

Physics Skills in Experimentation

Stage 6 Physics – Working Scientifically

Activity book

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Introduction

This activity book is intended for students of the Stage 6 Physics course in New South Wales. It draws upon resources published by the NSW Department of Education in their *Working Scientifically* support documents as well as those from the First Year Physics Unit at the University of New South Wales, Sydney.

In this activity book, definitions and procedures used in physics research and tertiary study are streamlined and adapted for students of physics at high school level. Some ideas and techniques have been simplified so that secondary school students can access skills for experimental work.

This activity book is in no way intended to be prescriptive; teachers should consider the learning requirements and outcomes for their students when using this activity book with their students.

– M de la Pena, 2020

References to the NSW Physics Stage 6 Syllabus

PH11/12-4 Processing data and information

A student selects and processes appropriate qualitative and quantitative data and information using a range of appropriate media.

PH11/12-5 Analysing data and information

A student analyses and evaluates primary and secondary data and information

(NSW Education Standards Authority, 2017)

1 – Units for measurement

Units add important information to measurements because a numerical value means nothing on its own.

A unit has to be an agreed quantity of a thing to be measured, because when we say or write down a measurement, we are actually giving a number of multiples of that unit. For example, when I tell you that the length of something is 3 metres, I am telling you that its length is 3×1 metre, and this will only be accurate if you agree with me about what a single metre is.

The agreed system of units used in science is the *International System of units* (SI units). The base units are:

Name	Symbol
Second	s
Metre	m
Kilogram	kg
Ampere	A
Kelvin	K
Mole	mol
Candela	cd

All other units are based on these seven base units.

Question 1

What base SI units are used for the following physical quantities?

Mass	
Amount of substance	
Electric current	
Luminous intensity	

Time	
Thermodynamic temperature	
Length	

Question 2

There are many other units used in physics, but they are actually derived from the base units.

For example, the units of speed are derived from the units for length and time (metres per second):

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1 \text{ m}}{1 \text{ s}} = 1 \text{ m/s} = 1 \text{ m} \cdot \text{s}^{-1}$$

What are the derived units for the following physical quantities? Relevant equations have been included to help you.

Quantity	Equation	Derived unit
Acceleration	$a = \frac{\Delta v}{t}$	
Force	$F = ma$	
Area	$A = l \times w$	
Volume	$V = l \times w \times h$	
Work	$W = Fd$	

Question 3

Some quantities are so special that their derived unit is given a name. For example, you might know the named unit for force – the newton (N). The unit for force is actually derived from the SI base units. Since force is determined by the equation

$$F = ma$$

the unit for force derived from base units is

$$1 \text{ newton} = 1 \text{ kg} \times 1 \text{ m} \cdot \text{s}^{-2} = 1 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}$$

Find out the named units for the quantities below. You can use the internet or references to find out.

Quantity	Derived unit in terms of base units	Named unit and symbol
Energy	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$	
Electric charge	$\text{A} \cdot \text{s}$	
Frequency	s^{-1}	
Pressure	$\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$	
Voltage	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \cdot \text{A}^{-1}$	

Question 4

Some quantities are so big or small in scale that the units used to measure them are too big or too small to make sense of their size. Sometimes, a prefix is added to the unit make the written or spoken value more manageable and easier to conceptualise.

For example, *3 thousand metres* is more conveniently written and spoken as *3 kilometres*.

Below is a table of SI prefixes and their multipliers.

Prefix name	Prefix symbol	Multiplier
giga-	G	10^9
mega-	M	10^6
kilo-	k	10^3
hect-	h	10^2
deca-	da	10
–	–	1
deci-	d	10^{-1}
centi-	c	10^{-2}
milli-	m	10^{-3}
micro-	μ	10^{-6}
nano-	n	10^{-9}

(a) Rewrite the following values in more convenient units

58 300 000 N	
0.00748 A	
101 300 Pa	

(b) Rewrite the following values in terms of their base-sized unit.

576 nm	
450 kN	
916 μ A	

Question 5

The units for mass have an interesting history. The fundamental unit for mass in the nineteenth-century metric system was the *gram* – a thousand grams being a *kilogram*. However, when SI units were established in 1960, the *kilogram* was chosen as the base unit because it was on the same kind of size scale as the other units, like the metre. Other units, like the *tonne*, also exist in metric systems of units.

Write the names of the convenient mass units based around the gram and kilogram.

0.000 001 kg	
0.001 kg	

1000 kg	
1 000 000 kg	

2 – Significant figures

Significant figures are important because they convey the accuracy and uncertainty in a value. The last significant figure in a number suggests that the value is accurate to within $\pm\frac{1}{2}$ of that place. For example:

- 120 m (2 significant figures) implies a length with uncertainty of 120 ± 5 m
- 123 m (3 significant figures) implies a length with uncertainty of 123.0 ± 0.5 m

Rules for significant figures – what counts as a significant figure?

- **Non-zero digits are significant.**
- **Trailing zeroes in a whole number are generally not significant** (these zeroes are used to keep the other figures in their correct value places).
 - 75000 m – the 7 and 5 are significant. There are 2 significant figures.
 - 75420 m – the 7, 5, 4 and 2 are significant. There are 4 significant figures. There is less uncertainty in this number.
- **Sometimes trailing zeroes are significant. They may be marked with an overbar or underlined.**
 - $532\bar{0}0$ – 5, 3, 2 and one of the zeroes are significant. There are 4 significant figures.
- **Leading zeroes are not significant** (these zeroes are used to keep the other figures in their correct value places).
 - 0.000832 kg – only 8, 3, and 2 are significant. There are 3 significant figures.
- **The zeroes between non-zero digits are significant.**
 - 90.04 s – each figure is significant. There are 4 significant figures.
- **The trailing zeroes in a decimal are significant.**
 - 8.30 L – each figure is significant. There are 3 significant figures.
 - 3.200 J – each figure is significant. There are 4 significant figures.

