BDI over the preceding three months.²¹

Equities. There has been a huge literature studying the stock return predictability. A long list of predictors have been previously utilized in the literature. In this study, we consider a set

²¹Baltic Dry Index is an assessment of the price of moving the major raw materials by sea. See, for example, Bakshi, Panayotov, and Skoulakis (2011) for a comprehensive analysis on the BDI as a reliable predictor for asset returns and global economic activities.

of predictors that have been extensively studied and, more importantly, confirmed in recent OOS studies.[See, among others, Welch and Goyal (2008) and Henkel, Martin, and Nardari (2011).] In addition to the short rate ($TB3$), long-term rate of return (LTR) and default spread (DFS) that are already defined in previous section, 3 new variables are included: the dividend price ratio (DP) defined as the difference between the log of the 12-month moving sum of dividends and the log of the S&P 500 index, the net payout yield (NPY) calculated as the difference of the log of the sum of dividends and repurchases less issuances and the log of the S&P500 index and the term spread measured (LTR) as the difference between the 10-year Treasury bond yield and the 90-day Treasury-bill. Again, we use the combination forecast method defined in Eq.(6) and Eq.(7) to forecast the next period OOS excess returns of S&P500 index $\hat{r}_{stk,t+1}^{comb}$.

Bonds. To forecast excess returns for the US Treasury bond index, we use two sets of predictive variables. The first set consists of forward rates calculated from 1-5 year US Treasury bond prices. [See Cochrane and Piazzesi (2005)] The second set includes 3 individual variables identified in Ilmanen (1995): the inverse relative wealth ($INVRELW$) as a proxy for time-varying risk aversion and two overall proxies for expected bond risk premium, namely the term spread ($TERM$) which is defined in the previous subsection and the real yield ($REALYLD$) measured as the difference between the long-term bond yield and the year-on-year inflation rate.

We follow, among others, Rapach, Strauss, and Zhou (2010) and employ the forecast combination method to produce one-period-ahead OOS forecasts for excess returns of each asset.²² The general rationale for using a forecast combination is that, while individual forecasts could suffer from model mis-specification and instabilities, combining the individual forecasts could exploit the valuable information carried by each predictor and, at the same time, achieve benefits from forecast diversification. From the diversification perspective, we pool the individual forecasts and thus reduce the “risk” - forecasting errors.

²²The method is originally pointed out by Bates and Granger (1969). Timmermann (2006) offers a comprehensive theoretical and empirical study on forecast combinations.

The forecasting model Eq.(5) - Eq.(7):

$$rx_{i,t} = \alpha_i + \beta_i z_{i,t-1} + \varepsilon_{i,t} \quad \text{where } i = 1 \cdots K \quad (5)$$

$$\hat{r}x_{i,t+1|t} = \hat{\alpha}_{i,t} + \hat{\beta}_{i,t} z_{i,t} \quad (6)$$

$$\hat{r}x_{t+1|t}^{comb} = \frac{1}{K} \sum_{i=1}^K \hat{r}x_{i,t+1|t} \quad (7)$$

where $z_{i,t-1}$ is the i^{th} individual predictor at time $t - 1$, and K is the total number of predictive variables.

We first run the univariate predictive regression defined in Eq.(5) to produce the IS estimates for parameters $\hat{\alpha}_{i,t}$ and $\hat{\beta}_{i,t}$ upto time t . Next, we follow Eq.(6) to produce the one-period-ahead forecasts $\hat{r}x_{i,t+1|t}$ using $\hat{\alpha}_{i,t}$, $\hat{\beta}_{i,t}$ and $z_{i,t}$. At last, we combine individual forecasts $\hat{r}x_{i,t+1|t}$ across the K individual forecasts with equal weights to obtain the next period OOS forecasts $\hat{r}x_{t+1|t}^{comb}$. We do not perform any statistic tests to determine combining weights, but simply average them to avoid estimating the weights which induces estimation error and may deteriorate the OOS performance.²³ The IS predictive regression is based on an expanding window and only uses information upto time t .

4.1.2 Forecasting covariance matrix

We next turn to estimate another key input in Eq.(3), namely the multi-period ahead conditional covariance matrix Σ_{t+1} . We employ the Dynamic Conditional Correlation (DCC) model proposed by Engle (2002) to obtain the OOS forecast.²⁴ Essentially, the one-period ahead covariance matrix estimator can be decomposed as:

$$\hat{\Sigma}_{t+1|t} = \hat{D}_{t+1|t} \hat{P}_{t+1|t} \hat{D}_{t+1|t} \quad (8)$$

²³Timmermann (2006) and Rapach, Strauss, and Zhou (2010) demonstrate the benefits of using equally weighted forecast combinations.

²⁴The DCC model and its variations have been used by several studies in asset allocation context. For example, Billio, Caporin, and Gobbo (2006), Huang and Zhong (2010), Case, Yang, and Yildirim (2012) and Della Corte, Sarno, and Tsiakas (2012).

where $\hat{D}_{t+1|t}$ is an $N \times N$ diagonal matrix with conditional standard deviation $\hat{\sigma}_{i,t+1|t}$ on the i^{th} diagonal, and $\hat{P}_{t+1|t} = \{\hat{\rho}_{ij,t+1|t}\}$ is an $N \times N$ matrix with ones on the diagonal and conditional correlations off the diagonal. Given the decomposition, we can employ a two-stage estimation procedure to obtain $\hat{\Sigma}_{t+1|t}$. In the first stage, we forecast each diagonal element ($\hat{\sigma}_{i,t+1|t}$) of $\hat{D}_{t+1|t}$ with a univariate GARCH(1,1) model. In the second stage, we first de-mean all return series and obtain their residuals. Then, we standardize the residual series by their conditional standard deviations and fit the standardized residuals into a multivariate GARCH(1,1) model to obtain the one-period-ahead conditional correlation matrix $\hat{P}_{t+1|t}$. All estimation are based on up to-time- t return data only (i.e. no additional predictive variables). The two-stage procedure is detailed in Appendix A.

4.1.3 Statistical Evaluation of Predictability

We statistically assess the forecastability of asset returns, volatility and correlations using the OOS R^2 statistics introduced in Campbell and Thompson (2008) and the Mincer-Zarnowitz (M-Z) regression proposed by Mincer and Zarnowitz (1969).

Excess Returns. The statistic measure for asset return predictability is computed as following:

$$\text{OOS } R_{rx}^2 = 1 - \frac{\sum_{t=1}^T (rx_t - \hat{r}x_{t|t-1})^2}{\sum_{t=1}^T (rx_t - \bar{r}x_{t-1})^2} \quad (9)$$

where rx_t is the ex post realized excess return, $\hat{r}x_{t|t-1}$ is the FWD estimator, and $\bar{r}x_{t-1}$ is the BWD estimator. In our case, the FWD estimator for each asset is the combination forecasts $\hat{r}x_{t|t-1}^{comb}$ defined in Eq.(7) using an expanding window estimation. For BWD estimators, we consider 60-, 120-month moving averages and historical means of monthly excess returns as benchmarks for the evaluation.

Volatility. There seems no universal agreement on the best measure evaluating the OOS forecasting performance of volatility. Therefore, in light of Andersen and Bollerslev (1998), we rely on the M-Z methodology to evaluate the alternative volatility forecasts by regressing the ex post

realized volatility computed from daily data on a constant and the various forecasts as follows:

$$rvol_t = \beta_0 + \beta_1 vol_{t|t-1}^* + \varepsilon_t \quad (10)$$

where $rvol_t$ is the ex post realized volatility calculated as the sum of squared daily returns over the period t , $vol_{t|t-1}^*$ is the evaluated volatility estimator using information upto time $t - 1$. In particular, $vol_{t|t-1}^* \equiv \bar{rvol}_{t-1}$ is the BWD sample volatility, i.e. the moving standard deviation of 60- or 120-month monthly returns, and $vol_{t|t-1}^* \equiv \hat{rvol}_{t|t-1}$ is the FWD volatility forecast, i.e. the estimator that exploits the time-varying properties. Following the standard MZ methodology, the regression defined in Eq.(10) with the β_0 closest to zero, the β_1 closest to one and the highest regression R^2 is viewed as the best forecasts.