The result of a calculation is only as accurate as the least accurate number used to compute it. When reporting the result of a calculation, the result must be rounded to the same number of figures as the smallest number of significant figures used in the calculation. For example

$$\text{Energy} = 3.457 \text{ W} \times 5.60 \text{ s} = 19.3292 \text{ J}$$

the answer can only be reported as 19.3 J because the smallest number of significant figures in the calculation was three.

Question 1

How many significant figures are given in the numbers below?

5 120 m	0.0024 kg	712 6 <u>0</u> 0 N
8.21×10^3 s	13.50 A	351.205 V

Question 2

Rewrite the following numbers to the required significant figures.

81 120 m (to 3 sig. fig.)		2.1456×10^6 kg (to 4 sig. fig.)	
617 960 K (to 4 sig. fig.)		0.0057143 C (to 3 sig. fig.)	
7.992 s (to 2 sig. fig.)		158 W (to 2 sig. fig.)	

Question 3

What is the uncertainty in the following numbers?

126 m	uncertainty = \pm	93.2 A	uncertainty = \pm
46.73 kg	uncertainty = \pm	0.850 N	uncertainty = \pm

Question 4

Perform these calculations and report the answer to the appropriate number of significant figures.

(a) Acceleration = $54.3 \text{ N} \div 32 \text{ kg}$

(b) Distance = $16.67 \text{ m.s}^{-1} \times 8.23 \text{ s}$

Question 5

In calculations that involve *only* addition or subtraction, results should be rounded to the smallest number of *decimal places*.

For example, $\Delta T = 299.3 \text{ K} - 276.731 \text{ K} = 22.569 \text{ K} = 22.6 \text{ K}$

Perform these calculations and report the answers with the appropriate number of digits.

(a) $\Delta m = 45.212 \text{ kg} - 22.37 \text{ kg}$

(b) $L_{\text{tot}} = 532.8 \text{ m} + 367.178 \text{ m}$

3 – Scientific notation & orders of magnitude

Scientific notation is a neat way of writing very large or very small numbers. This is where numbers are written as a product of powers of ten, also called *orders of magnitude*. For example:

- 4 250 000 m can be written as 4.25×10^6 m
- 0.0000327 m can be written as 3.27×10^{-5} m

These numbers are 11 orders of magnitudes apart, since there are 11 powers of ten between 10^{-5} and 10^6 .

Orders of magnitude are determined by the nearest power of ten. For example

- 2.4×10^6 m has 6 orders of magnitude, but
- 8.6×10^6 m has 7 orders of magnitude (because it rounds up to $10 \times 10^6 = 10^7$)

Question 1

Write these numbers in their natural form.

1.5×10^4 N	
9.15×10^{-6} m	

3.145×10^{-3} J	
7.2×10^9 V	

Question 2

Write these numbers in scientific notation, to three significant figures.

9215 m s ⁻¹	
0.1274 A	

0.00032498 C	
7 231 485 m ²	

Question 3

What orders of magnitude are these numbers in?

8 425 000 s	
7.45×10^{-4} cd	

0.00256 Pa	
9.45×10^6 kg	

Question 4

How many orders of magnitude are between these numbers?

153 J and 2 J	
261 N and 5.4×10^4 N	

3.2×10^{-3} V and 0.71 V	
0.082 m and 0.00045m	

4A – Linear regression

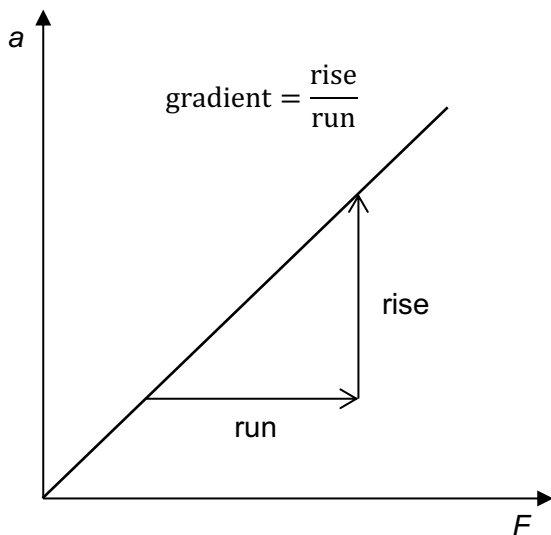
Linear regression is a technique used to analyse data by fitting them to a straight line. By examining the slope of a straight line graph, physical relationships and values can be extracted.

Linear graphs are used because the way they behave is well known and easy to analyse. Recall that the equation of a straight line is

$$Y = AX + B$$

where A is the gradient of the line and B is the Y -intercept. Y is the dependent variable and X is the independent variable, and Y and X are directly proportional to each other.

As an example, below is a graph of acceleration versus force applied to an object.



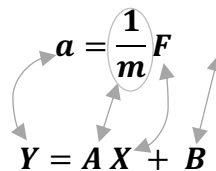
Why use linear regression?

Linear regression is preferable to simply substituting a data pair into an equation and solving for unknowns because:

- a gradient measures the relative changes in each variable, and not an absolute value – this reduces the effects of systematic errors.
- a gradient is like an average ratio between the dependent and independent variables – this reduces the effects of random errors.

We know of a relationship between force and acceleration, given by Newton's second law: $a = \frac{F}{m}$. We could rewrite this equation to get it into the form of a straight-line equation, like this:

$$a = \frac{1}{m} F$$

$$Y = AX + B$$


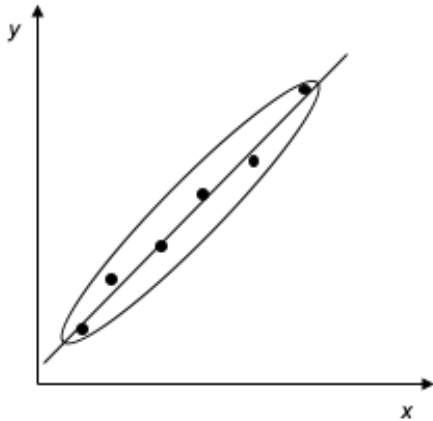
We can see the variables and values that correspond with those in the general equation for a straight line. When acceleration is the Y -variable and force is the X -variable, then the gradient is equal to $\frac{1}{\text{mass}}$ and there is not expected to be a Y -intercept.

If you can calculate the gradient, then you can determine or verify the mass of the object.

$$\text{gradient} = \frac{1}{\text{mass}} \Rightarrow \text{mass} = \frac{1}{\text{gradient}}$$

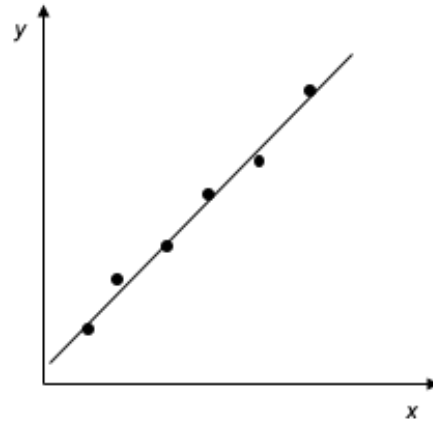
Methods for drawing a line of best fit by hand

Data is always affected by experimental error. That is why, even though theory would expect a straight-line graph, the data rarely lies on one. The aim of drawing a line of best fit is to estimate what the line would be, if it were not for the errors. You can do this by hand, but there are computer applications that can do this quite effectively (such as spreadsheets).



Using an ellipse

Draw an ellipse around all the data points, and then draw a line that bisects this ellipse, lengthways.



Just eyeballing it

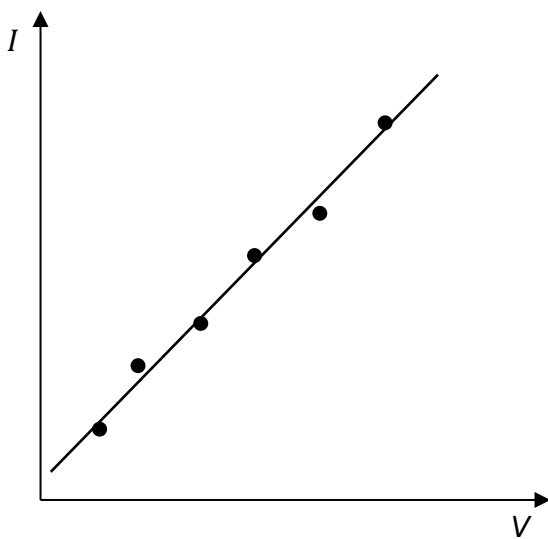
Line up your ruler along the plotted data points and wiggle it around until you have a line that follows the trend down the middle. The data points should be roughly evenly distributed either side of the line.

Importantly, do not force your line so that it goes through the origin. A non-zero Y-intercept, even when you don't expect one, can sometimes be an important piece of information – it can be an indicator of systematic error, for example.

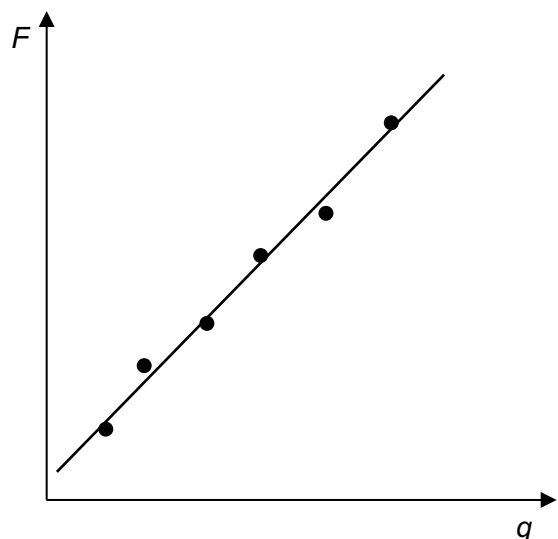
Question 1

Rearrange the equations into the linear form shown by these graphs.