Correlations. The OOS forecasting performance of correlations is evaluated in the same way as volatility.

$$\rho_{ij,t} = \beta_0 + \beta_1 \rho_{ij,t|t-1}^* + \varepsilon_t \quad (11)$$

where ρ_t is the ex post realized correlation between asset i and j over the period t , $\rho_{ij,t|t-1}^*$ is the evaluated correlation estimator using information upto time $t - 1$. In particular, $\rho_{ij,t|t-1}^* \equiv \bar{\rho}_{ij,t-1}$ is the BWD sample correlation (i.e. the moving correlation of 60- or 120-month monthly returns), and $\rho_{ij,t|t-1}^* \equiv \hat{\rho}_{ij,t|t-1}$ is the FWD correlation forecast using the DCC model.

4.1.4 Economic Evaluation of MV Strategy Performance

With the BWD ($\bar{\mu}_t$ and $\bar{\Sigma}_t$) and the FWD estimators ($\hat{\mu}_{t+1|t}$ and $\hat{\Sigma}_{t+1|t}$) of expected returns, volatility and correlation matrix, we can then determine the optimal weight vector (ω_t) as expressed in Eq.(3). We use these weights and the ex post realized asset returns (r_{t+1}) to compute the portfolio return in time period $t + 1$. Repeating these steps, we obtain the OOS portfolio return series $\{r_{p,t}\}_{t=\tau+1,\dots,T}$ for economic assessment.

We compute a number of portfolio performance measures that have been widely used in the asset allocation literature to assess the economic performance of alternative strategies: the Sharpe

Ratio (SR), the Sortino’s Upside Potential Ratio (UP), the Certainty Equivalent Return (CEQ) and the Portfolio Turnover (TO). We detail these metrics in the following.

We compute the OOS *Sharpe Ratio*²⁵ as:

$$\text{SR}_{\tau+1:T} = \frac{\bar{\mu}_{\tau+1:T}}{\bar{\sigma}_{\tau+1:T}} \quad (12)$$

where $\bar{\mu}_{\tau+1:T}$ and $\bar{\sigma}_{\tau+1:T}$ are respectively the sample mean and standard deviation of realized portfolio excess returns over the OOS period $[\tau+1:T]$. To test whether the SRs of two strategies are statistically distinguishable, we also compute the p-value of the difference, using the approach suggested by Jobson and Korkie (1981) after making the correction pointed out in Memmel (2003).

We also calculate an additional ratio measure, namely the *Sortino’s Upside Potential Ratio* (UP) originally proposed by Sortino, Meer, and Plantinga (1999). In contrast to SR, UP is calculated as the ratio of the sample mean of positive portfolio returns divided by the “bad” standard deviation by using negative returns only:²⁶

$$\text{UP}_{\tau+1:T} = \frac{\frac{1}{T-\tau} \sum_{t=\tau+1}^T \text{Max}(\tilde{r}_{p,t}, 0)}{\left[\frac{1}{T-\tau} \sum_{t=\tau+1}^T \text{Min}^2(\tilde{r}_{p,t}, 0) \right]^{\frac{1}{2}}} \quad (13)$$

Moreover, we compute the *Certainty Equivalent Return* (CEQ), which can be interpreted as the risk-free rate that an investor is willing to accept rather than adopting a particular portfolio strategy.²⁷ In particular, the CEQ is computed as:

$$\text{CEQ}_{\tau+1:T} = \bar{\mu}_{\tau+1:T} - \frac{\gamma}{2} \bar{\sigma}_{\tau+1:T}^2 \quad (14)$$

²⁵SR measures a trading strategy’s risk-adjusted performance by excess return (over the risk-free rate) per unit of deviation.

²⁶It is well known that SR is subject to the assumption that portfolio returns are normally distributed, thus, relies on symmetric distributions to provide reliable assessments. However, in case that portfolio returns exhibit excess skewness, SR could result in misleading evaluations if one fails to take into account such asymmetries. Lien (2002) demonstrates that UP could provide opposite but more plausible rankings among alternative investment strategies than SR when the returns exhibit positive skewness, but also the same rankings as SR when the returns are distributed normally. Therefore, we consider DOWN and UP as superior portfolio performance measures than SR.

²⁷CEQ has been widely adopted to assess portfolio performance in the asset allocation literature. See, for instance, Kandel and Stambaugh (1996), Campbell and Viceira (1999), Ang and Bekaert (2002), Campbell and Thompson (2008) and Rapach, Strauss, and Zhou (2010).

where γ is the risk aversion coefficient, $\bar{\mu}_{\tau+1:T}$ and $\bar{\sigma}_{\tau+1:T}^2$ is sample mean and variance of realized portfolio returns, respectively. It is worth noting that the difference of CEQ's can be interpreted as the Performance Fee²⁸ (a.k.a. opportunity cost) that is adopted in a number of studies, e.g. Fleming, Kirby, and Ostdiek (2001). Given the simple relationship between performance fee and CEQ, we only consider CEQ in our studies.

Lastly, to measure the amount of trading required to implement the strategy, we follow DeMiguel, Garlappi, and Uppal (2009) and compute the *Portfolio Turnover*, which can be interpreted as the average percentage of wealth traded at each period, as:

$$TO_{\tau+1:T} = \frac{1}{T - \tau} \sum_{t=\tau}^{T-1} \sum_{j=1}^K (|\omega_{j,t+1} - \omega_{j,t+}|) \quad (15)$$

where $\omega_{j,t+1}$ is the desired portfolio weight in asset j at time $t + 1$; $\omega_{j,t+}$ is the portfolio weight in asset j before rebalancing at time $t + 1$.

Consistent with DeMiguel, Garlappi, and Uppal (2009), we incorporate a proportional transaction cost into all portfolio returns. When a portfolio of N assets is rebalanced at time $t + 1$, the magnitude of trading asset j is $|\omega_{j,t+1} - \omega_{j,t+}|$. Provided a proportional transaction cost c , the trading cost of the entire portfolio is $c \cdot \sum_{j=1}^N |\omega_{j,t+1} - \omega_{j,t+}|$. Therefore, we can write the portfolio return net of transaction cost as

$$r_{p,t+1}^{net} = (1 + r_{p,t+1})(1 - c \cdot \sum_{j=1}^N |\omega_{j,t+1} - \omega_{j,t+}|) - 1 \quad (16)$$

where $r_{p,t+1} = \sum_{j=1}^N r_{j,t+1}\omega_{j,t}$ is the portfolio return with zero transaction cost.

4.2 The Non-MV (4-Moment) Case

Departing from the classic Markowitz paradigm, we next investigate the impact of higher moments of asset returns on optimal multi-asset strategies. To capture investors' preferences for the first

²⁸The difference of CEQ's (i.e. Performance Fee) can be interpreted as the premium that needs to be added to the benchmark portfolio return so that the investor becomes indifferent between the competing and benchmark portfolios.