Current vs voltage across a resistor



Force experience by charges in an electric field.



$$V = IR \rightarrow$$

$$E = \frac{F}{q} \rightarrow$$

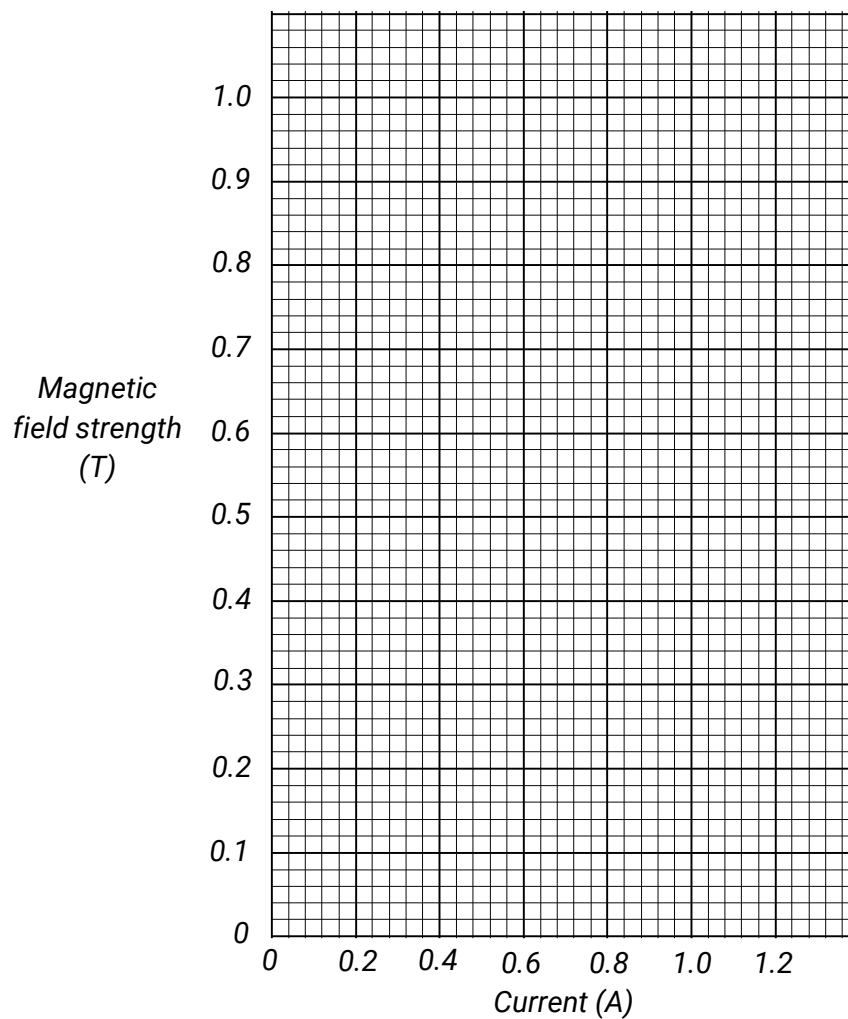
Question 2

(a) Plot the following data on the grid and draw a line of best fit.

Coil current and magnetic field strength inside a solenoid

<i>Independent variable</i>	<i>Dependent variable</i>
Current, I (A)	Magnetic Field, B (T)
0.2	0.22
0.4	0.29
0.6	0.55
0.8	0.58
1.0	0.71
1.2	1.04

Magnetic field strength vs coil current inside a solenoid



(b) The equation that relates the magnetic field strength and coil current in this solenoid is

$$B = \mu I$$

where μ is a *magnetic permeability constant* – a measure of the resistance of a material (such as one placed inside the solenoid) against the formation of a magnetic field.

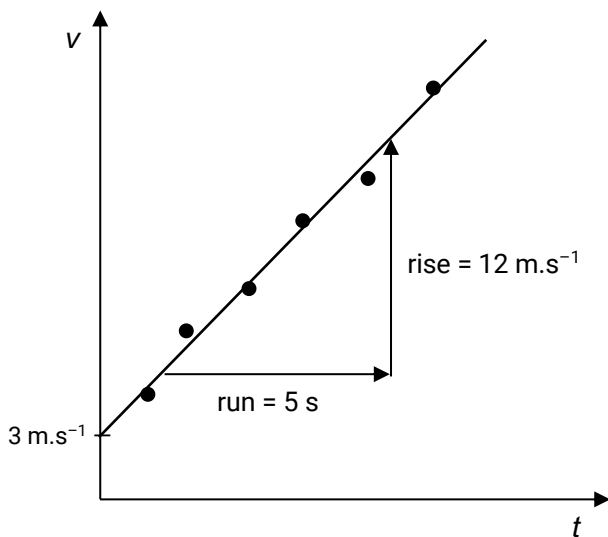
Given that the equation of a straight line is $Y = AX + B$, identify the term in the equation that represents the:

Y-variable	
X-variable	
Gradient	
Y-intercept	

(c) Calculate the gradient of the line of best fit.

Question 3

Velocity vs time graph for a car



(a) The equation of motion relevant to the graph on the left is $v = u + at$. Rearrange this equation so that it matches the straight-line equation.

(b) What does the gradient of this graph represent? Calculate this value.

(c) What does the Y-intercept of the graph represent? Write down this value.