4 moments of portfolio returns, we consider a non-MV investor characterized by a power utility function:²⁹

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma} \quad (17)$$

where W_{t+1} is the investor's wealth at time $t + 1$ and γ ($\gamma > 0$ and $\gamma \neq 0$) measures her constant relative risk aversion (CRRA). Hence, the investor's portfolio optimization problem (described in Eq.(1)) is given by:

$$\max_{\{\omega_t\}} E_t \left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right] \quad (18)$$

Following the literature [see, among others, Harvey and Siddique (2000), Dittmar (2002), and Jondeau and Rockinger (2006, 2012)] we approximate the expected power utility function as a fourth-order Taylor series expanded around W_{t+1} :

$$E_t \left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right] \approx \phi_0 + \phi_1 m_{p,t+1}^{(1)} + \phi_2 m_{p,t+1}^{(2)} + \phi_3 m_{p,t+1}^{(3)} + \phi_4 m_{p,t+1}^{(4)} \quad (19)$$

where $\phi_0 = \frac{1}{1-\gamma}$, $\phi_1 = 1$, $\phi_2 = -\frac{\gamma}{2}$, $\phi_3 = \frac{\gamma(\gamma+1)}{6}$, $\phi_4 = -\frac{\gamma(\gamma+1)(\gamma+2)}{24}$, and $m_{p,t+1}^{(i)}$ is the i^{th} -order expected non-central moment of portfolio returns. Even though there is no widely accepted rule for selecting the optimal order of truncation, this 4-Moment framework is economically appealing: the investor favors expected return and positive skewness (ϕ_1 & $\phi_3 > 0$) but dislikes variance and kurtosis (ϕ_2 & $\phi_4 < 0$), in accordance with the economic theories suggested by Scott and Horvath (1980) and Dittmar (2002).

Hence, the non-MV investor's time-t optimal 4-Moment portfolio weights are defined as:

$$w_t^* = \arg \max \{ \phi_0 + \phi_1 m_{p,t+1}^{(1)} + \phi_2 m_{p,t+1}^{(2)} + \phi_3 m_{p,t+1}^{(3)} + \phi_4 m_{p,t+1}^{(4)} \} \quad (20)$$

²⁹Campbell and Viceira (2003) point out that power utility implies that absolute risk aversion is declining in wealth, while relative risk aversion is a constant (γ). As absolute risk aversion should decline, at the very least should not increase, with wealth, this rules out the assumption of quadratic utility and favors power utility over exponential utility.

4.2.1 Implementing 4-Moment Strategies

To solve the 4-Moment portfolio optimization problem, we need to estimate the first four non-central moments of portfolio returns (i.e. $\{m_{p,t+1}^{(i)}\}_{i=1,\dots,4}$). Jondeau and Rockinger (2012) show that these moments can be analytically expressed as functions of portfolio weights, expected returns, covariance, co-skewness and co-kurtosis of asset returns.

Thus, to solve the 4-Moment optimal portfolio problem defined in Eq.(20), one needs one-step-ahead forecasts for all those return moments. Notation wise, one needs to obtain estimators for $\{\mu_{t+1}, \Sigma_{t+1}, S_{t+1}, K_{t+1}\}$, where μ_{t+1} , Σ_{t+1} , S_{t+1} and K_{t+1} are expected returns, covariance, co-skewness and co-kurtosis matrices, respectively.

Consistent with the implementation of MV strategies, we consider two types of estimators in the 4-Moment case, namely the BWD set denoted by $\{\bar{\mu}_t, \bar{\Sigma}_t, \bar{S}_t, \bar{K}_t\}$ and the FWD set denoted by $\{\hat{\mu}_{t+1|t}, \hat{\Sigma}_{t+1|t}, \hat{S}_{t+1|t}, \hat{K}_{t+1|t}\}$. The BWD estimators can be easily computed from sample returns using information available upto time t. We have already introduced estimating $\hat{\mu}_{t+1|t}$ in Section 4.1. In estimating $\{\hat{\Sigma}_{t+1|t}, \hat{S}_{t+1|t}, \hat{K}_{t+1|t}\}$ we follow the procedure proposed by Jondeau and Rockinger (2012). We outline the entire procedure in Appendix B.

With the moment estimators, either $\{\bar{\mu}_t, \bar{\Sigma}_t, \bar{S}_t, \bar{K}_t\}$ or $\{\hat{\mu}_{t+1|t}, \hat{\Sigma}_{t+1|t}, \hat{S}_{t+1|t}, \hat{K}_{t+1|t}\}$, we can numerically solve the non-MV investor's optimization problem for the optimal portfolio weights ω_t^* in Eq.(20). By repeating this optimizing procedure at each time $t = \tau, \dots, T - 1$, we can obtain the optimal portfolio weights $\{\omega_t^*\}_{t=\tau, \dots, T-1}$.

4.2.2 Evaluating Non-MV Allocation Strategies

As SR might not be a reliable measure for investors with higher moment preferences, we drop the SR and consider the following measures assessing the performance of 4-Moment portfolio strategies: the UP defined as Eq.(13) in section 4.1.4, the power utility-based (CEQ) adopted by Cenesizoglu and Timmermann (2012), and two additional risk-only measures, the 1% Value-at-Risk (1% VaR) and the 1% Expected Shortfall (1%ES).

Specifically, the CEQ based on power utility is computed as:

$$\text{CEQ}_{\tau+1:T} = [(1 - \gamma)\bar{U}_{\tau+1:T}(W_t)]^{\frac{1}{1-\gamma}} - 1 \quad (21)$$

where $\bar{U}_{\tau+1:T}(W_t)$ is mean realized power utility. Similar to the CEQ in the MV case, the difference of power utility-based CEQ's can also be interpreted as the performance fee an investor is willing to pay to switch from a benchmark strategy to the competing strategy.

Following Patton (2004) and Jondeau and Rockinger (2012), we consider two additional tail risk measures, namely the 1% Value-at-Risk (1% VaR) and 1% Expected Shortfall (1% ES). In particular, 1% VaR, defined as the first empirical percentile of the realized returns, is calculated as:

$$1\%VaR = \hat{F}_n^{-1}(0.01) \quad (22)$$

where \hat{F}_n is the empirical distribution of portfolio returns using the n OOS observations. The 1% ES is computed as the average return on a portfolio given that the return has exceeded its 1% VaR:

$$1\%ES = -E_n[r_{p,t} | r_{p,t} \leq 1\%VaR] \quad (23)$$

where E_n is the sample average.

5 Data and Descriptive Statistics

5.1 Data

Our data are compiled from various standard sources, including Datastream, CRSP, Federal Reserve Economic Database and the data libraries on Amit Goyal's and Ken French's websites.

We use both daily and monthly major return indexes measuring the three asset classes: U.S. Equity, U.S. Bond and Commodity. Specifically, the equity class is proxied by the S&P500 Total Return Index (SP500). The monthly data from CRSP go back to 1946:01 and the daily data

from Datastream start from 07/03/1962. The bond class is measured by the Barclays Capital U.S. Aggregate Bond Index (Bond). The monthly and daily index data downloaded from Datastream start from 1976:01 and 01/01/1989, respectively. The commodity class is represented using the world-production weighted S&P Goldman Sachs Commodity Index (GSCI Total Return)³⁰. The monthly and daily index data are obtained from Datastream, and respectively start from 1970:01 and 03/31/1970. We use the 30-day U.S. Treasury Bill as cash and the data are from Ken French’s data library. All data series end on 12/31/2012. The reason we consider the GSCI instead of alternative commodities indexes or individual futures is two-folds: 1. GSCI is one of the leading investable commercial commodity indexes followed by a number of exchange-traded products and also has the longest data history; 2. a majority of professional asset managers choose to gain their exposure in commodity asset class through investment vehicles such as ETFs, ETNs and mutual funds, which are index-based passive investment strategies.³¹

The predictive variables used in this study for forecasting monthly asset returns are mostly macro economic variables and have been extensively studied in the literature.³² Specifically, a majority of the predictors are available for download at Amit Goyal’s website. In addition, we download the Baltic Dry Index data from Datastream, and obtain 1- to 30-year U.S. Treasury Bond price data from the Fama-Bliss bond dataset on CRSP.