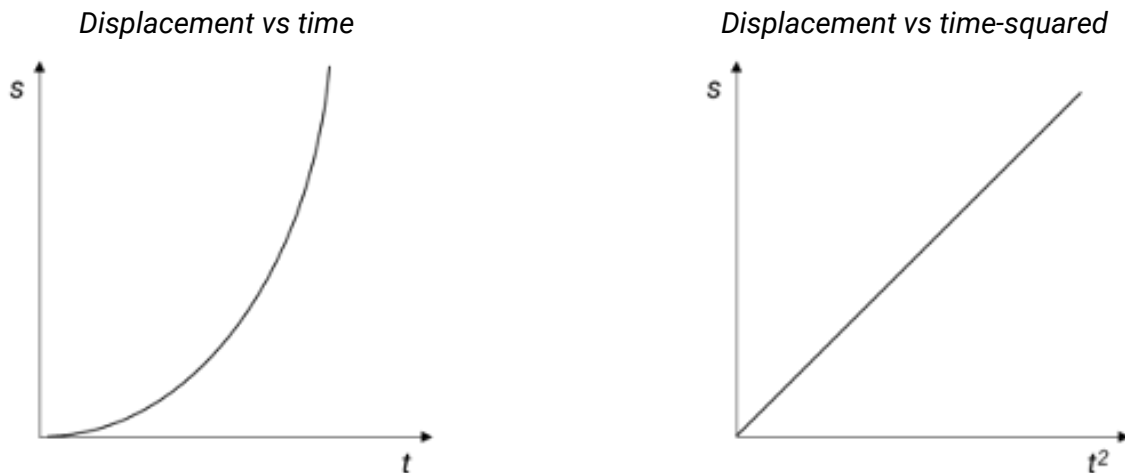
4B – Advanced linear regression

Many physical relationships are not linear, but we can still use the techniques of linear regression to establish or verify those relationships – we can *linearise* non-linear relationships.

By way of example, let's look at the displacement of an object as it accelerates uniformly with time:

$$s = \frac{1}{2}at^2$$

A graph of displacement against the raw time data (below, left) is not linear – in fact, it is parabolic. Displacement is not proportional to time.



If we reinterpret the original equation, realise that we can say *displacement is proportional to the square of time* ($s \propto t^2$). We can reframe the equation like this:

$$s = \frac{1}{2}a t^2$$

$Y = AX + B$

The diagram shows arrows mapping the terms of the equation to the linear form. An arrow points from s to Y . An arrow points from $\frac{1}{2}a$ to A . An arrow points from t^2 to X . An arrow points from the constant term (implied 0) to B .

This tells us to plot displacement s on the Y-axis and time-squared t^2 on the X-axis – we must *square* each value of time before we plot it. This graph (above, right) *will* give us a straight line.

The gradient of the *displacement versus time-squared graph* is equivalent to $\frac{1}{2}a$, or half the acceleration.

Thus:

$$\text{gradient} = \frac{1}{2}a \rightarrow \text{acceleration} = 2 \times \text{gradient}$$

Question 1

The table below shows the kinetic energy of an object at different speeds.

Kinetic energy of an object at different speeds

Independent variable	Dependent variable	Processed data
Speed, v (m.s ⁻¹)	Kinetic energy, K (J)	
0.45	2.0	
0.63	4.0	
0.77	6.0	
0.89	8.0	
0.99	10.0	

(a) Given that the equation that relates kinetic energy and speed is

$$K = \frac{1}{2}mv^2$$

rearrange this equation to make it fit $Y = AX + B$.

(b) Using your equation in (a), determine the variables that you should graph so that you can get a straight line.

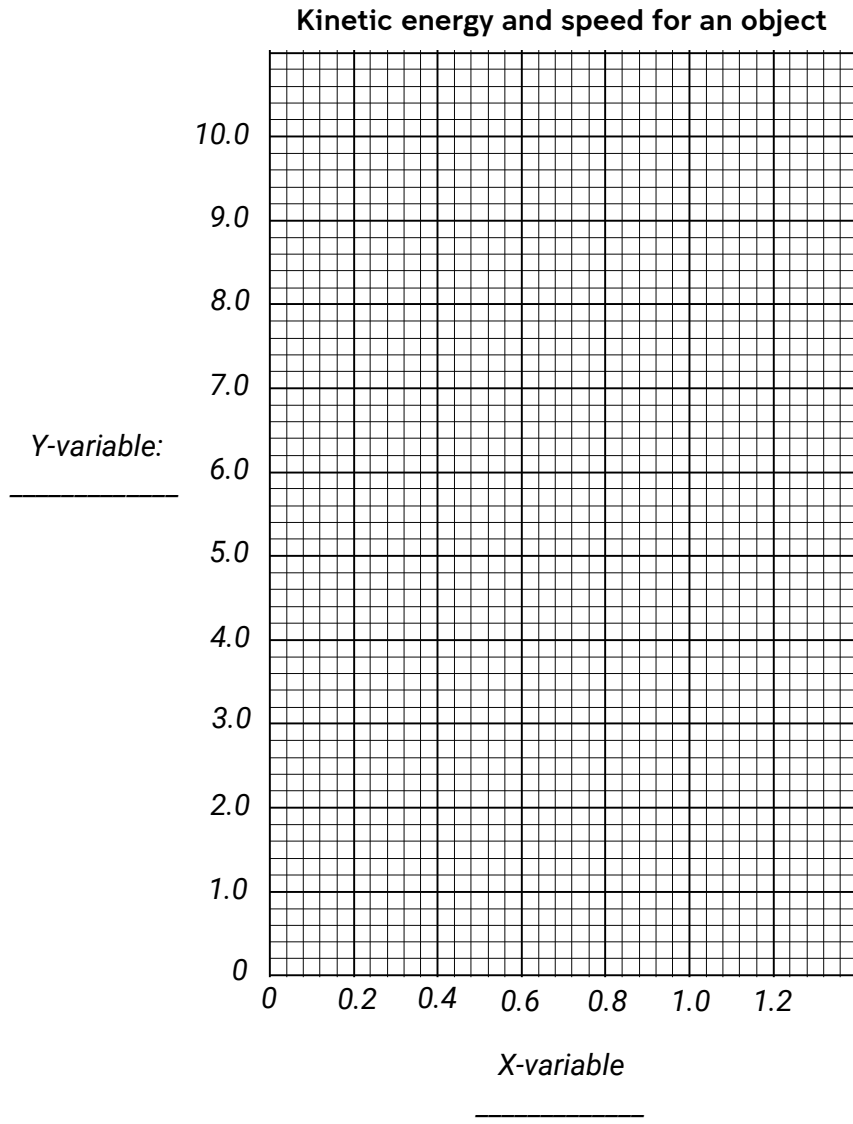
Y-variable	
X-variable	

(c) If you need to process the data for one of the variables so that you can obtain a straight line, do this. Write the new variable name and values in the *Processed data* column of the table.

(d) Identify the quantity that you can determine from the gradient of this graph. Write an equation involving the gradient that you can use to calculate this quantity.

(e) Do you expect there to be a Y-intercept? Why or why not?

(f) Plot the data from the previous page so that you get a straight-line graph.



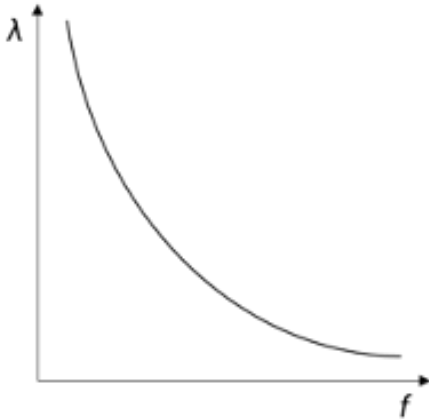
(g) Use the graph to calculate the mass of the object used in the experiment.

Question 2

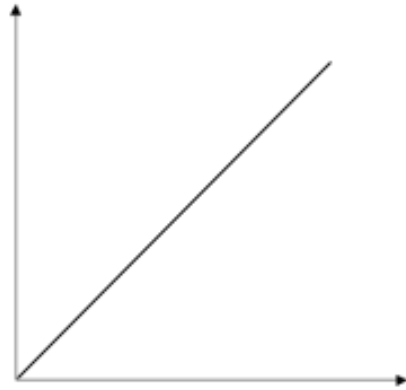
Waves on the water with a constant speed v were observed – their frequencies and their wavelengths were measured. The equation that relates wavelength and frequency is $v = f\lambda$.

The graph below, left, shows the relationship between frequency and wavelength.

Wavelength vs frequency for water waves



Title: _____



- (a) Rearrange the formula so that it is in the form $Y = AX + B$. (Hint: The Y-variable is a term that involves λ , and the X-variable is a term that involves f).

- (b) Using your equation in (a), determine the variables that you would graph to obtain a straight line. You can use your answers here to label the graph axes in the graph above, right.

Y-variable	
X-variable	

- (c) What quantity does the gradient of the straight line represent?

Question 3

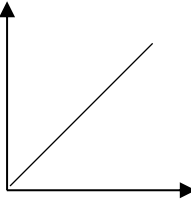
For each experiment that has been described, think about how you would graph the variables that have been mentioned.

Rewrite the equations in linear form, determine and label the graph axes, and write down the terms corresponding to the line's gradient.

- (a) You measure the intensity of light I at several distances from a lamp, d . We know that the intensity of light decreases with distance by the inverse square law:

$$I = \frac{I_0}{d^2}$$

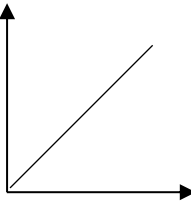
where I_0 is the reference intensity of light (taken at $d = 1$ m).

Equation in linear form $Y = AX + B$	Graph axes	Graph	Gradient
	Y-axis: X-axis:		

- (b) You point a laser at a glass block, and you measure the incident angle θ_1 and refracted angle θ_2 . We know the angles are related by Snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

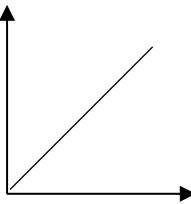
where n_1 and n_2 are the refractive indices of the air and the glass respectively.

Equation in linear form $Y = AX + B$	Graph axes	Graph	Gradient
	Y-axis: X-axis:		

- (c) As time t progresses, radioactive material decays and the amount left N decreases. This is given by the equation

$$\frac{N}{N_0} = e^{-\lambda t}$$

where N_0 is the original amount of material and λ is a decay constant. (Hint: keep $\frac{N}{N_0}$ together)

Equation in linear form $Y = AX + B$	Graph axes	Graph	Gradient
	Y-axis: X-axis:		



5 – Accuracy and error

Accuracy is the closeness of an observed value to its “true” value (a true value could be some theoretical value, or a value accepted by physicists and tabulated in secondary sources).

On some level, every measurement is limited in its accuracy. One reason is that there are limitations in the instruments we use for measuring – they may lack sensitivity, or the graduations on them (their *resolution*) might not be fine enough.



Environmental factors can also interfere with making measurements, and we as humans have our limitations in making and reading measurements, too.

The difference between an observed value and its true value is called *error* (for us, it does not mean “mistake”, as is its common meaning).