5.2 Descriptive Statistics

Panel A of Table 1 reports the summary statistics of asset annualized excess returns during the full sample periods. We can see that Equity has the highest mean excess return (4.56%) and a standard deviation of 14.9% and exhibits most extreme negative skewness (-0.56) and relatively

³⁰The total return index measures the returns accrued from investing in fully-collateralized nearby commodity futures, and the spot index measures the level of nearby commodity prices. Thus, the excess return and total return indices provide useful representations of returns available to investors from investing in the S&P GSCI.

³¹According to Morningstar & Barron’s Alternative Investment Survey 2011, over 60% of institutional investors choose commodity ETFs, ETNs and mutual funds as their primary vehicles to access the commodity investment.

³²For Equity, see, among others, Campbell and Thompson (2008), Welch and Goyal (2008), Rapach, Strauss, and Zhou (2010), Henkel, Martin, and Nardari (2011). For Bond, Cochrane and Piazzesi (2005), Ilmanen (1995), Ludvigson and Ng (2009). For Commodity, Bessembinder and Chan (1992), Bakshi, Panayotov, and Skoulakis (2011), Gargano and Timmermann (2012), Hong and Yogo (2012).

high excess kurtosis (1.08). Commodity has a mean excess return of 4.06%, the highest standard deviation (21.19%) and distributes with negative skewness (-0.17) and the most extreme excess kurtosis (2.0). Bond has the lowest mean (3.62%), the lowest standard deviation (3.73%), a modest negative skewness (-0.28) and the lowest excess kurtosis (0.57).

Panel B of Table 1 reports the unconditional correlation matrix of the three monthly excess return series. We find that Commodity is positively correlated with Equity (0.17), but uncorrelated with Bond (-0.03).

[Table 1 goes here...]

6 Empirical Results

6.1 Results for MV Case

In this section, we report the empirical results of the MV case. We first present the statistical evaluation on forecastability of excess returns, volatility and correlations for each asset class. Next, we discuss the economic assessment on the competing multi-asset investment strategies specified in Section 4.

6.1.1 Statistical Evaluation of Forecastability

Table 2 reports the statistical results of evaluating the FWD estimators relative to various BWD benchmarks during the 1995:01 - 2012:12 OOS period. Panel A shows the OOS R^2 statistics of predicting excess returns, and Panel B presents the results of M-Z regressions for volatility and correlations defined in Eq.(10) and Eq.(11), respectively. For excess returns, a positive OOS R^2

indicates that the FWD estimator is superior to its corresponding BWD peer, evaluating based on the ex post realized excess returns. For volatility and correlations, the benchmark, i.e. the left-hand side variable in E-Z regressions, is the ex post realized volatility and correlations computed from daily returns.³³

In Panel A of Table 2, we compare the FWD forecasts of excess returns from an expanding window estimation with three BWD ones, namely 5- and 10-year rolling mean and full-sample historical mean. Consistent with Welch and Goyal (2008), we find that the FWD estimator yields the lowest R^2 relative to the historical mean, but it does generate more accurate forecasts than all BWD estimates except the historical mean of SP500. Turning to the results of volatility and correlations shown in Panel B of Table 2, the main finding is that almost all FWD estimators, in comparison with their BWD peers, generate closer-to-zero β_0 's, closer-to-one β_1 's and much higher R^2 's. In summary, FWD estimators tend to statistically outperform BWD peers by meaningful margins, especially for volatilities and correlations.

[Table 2 goes here...]

6.1.2 Economic Evaluation of Strategy Performance

In this section, we evaluate the MV investor's multi-asset investment strategies from Section 4.1. We first compare the BWD S-B strategy with the BWD S-B-C strategy, neither of which take into account the time-varying properties of asset return moments. Then, we report the comparison among strategies that exploit the predictability in asset returns, volatility and correlations. Lastly, we source the economic gains by switching from BWD to FWD and identify the magnitude

³³Even though volatility and correlations are latent and thus unobservable, Andersen and Bollerslev (1998) suggest that realized volatility calculated from higher-frequency data is a reliable measure of ex-post volatility.

associated with each dimension of predictability.

We report empirical results under three constraints on the weights of risky assets that have been widely imposed in the asset allocation literature: 1. no short-selling ($0 < w$); 2. no short-selling and limited leverage ($0 < w < 1$), 3. no-short selling and no-leverage ($0 < w < 0.33$) As the empirical implementation requires a specification of the investor's risk attitude, following the extant literature, we consider alternative levels of relative risk aversion coefficient, $\gamma = 3, 6, 10$ that respectively represent low, moderately and extremely risk averse investors.³⁴ As the FWD strategies generate higher portfolio turnover than the BWD ones, we include an estimate of transaction costs for the alternative strategies. We set the proportional transaction cost equal to 50 bps for trading all asset class indexes and report the net-transaction-cost results below.³⁵ We also consider passive strategies that rebalance the portfolio back to a predetermined fixed weight (e.g. 60% stock - 40% bond or 60% stock - 30% bond - 10% commodity). Because these passive strategies yield significant underperformance relative to the active strategies reported here, we do not include those results but they are available upon request.

Panel A of Table 3 reports the performance measures for BWD S-B and BWD S-B-C strategies. We will focus on the comparison of CEQ in the discussion for two reasons: 1. CEQ has explicit economic meanings that help us interpret the results; 2. SR does not appropriately measure the conditional performance of an active strategy, because the ex post (unconditional) standard deviation is an inappropriate measure for the (conditional) risk the investor is facing at each point in time.[See Marquering and Verbeek (2004)] Across both sets of portfolio constraints and all levels of risk aversion, the S-B-C strategies consistently underperform the S-B ones. Meanwhile, adding Commodity to S-B portfolios generates substantially higher turnovers and, hence, higher transactions costs. In particular, a moderately risk-averse MV investor ($\gamma = 6$), in a world without

³⁴Even though the literature has not yet agreed upon a commonly accepted estimate of the coefficient of relative risk aversion, many studies suggest a plausible value could be ranging from as low as 0.1 to 10. See, among others, Mankiw (1985), Friend and Blume (1975) and Gordon and St-Amour (2004).

³⁵DeMiguel, Garlappi, and Uppal (2009) assume 50 bps per transaction based on the studies for individual stocks on the NYSE by Stoll and Whaley (1983), Bhardwaj and Brooks (1992) and Lesmond, Ogden, and Trzcinka (1999). Daskalaki and Skiadopoulos (2011) assume 50 bps transaction cost for stock and bond indexes and 35 bps for commodity index based on their discussion with practitioners in the commodity markets.

predictability of asset return moments, would pay up to 30 bps per year to drop commodities from her traditional asset portfolio. This result is consistent with the findings in Daskalaki and Skiadopoulos (2011) who also employ the BWD estimators for their analysis.

Turning to Panel B of Table 3, we can see that almost all measures increase dramatically. This indicates that switching from BWD to FWD strategies adds sizable value to both S-B and S-B-C investors. Zooming into the comparisons between S-B and S-B-C, we can see that the commodity-augmented strategies dominate the traditional asset ones across both weight constraints and all levels of risk aversion. For instance, a moderately risk-averse investor facing a short-selling constraint would pay 101 bps annually in order to incorporate commodity into her traditional S-B portfolio. In other words, these results indicate that investors not only benefit from employing FWD investment strategies, but also gain extra value by including Commodity in their S-B-C portfolios. As we mentioned earlier, implementing FWD strategies does come with cost - high portfolio turnover. On average, FWD strategies need to rebalance 8-20% of wealth each month, approximately 5-8 times as much as the BWD ones do. However, all FWD strategies survive a transaction cost of 50 bps. The comparisons of the Upside Potential Ratio (UP) broadly confirms the conclusions from the analysis of CEQ.

[Table 3 goes here...]

Given the important differences that arise from comparing S-B and S-B-C under the BWD and FWD scenarios, we next investigate the sources of such difference. To do so, we use the BWD strategies as benchmark and add one FWD moment at one time to explore the changes in comparisons. Table 4 reports the results that compare BWD and FWD on each dimension. Due to space limitation, we only present the $\gamma = 6$ case, but the results are qualitatively the same

with other risk aversion coefficients. We first compare the first rows (BWD & BWD) in the S-B and S-B-C panels, the benchmark case. We can see that the commodity-augmented strategy underperforms the traditional asset strategy across all metrics. Adding the FWD expected returns only (FWD & BWD), the performance of both strategies increases relative to their benchmarks, but the S-B-C still underperform the S-B. However, replacing the BWD covariance matrix with the FWD one (BWD & FWD), the S-B-C portfolio outperforms the S-B by 128 bps in CEQ. Combining FWD expected returns and FWD covariance matrix (FWD & FWD), both S-B and S-B-C yield the highest value and S-B-C dominates S-B by a margin of 101 bps in CEQ. The results are slightly different in the case of no short-selling and limit leverage constraints ($0 < w < 1$) that, in both S-B and S-B-C panels, the best performance comes out at BWD & FWD instead of FWD & FWD. However, it is still the case that the S-B-C strategy dominates the S-B portfolio: after accounting for transactions costs, our estimate indicate that an investor would pay up to 71 basis points in order to augment his S-B portfolio with a dynamic exposure to commodities. In summary, the results indicate that, to an investor facing no short-selling constraint, forecasting asset returns and covariance matrix individually could add significant economic value of her portfolio, and her portfolio value is maximized by forecasting both. On the other hand, to an investor facing both no short-selling and limited leverage constraints, her portfolio value is maximized by employing FWD covariance only. Furthermore, the majority of added value by going from BWD to FWD strategies comes from covariance forecasting.

[Table 4 goes here...]

We can see from Table 3 that, as investors' risk aversion (Γ) increases, portfolio performance tends to decrease. This, as shown in Eq.(3), is because more risk averse investors tend to

hold less risky assets and more cash. However, we also observe that the outperformance margins of S-B-C relative to S-B has also decreased as risk aversion increases. To investigate this issue, we show the average weights and weight changes of S-B-C strategies for different risk aversion levels in Table 5. We can see that, by increasing Gamma from 3 to 10, the weights in stocks, bonds and commodities across all strategies drop by 31%, 3% and 38% on average, respectively. In contrast, the weight in cash increases by 72%. Furthermore, the magnitudes of reduction are positively associated with the risk levels of asset classes. Specifically, the average weight reduction in commodities is 7% more than that in stocks each month. In other words, as an investor becomes more risk averse, she tends to scale down her position in commodities more than that in other risky assets, which, in turn, makes her S-B-C portfolio closer to the S-B portfolio. Such disproportionate reductions in the allocation to commodities could potentially explain the shrinking outperformance margins of S-B-C versus S-B strategies as an investor’s risk aversion increases.

[Table 5 goes here...]

6.1.3 Robustness of Results

To assess the robustness of the results presented in the previous section, we perform several additional tests.

First, we employ different estimation window sizes for all BWD and FWD moment estimators, namely 5yr-, 10yr-rolling and full sample expanding windows. We find that our conclusions remain valid across all window sizes.

Second, we use alternative forecasting methods for volatility and correlations. Specifically, instead of the GARCH-type volatility model, we employ autoregressive specifications to forecast

monthly realized volatility computed from daily returns. We also consider the asymmetric DCC framework proposed by Engle and Colacito (2006) to forecast conditional correlation matrices. We find that these sophisticated models do fit data better in-sample. However, they tend to slightly underperform their simpler versions in the out-of-sample context.

Third, as an alternative to the GSCI index, we repeat the analysis using the UBS-Dow Jones Commodity index. Fourth, instead of using the broad-based commodity index (GSCI), we repeat our analyses with sub-sector commodity indexes, including agriculture, energy, industrial metals and precious metals.

6.2 Results for Non-MV Case

In this section, we evaluate the MV investor's multi-asset investment strategies from Section 4.2.

6.2.1 Statistical Evaluation of Predictability

TO BE ADDED

6.2.2 Economic Evaluation of Portfolio Performance

TO BE ADDED

7 Concluding Remarks

In this paper, we examine the economic value of adding commodities into a traditional (stock-bond-cash) portfolio in an out-of-sample context. In contrast to existing literature, which conducts the analysis using backward looking estimators for asset returns moments when forming optimal portfolios, we model and forecast the dynamics of return moments in a forward looking framework. This allows us to exploit the predictability of mean, volatility, correlations, skewness and kurtosis

of returns that previous studies did not account for when assessing the role of commodities in multi-asset portfolios. We perform the analysis for both MV and non-MV investors.

For the MV case, we find that, by exploiting the predictability of asset return moments, the addition of commodities to a traditional asset portfolio generates significant out-of-sample economic value. We next examine the sources of the economic gains, and find that predicting excess returns and covariance matrix separately adds significant value to MV investors, but most of the added value comes from predicting the covariance matrix.

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Appendices

A Covariance Matrix Forecasting

We follow the two-stage DCC model proposed by Engle (2002) and the multi-period ahead forecasting method introduced in Engle and Sheppard (2001) to estimate and forecast the conditional covariance matrix over the multi-period $[t + 1 : t + n]$:

$$\hat{\Sigma}_{t+1:t+n|t} = \hat{D}_{t+1:t+n|t} \hat{P}_{t+1:t+n|t} \hat{D}_{t+1:t+n|t} \quad (\text{A.1})$$

where $\hat{\Sigma}_{t+1:t+n|t}$ is the covariance matrix forecasts, $\hat{D}_{t+1:t+n|t}$ is a diagonal matrix with the volatility forecasts, and $\hat{P}_{t+1:t+n|t}$ is the correlation forecasts.

Step 1: Forecast the conditional volatility diagonal matrix $\hat{D}_{t+1:t+n|t}$.

Estimate the IS GARCH(1,1) model using data upto time t :

$$r_t = u_t + \varepsilon_t \quad (\text{A.2})$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (\text{A.3})$$

where r_t is asset return at time t , u_t is the mean of return upto time t , and ε_t is the residuals from the mean equation (A.2).

Produce the one-period ahead variance forecasts $\hat{\sigma}_{t+1|t}^2$:

$$\hat{\sigma}_{t+1|t}^2 = \hat{\omega}_t + \hat{\alpha}_t \varepsilon_t^2 + \hat{\beta}_t \hat{\sigma}_t^2 \quad (\text{A.4})$$

where ω_t , α_t and β_t are coefficients estimated using data upto time t from the model specified in Eq.(A.2-A.3).

Aggregate one-period ahead forecasts into multi-period forecasts:

$$\hat{\sigma}_{t+1:t+n|t}^2 = n * \hat{\sigma}_{t+1|t}^2 \quad (\text{A.5})$$

where n is the number of trading days during the period $[t + 1 : t + n]$.

The OOS diagonal volatility matrix forecast $\hat{D}_{t+1:t+n|t}$ is given by:

$$\hat{D}_{t+1:t+n|t} = \begin{pmatrix} \hat{\sigma}_{1,t+1:t+n|t} & 0 & \cdots & 0 \\ 0 & \hat{\sigma}_{2,t+1:t+n|t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\sigma}_{K,t+1:t+n|t} \end{pmatrix}; \quad 1 \cdots K \text{ assets}$$

Step 2: Forecast the conditional correlation matrix $P_{t+1:t+n|t}$.