There are three ways that we can quantify error and accuracy:

Absolute error	$ \text{true value} - \text{observed value} $
Relative error (also called percentage error when expressed as %)	$\frac{ \text{true value} - \text{observed value} }{\text{true value}}$
Accuracy	100% – percentage error

High accuracy measurements have small errors, and low accuracy measurements have large errors. You could set an arbitrary limit on what you call “accurate”, say for example, “*less than 5% error*”.

Question 1

Calculate the absolute and percentage errors for the following results. Using the arbitrary criteria for accuracy as being >95% accurate, assess whether the results are accurate or inaccurate.

True value	Observed value	Absolute error	Percentage error	Accuracy	Accurate?
2.638 m	2.715 m				
15.26 s	13.98 s				
83 400 kg	84 200 kg				
45 N	41 N				

Systematic and random errors

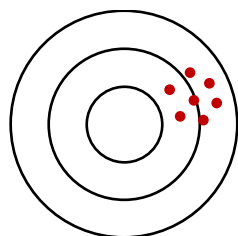
When you make multiple measurements and compute the errors, you might start to recognise patterns in how and when they occur. Because of this, errors can be put into one of two categories depending on how they behave:

- **Systematic errors** – When repeated, observed values are displaced in same direction from the true value. That is, the observed values might read consistently higher or consistently lower than the true value.

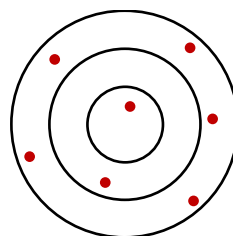
These types of errors are often caused by improperly calibrated measuring instruments, or “zero” errors (such as when an electronic balance shows a non-zero reading when there is nothing on its pan – every reading will be higher than it should be).

- **Random errors** – When repeated, observed values are scattered randomly above *and* below the true value.

These types of errors are often caused by random fluctuations in the ambient conditions or uncontrolled variables.



Systematic errors shift all measurements in the same direction.



Random errors cause measurements to spread randomly in all directions.

Question 2

Assess whether the following situations represent *systematic* or *random* errors.

- (a) The Royal Australian Mint states that the mass of a 50 cent coin is 15.55 g, so some students choose to measure some for themselves. First, they measure the mass of one coin, and then of two coins, and so on.

Predict the “true” values and compare them with the observations.

Number of coins	“True” values	Observed values
1		14.61 g
2		31.06 g
3		45.68 g
4		61.58 g
5		77.14 g

What type of error is demonstrated here?

- (b) Some students are interested in the boiling temperature of water. They go away in groups and heat equal amounts of water drawn from the same laboratory tap. They measure the water temperature with thermometers once boiling. Below are their results.

Group 1	Group 2	Group 3	Group 4	Group 5
102°C	99°C	103°C	101°C	98°C

What type of error is demonstrated here?

Improving accuracy (that is, reducing errors!)

To increase accuracy, we therefore need to *reduce error*. We can do that by modifying experimental techniques or procedures to make the error *absolutely* smaller, or by making the error smaller *relative* to the value we are measuring.

Question 3

How does the use of measuring instruments with appropriate sensitivity and resolution improve accuracy?

Question 4

The following procedure reduces the *relative* error of a measurement:–

- Instead of measuring the relatively quick period of a short pendulum, you could measure the relatively slow period for a long pendulum.

Explain how this sort of technique improves accuracy.

Question 5

Explain how pressing the “tare” (or “zero”) button on an electronic balance before measuring a mass reduces systematic error. What might happen if you did not press the tare button?

Question 6

Explain how taking measurements in a series of repeated trials, and then calculating an average, reduces random error.

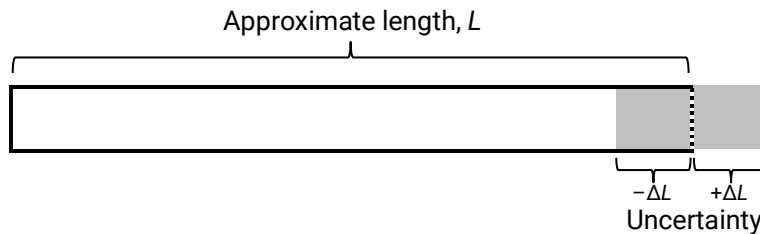


6 – Estimating uncertainties in measurements

While the concept of *error* compares measurements against values assumed to be “true”, there are many more measurements that cannot be compared to known or accepted values. For example, if we measured the length of a bit of string that we have, what is our error? We cannot compare our measurement with that of an expert – they don’t have our string!

Instead, what we should do is report our measurements with some indication of the certainty we have in it.

Remember, every measurement we make is, on some level, an approximation. To communicate how precise we think our measurement is, we can cite a margin of error which we call *uncertainty*.



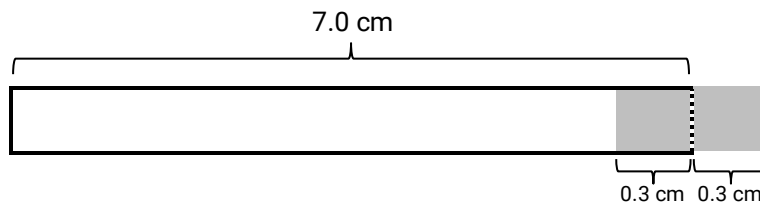
If we have a measurement x , then the uncertainty in that measurement is Δx . When we communicate the measurement to others, we write it in the form $x \pm \Delta x$. The uncertainty has the same units as the measurement.