Remove the conditional mean from the K asset return series r_t and obtain their residuals $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}, \dots, \varepsilon_{K,t}]'$ and standardize the residuals using the conditional standard deviation D_t :

$$s_t = D_t^{-\frac{1}{2}} \varepsilon_t \quad (\text{A.6})$$

Fit s_t into the following multivariate model:

$$Q_t = (1 - \alpha - \beta)\Omega + \alpha s_{t-1} s_{t-1}' + \beta Q_{t-1} \quad (\text{A.7})$$

where Ω is the unconditional covariance matrix of s_t and Q_t is the conditional covariance matrix.

Produce the one-period-ahead forecasts:

$$\hat{Q}_{t+1|t} = \hat{\Omega}_t + \hat{\alpha}_t s_t s_t' + \hat{\beta}_t \hat{Q}_t \quad (\text{A.8})$$

where $\hat{\Omega}_t$, $\hat{\alpha}_t$ and $\hat{\beta}_t$ are estimates from Eq.(A.7).

By making the approximation that $E_t[s_{t+1}s'_{t+1}] \approx Q_{t+1}$, the n-period ahead forecasts of Q can be generated:

$$\hat{Q}_{t+n|t} = \sum_{i=0}^{n-2} (1 - \alpha - \beta)\Omega(\alpha + \beta)^i + (\alpha + \beta)^{n-1}\hat{Q}_{t+1|t} \quad (\text{A.9})$$

Aggregate single-period forecasts into multi-period forecasts:

$$\hat{Q}_{t+1:t+n|t} = \sum_{i=1}^n \hat{Q}_{t+i|t} \quad (\text{A.10})$$

Obtain the multi-period correlation matrix forecasts:

$$\hat{P}_{t+1:t+n|t} = \hat{Q}_{t+1:t+n|t}^{*-\frac{1}{2}} \hat{Q}_{t+1:t+n|t} \hat{Q}_{t+1:t+n|t}^{*-\frac{1}{2}} \quad (\text{A.11})$$

where $\hat{Q}_{t+1:t+n|t}^*$ denotes the $n \times n$ diagonal matrix composed of the diagonal elements of $\hat{Q}_{t+1:t+n|t}$.

Plugging $\hat{D}_{t+1:t+n|t}$ and $\hat{P}_{t+1:t+n|t}$ into Eq.(A.1), we can obtain the conditional covariance matrix forecast $\hat{\Sigma}_{t+1:t+n|t}$.

B Forecasting higher moments

In what follows we report the details of the procedure proposed by Jondeau and Rockinger (2012) in order to forecast the first four moments of individual asset and portfolio returns that are necessary to solve the portfolio optimization problem for a non-MV investor.

A portfolio's first four non-central moments in Eq.(20) can be written as:

$$\begin{aligned}
m_{p,t+1}^{(1)} &= \mu_{p,t+1} \\
m_{p,t+1}^{(2)} &= \sigma_{p,t+1}^2 + \mu_{p,t+1}^2 \\
m_{p,t+1}^{(3)} &= u_{p,t+1}^{(3)} + 3\mu_{p,t+1}\sigma_{p,t+1}^2 + \mu_{p,t+1}^3 \\
m_{p,t+1}^{(4)} &= u_{p,t+1}^{(4)} + 4u_{p,t+1}^{(3)}\mu_{p,t+1} + 6\sigma_{p,t+1}^2\mu_{p,t+1}^2 + \mu_{p,t+1}^4
\end{aligned} \tag{B.1}$$

where $\mu_{p,t+1}$, $\sigma_{p,t+1}^2$, $u_{p,t+1}^{(3)}$ and $u_{p,t+1}^{(4)}$ are the portfolio's expected return, variance, third and fourth central moments, respectively. For a given portfolio weights ω_t , the expected portfolio return, variance, third and fourth central moments are expressed as:

$$\begin{aligned}
\mu_{p,t+1} &= \omega_t' \mu_{t+1} \\
\sigma_{p,t+1}^2 &= \omega_t' \Sigma_{t+1} \omega_t \\
u_{p,t+1}^{(3)} &= \omega_t' S_{t+1} (\omega_t \otimes \omega_t) \\
u_{p,t+1}^{(4)} &= \omega_t' K_{t+1} (\omega_t \otimes \omega_t \otimes \omega_t)
\end{aligned} \tag{B.2}$$

where μ_{t+1} , Σ_{t+1} , S_{t+1} and K_{t+1} are expected asset returns, covariance, co-skewness and co-kurtosis matrices, respectively.

Provided the expected third and fourth central moments (i.e. $u_{i,t+1}^{(3)}$ and $u_{i,t+1}^{(4)}$) exist for each asset return series $r_{i,t}$, we can obtain the $n \times n^2$ conditional co-skewness matrix:

$$S_{t+1} = E_t[(r_{t+1} - \mu_{t+1})(r_{t+1} - \mu_{t+1})' \otimes (r_{t+1} - \mu_{t+1})'] = \{s_{ijk,t+1}\} \tag{B.3}$$

$$\text{with component } (i, j, k) : \quad s_{ijk,t+1} = \sum_{r=1}^N \sigma_{ir,t+1} \sigma_{jr,t+1} \sigma_{kr,t+1} u_{r,t+1}^{(3)}$$

where $\sigma_{ij,t+1}$ is the element of the ‘‘square root’’ of the covariance matrix, $\Sigma_{t+1}^{\frac{1}{2}}$, which can be

obtained using Eigen decomposition. The conditional co-kurtosis matrix:

$$K_{t+1} = E_t[(r_{t+1} - \mu_{t+1})(r_{t+1} - \mu_{t+1})' \otimes (r_{t+1} - \mu_{t+1})' \otimes (r_{t+1} - \mu_{t+1})'] = \{k_{ijkl,t+1}\} \quad (\text{B.4})$$

with component $(i, j, k, l) : k_{ijkl,t+1} = \sum_{r=1}^N \sigma_{ir,t+1} \sigma_{jr,t+1} \sigma_{kr,t+1} \sigma_{lr,t+1} u_{r,t+1}^{(4)} + \sum_{r=1}^N \sum_{s \neq r} \psi_{rs,t+1}$
where $\psi_{rs,t+1} = \sigma_{ir,t+1} \sigma_{jr,t+1} \sigma_{ks,t+1} \sigma_{ls,t+1} + \sigma_{ir,t+1} \sigma_{js,t+1} \sigma_{kr,t+1} \sigma_{ls,t+1} + \sigma_{is,t+1} \sigma_{jr,t+1} \sigma_{kr,t+1} \sigma_{ls,t+1}$.

Therefore, to derive the first four moments of the 4-Moment FWD strategy described Eq.(B.1) - Eq.(B.4), the first step is to obtain the FWD estimates for $sk_{i,t+1}$ and $ku_{i,t+1}$, namely the third and fourth central moments for each asset return. The following steps detail the estimation procedure for $u_{i,t+1}^{(3)}$ and $u_{i,t+1}^{(4)}$:

Step 1: Estimate the following model using information available upto time t and obtain the IS parameter estimates.

The n asset excess returns, $r_t = [r_{1,t}, \dots, r_{n,t}]$:

$$r_t = \mu_t + \varepsilon_t \quad (\text{B.5})$$

$$\varepsilon_t = \Sigma_t^{\frac{1}{2}} z_t \quad (\text{B.6})$$

$$z_t \sim g(z_t | \lambda_t, \eta_t) \quad (\text{B.7})$$

Eq.(B.5) decomposes the excess return vector, r_t , into two parts: the expected excess returns, μ_t , and the unexpected excess returns, ε_t . Eq.(B.6) describes the unexpected returns ε_t , where z_t denotes the independent innovation vector with zero mean and unit variance, and $\Sigma_t = D_t P_t D_t'$ denotes the conditional covariance matrix, where D_t is a diagonal matrix with standard deviations on the diagonal and P_t is the symmetric conditional correlation matrix. Eq.(B.7) specifies that the marginal distribution of innovations z_t follows Hansen's generalized Skew-t distribution $g(z_t | \lambda_t, \eta_t)$, where λ_t and η_t respectively capture the time-varying asymmetries and fat-tailedness.

Furthermore, we follow Jondeau and Rockinger (2012) and adopt the GJR-GARCH specification to model conditional variance for each asset returns, $\sigma_{i,t}^2$:

$$\sigma_{i,t}^2 = \omega_{i,t} + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \psi_i \varepsilon_{i,t-1}^2 \cdot \mathbf{1}_{(\varepsilon_{i,t-1} < 0)} \quad (\text{B.8})$$

Moreover, we employ the DCC specification of Engle(2002) to model the timing varying correlation matrix, P_t :

$$P_t = (\text{diag}(Q_t))^{-1/2} Q_t (\text{diag}(Q_t))^{-1/2} \quad (\text{B.9})$$

$$Q_t = (1 - a - b)\bar{Q} + a(s_{t-1}s'_{t-1}) + bQ_{t-1} \quad (\text{B.10})$$

where a and b are scalars, and $s_t = D_t^{-1}\varepsilon_t$ is standardized residuals.

The dynamics of conditional variance and correlations are driven by the innovation vector, z_t , which is drawn from the multivariate Skew-t distribution with time-varying individual asymmetry parameter and the degree of freedom, $\lambda_{i,t}$ and $\eta_{i,t}$ respectively. The dynamic process of the two shape parameters are modeled as:

$$\lambda_{i,t} = -1 + \frac{2}{1 + e^{-\tilde{\lambda}_{i,t}}} \quad (\text{B.11})$$

$$\tilde{\lambda}_{i,t} = d_0 + d_1^- z_{i,t-1} \cdot \mathbf{1}_{(z_{i,t-1} \leq 0)} + d_1^+ z_{i,t-1} \cdot \mathbf{1}_{(z_{i,t-1} > 0)} + d_2 \tilde{\lambda}_{i,t-1} \quad (\text{B.12})$$

$$\eta_{i,t} = -4 + \frac{26}{1 + e^{-\tilde{\eta}_{i,t}}} \quad (\text{B.13})$$

$$\tilde{\eta}_{i,t} = c_0 + c_1^- |z_{i,t-1}| \cdot \mathbf{1}_{(z_{i,t-1} \leq 0)} + c_1^+ |z_{i,t-1}| \cdot \mathbf{1}_{(z_{i,t-1} > 0)} + c_2 \tilde{\eta}_{i,t-1} \quad (\text{B.14})$$

Eq.(B.12) and Eq.(B.14) respectively specify the temporal dynamic process of the asymmetry parameter $\lambda_{i,t}$ and the degree of freedom $\eta_{i,t}$. Eq.(B.11) and Eq.(B.13) respectively denote the logistic mappings from estimated parameters to true Skew-t shape parameters that satisfy the theoretical restrictions pointed out by Hansen (1994).³⁶ The model Eq.(B.5) - Eq.(B.14) can be estimated using the Maximum Likelihood method. The parameter estimates obtained from the estimation based on asset return data available upto time t are:

$$\{\hat{\omega}_{i,t}, \hat{\alpha}_{i,t}, \hat{\beta}_{i,t}, \hat{a}_t, \hat{b}_t, \hat{c}_{0,t}, \hat{c}_{1,t}^-, \hat{c}_{1,t}^+, \hat{c}_{2,t}; \hat{d}_{0,t}, \hat{d}_{1,t}^-, \hat{d}_{1,t}^+, \hat{d}_{2,t}\} \quad (\text{B.15})$$

Step 2: Obtain one-period ahead OOS parameter forecasts using the estimates in Eq.(B.15).

$$\hat{\lambda}_{t+1|t} = \hat{d}_{0,t} + \hat{d}_{1,t}^- z_t \cdot \mathbf{1}_{(z_t \leq 0)} + \hat{d}_{1,t}^+ z_t \cdot \mathbf{1}_{(z_t > 0)} + \hat{d}_{2,t} \hat{\lambda}_t \quad (\text{B.16})$$

³⁶To ensure the first four moments exist, the two restrictions need to be maintained: $-1 < \lambda_{i,t} < 1$ and $\eta_{i,t} > 4$.

$$\hat{\lambda}_{t+1|t} = -1 + \frac{2}{1 + e^{-\hat{\lambda}_{t+1|t}}} \quad (\text{B.17})$$

$$\hat{\eta}_{t+1|t} = \hat{c}_{0,t} + \hat{c}_{1,t}^- |z_t| \cdot \mathbf{1}_{(z_t \leq 0)} + \hat{c}_1^+ |z_t| \cdot \mathbf{1}_{(z_t > 0)} + \hat{c}_{2,t} \hat{\eta}_t \quad (\text{B.18})$$

$$\hat{\eta}_{t+1|t} = -4 + \frac{26}{1 + e^{-\hat{\eta}_{t+1|t}}} \quad (\text{B.19})$$

where $\mathbf{1}_{(z_t \leq 0)}$ is an indicator function, and $\hat{\eta}_{t+1|t}$ and $\hat{\lambda}_{t+1|t}$ are the next period shape parameter OOS forecasts.

Step 3: Forecast one period ahead central moments for each asset:

$$\hat{\mu}_{t+1|t}^{(3)} = \hat{M}_{3,i,t+1|t} - 3\hat{M}_{1,t+1|t}\hat{M}_{2,t} + 2\hat{M}_{1,t+1|t}^3 \quad (\text{B.20})$$

$$\hat{\mu}_{t+1|t}^{(4)} = \hat{M}_{3,t+1|t} - 4\hat{M}_{1,t+1|t}\hat{M}_{2,t+1|t} + 6\hat{M}_{1,t+1|t}^2\hat{M}_{2,t+1|t} - 3\hat{M}_{1,t+1|t}^4 \quad (\text{B.21})$$

where

$$\hat{M}_{r,t+1|t} = \frac{\Gamma(\frac{\hat{\eta}_{t+1|t}-r}{2})\Gamma(\frac{r+1}{2})(\hat{\eta}_{t+1|t}-2)^{\frac{r+1}{2}}}{\sqrt{\pi(\hat{\eta}_{t+1|t}-2)}\Gamma(\frac{\hat{\eta}_{t+1|t}}{2})} \cdot \frac{\hat{\lambda}_{t+1|t}^{r+1} + \frac{(-1)^r}{\hat{\lambda}_{t+1|t}^{r+1}}}{\hat{\lambda}_{t+1|t} + \frac{1}{\hat{\eta}_{t+1|t}}}$$

is the r^{th} raw moment of z_t . Clearly, the third and fourth central moments of z_t are non-linear functions of the shape parameter estimates $\hat{\eta}_{i,t+1|t}$ and $\hat{\lambda}_{i,t+1|t}$.

Table 1**Descriptive Statistics**

Panel A presents the summary statistics of monthly excess returns of three major asset class indexes: S&P500, Barclays Aggregate Bond Index and S&P GSCI. Panel B reports the unconditional correlation matrix. The datasets span the period 01/1989 - 12/2012.

Panel A: Summary Statistics

	N	Mean	Stdev	Median	Min	Max	Skew	X-Kurt
SP500	288	.0456	.1490	.1004	-.89	2.42	-.56	1.08
Bond	288	.0362	.0373	.0434	-.34	.54	-.28	.57
GSCI	288	.0406	.2119	.0523	-.98	10.12	-.17	2.00

Panel B: Unconditional correlation matrix

SP500	1		
Bond	0.13	1	
GSCI	0.17	-0.03	1

Table 2**Statistical Evaluation of Predictability**

The table reports the forecasting performance of monthly excess returns, volatility and correlation. Panel A represents the out-of-sample R² statistics (in percentage), and the columns represent the sample window sizes of benchmark backward-looking estimators. Panel B shows the results from running Mincer-Zarnowitz regressions. The sample period spans 01/1995 - 12/2012.

Panel A: OOS R ² for Excess Return Forecasting								
Bwd window	5Yr	10Yr	HM					
SP500	3.0	0.8	-0.2					
Bond	6.7	4.1	5.1					
GSCI	2.9	2.4	1.2					

Panel B: Mincer-Zarnowitz Regression Evaluation for Volatility and Correlation Forecasting									
	Estimator	Volatility				Correlation			
		β_0	β_1	R ² (%)		Estimator	β_0	β_1	R ² (%)
SP500	BWD-5Yr	0.05 [5.59]	-0.02 [-0.11]	0.01	SP500 & Bond	BWD-5Yr	-0.15 [-5.06]	0.54 [5.87]	13.88
	BWD-10Yr	0.12 [4.43]	-1.62 [-2.58]	3.01		BWD-10Yr	-0.19 [-6.52]	0.89 [7.47]	20.68
	FWD	0.00 [1.67]	0.89 [17.76]	59.58		FWD	-0.05 [-2.62]	1.00 [16.06]	54.65
Bond	BWD-5Yr	0.02 [5.47]	-0.54 [-1.91]	1.68	SP500 & GSCI	BWD-5Yr	0.04 [1.71]	0.76 [9.6]	30.09
	BWD-10Yr	0.01 [2.91]	0.28 [1.18]	0.65		BWD-10Yr	0.09 [4.88]	1.19 [9.95]	31.63
	FWD	0.00 [-0.09]	0.97 [11.43]	37.91		FWD	0.05 [2.62]	0.99 [12.43]	41.92
GSCI	BWD-5Yr	0.02 [3.83]	0.61 [5.96]	14.24	Bond & GSCI	BWD-5Yr	-0.11 [-5.19]	0.50 [1.82]	1.52
	BWD-10Yr	0.01 [1.42]	0.86 [5.93]	14.12		BWD-10Yr	-0.09 [-4.9]	-0.08 [-0.36]	0.06
	FWD	0.01 [1.33]	0.92 [14.35]	49.06		FWD	-0.03 [-1.41]	0.70 [5.36]	11.83

Table 3**Measures of Strategy Performance: Stock-Bond vs. Stock-Bond-Commodity**

The table reports the strategy performance for the Stock-Bond (S-B) versus Stock-Bond-Commodity (S-B-C) and Backward- versus Forward-looking estimators. The reported metrics include: the annualized Sharpe ratio (SR), the annualized Sortino's UP ratio (UP), the annualized Certainty Equivalent Return (CEQ) and the Turnover (TO). The results for various levels of risk aversions, portfolio constraints and sub-periods are also reported. Full period: 01/1995 - 12/2012. Transaction cost: 50bps.

Constraint:	0 < w				0 < w < 1				0 < w < 0.333			
Metrics:	SR	UP	CEQ	TO(%)	SR	UP	CEQ	TO(%)	SR	UP	CEQ	TO(%)
Panel A: BWD												
Gamma = 3												
S-B	.97	4.45	.1411	1	.39	2.53	.0220	3	.09	2.43	.0029	4
S-B-C	.92	4.82	.1384	2	.36	3.44	.0204	4	.01	2.57	-.0017	4
Gamma = 6												
S-B	.92	4.46	.0653	1	.66	2.78	.0261	2	.14	2.56	.0047	3
S-B-C	.87	4.43	.0623	2	.53	2.85	.0236	3	.05	2.56	.0011	4
Gamma = 10												
S-B	.90	3.79	.0378	1	.82	3.95	.0259	2	.22	2.53	.0075	2
S-B-C	.84	3.76	.0355	2	.70	3.62	.0239	2	.13	2.52	.0059	3
Panel B: FWD												
Gamma = 3												
S-B	1.48	6.94	.3155	8	.44	3.90	.0287	15	.21	3.38	.0080	10
S-B-C	1.52	9.82	.3616	16	.50	4.60	.0414	17	.31	4.03	.0224	14
Gamma = 6												
S-B	1.33	5.79	.1346	8	.65	3.90	.0291	14	.22	3.51	.0077	11
S-B-C	1.33	7.78	.1447	16	.66	4.74	.0362	16	.29	4.17	.0188	17
Gamma = 10												
S-B	1.26	5.12	.0742	8	.84	4.56	.0277	11	.27	3.61	.0084	13
S-B-C	1.24	6.62	.0760	16	.81	5.08	.0318	13	.26	3.95	.0138	20

Table 4**Measures of Strategy Performance: BWD vs. FWD**

The table reports the strategy performance for the Stock-Bond (S-B) versus Stock-Bond-Commodity (S-B-C) and Backward- versus Forward-looking estimators. The reported metrics include: the annualized Sharpe ratio (SR), the annualized Sortino's UP ratio (UP), the annualized Certainty Equivalent Return (CEQ) and the Turnover (TO). The results for various levels of risk aversions, portfolio constraints and sub-periods are also reported. Full period: 01/1995 - 12/2012. Transaction cost: 50bps. Risk aversion coefficient: 6.

Constraint:	0 < w				0 < w < 1				0 < w < 0.333			
Metrics:	SR	UP	CEQ	TO	SR	UP	CEQ	TO	SR	UP	CEQ	TO
S-B												
Bwd & Bwd	.92	4.46	.0653	1	.66	2.78	.0261	2	.14	2.56	.0047	3
Fwd & Bwd	.96	4.96	.0751	9	.53	2.48	.0205	11	.19	3.04	.0037	13
Bwd & Fwd	1.12	5.71	.0950	6	.65	3.60	.0241	14	.22	3.48	.0050	12
Fwd & Fwd	1.33	5.79	.1346	8	.55	3.90	.0291	15	.22	3.51	.0077	11
S-B-C												
Bwd & Bwd	.87	4.43	.0623	2	.53	2.85	.0236	3	.05	2.56	.0011	4
Fwd & Bwd	.94	4.94	.0690	16	.44	2.46	.0146	16	.28	3.79	.0071	17
Bwd & Fwd	1.16	7.26	.1078	9	.66	4.07	.0294	16	.30	3.59	.0085	14
Fwd & Fwd	1.33	7.78	.1447	16	.60	4.74	.0362	20	.29	4.17	.0188	17

Table 5**Relative Risk Aversions and Average Weights of S-B-C Strategic Portfolios**

This table shows the average weights of asset classes in each strategic S-B-C portfolio and the reductions in weights from increasing risk aversion. Specifically, the first panel reports the average weights for the investors with relatively low risk aversion (Gamma = 3); the second panel reports the average weights for the investors with relatively high risk aversion (Gamma = 10); the third panel reports the average weight changes in Gamma. The negative values mean weight reduction and the positive values mean weight increases. Constraints for weights: $0 < w < 1$.

Strategy	Stock(%)	Bond(%)	Commodity(%)	Cash(%)
GAMMA=3				
Bwd & Bwd	43	100	55	-98
Fwd & Bwd	40	98	56	-94
Bwd & Fwd	56	100	61	-117
Fwd & Fwd	54	99	63	-116
GAMMA=10				
Bwd & Bwd	13	100	18	-30
Fwd & Bwd	12	92	19	-24
Bwd & Fwd	21	99	21	-42
Fwd & Fwd	22	95	24	-40
Changes in weights				
Bwd & Bwd	-30	0	-37	67
Fwd & Bwd	-28	-7	-36	70
Bwd & Fwd	-35	-1	-40	75
Fwd & Fwd	-32	-5	-39	76
Avg. Changes	-31	-3	-38	72

Figure 1: Dynamic Correlations between S&P500, Bond and GSCI indexes. Sample period: 1993:01 – 2012:12.