For example, the bar in the diagram above could have length 6.0 ± 0.5 cm.

So when we make measurements, how do we know how big our uncertainty is? Well, we will answer that later in this unit.

Question 1

What is the length and uncertainty of the bar below?



Length =

A smaller uncertainty means that we can have higher confidence in a measurement. Compare this bar to the one earlier, in the notes ($L = 6.0 \pm 0.5$ cm). Which measurement can we have more confidence in?

Writing uncertainties

Recall how significant figures communicate the uncertainty in a number. For example, 1234 m (4 significant figures, has uncertainty $\pm 0.5\text{ m}$) has less uncertainty than 1200 m (2 significant figures, has uncertainty $\pm 50\text{ m}$). Because of this, there is a link between the number of digits that we report for a measurement and its uncertainty.

So to properly present a number with its uncertainty:

- *Step 1:* Round the uncertainty to one significant figure.
- *Step 2:* Round the measurement to the same place value (decimal place).

If the measurement's and uncertainty's lowest place values don't match, then the extra digits are meaningless – a relatively large uncertainty swamps the small value added by the extra digits.

Examples:

- $9.61482 \pm 0.0372\text{ m}\cdot\text{s}^{-2}$ should be rounded to $9.61 \pm 0.03\text{ m}\cdot\text{s}^{-2}$
- $1522.1 \pm 68.34\text{ km}$ should be rounded to $1520 \pm 70\text{ km}$

Question 2

Rewrite these uncertain quantities with the appropriate digits.

$51.784 \pm 0.0812\text{ m}$		$0.2874 \pm 0.0053\text{ A}$	
$841\ 063 \pm 462\text{ kg}$		$(6.322 \pm 0.48) \times 10^4\text{ V}$	

Just like with errors, we can express uncertainties in absolute terms, or in relative terms.

Absolute uncertainty (has the same units as the measurement)	Δx
Relative uncertainty (also called percentage uncertainty when expressed as %)	$\frac{\Delta x}{x}$

For example, the resistance of an electrical component could be written in absolute terms as $12.2 \pm 0.3\ \Omega$ or in percentage terms as $12.2\ \Omega \pm 2\%$. We can arbitrarily set a criteria for how small an uncertainty counts as being "high precision", i.e. $<5\%$.

Question 3

Compute the percentage uncertainties in the following values.

$9.79 \pm 0.09\text{ m}\cdot\text{s}^{-2}$		$0.243 \pm 0.008\text{ A}$	
$(3.7 \pm 0.5) \times 10^{-6}\text{ T}$		$16700 \pm 500\text{ lx}$	

How big should my uncertainty be?

There is no single way to estimate the uncertainty in a measurement – it depends on how you have made your measurement. Next, we will focus on the uncertainty in a single direct measurement, and then the uncertainty in an average from repeated trials.

– Uncertainty in direct measurements

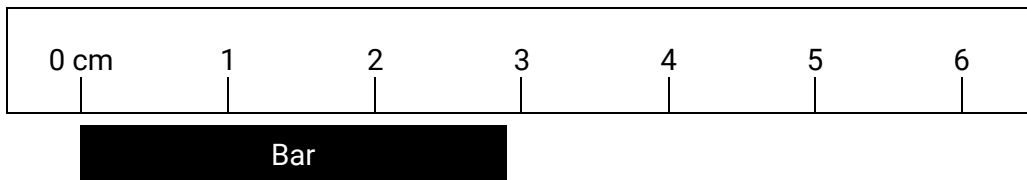
The uncertainty in a direct measurement comes from our ability to make readings with measuring instruments.

Digital Caliper	
Resolution:	0.01 mm
Accuracy:	±0.02 mm

Some measuring instruments are labelled with the accuracy or uncertainty that you can expect when you use them. In this case, ±0.02 mm.

For all other instruments, the absolute smallest uncertainty they provide is limited by the graduations marked on them:

Uncertainty due to measuring instrument	$\pm \frac{1}{2} \times (\text{instrument graduation})$
---	---



This ruler is graduated in 1.0 cm increments. The uncertainty in direct measurements made with this ruler would be ±0.5 cm.

In the diagram above, we can read that the bar is near to 3.0 cm (the nearest centimetre mark) but we cannot confidently estimate the length of the bar any finer than that.

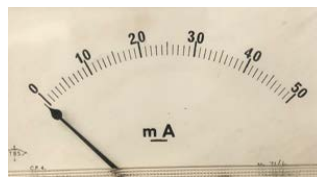
We would report this measurement as 3.0 ± 0.5 cm (notice the “.0” in the measurement – so that the number of decimal places in the uncertainty matches).

Question 4

(a) What is the smallest uncertainty that you can expect with these instruments?



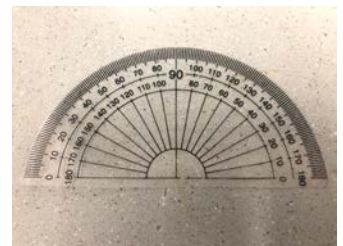
Stopwatch



Milliammeter



Electronic balance



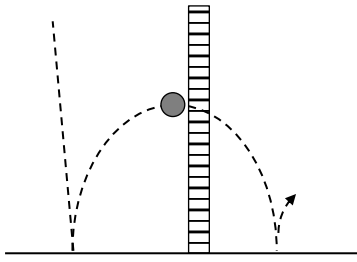
Protractor

±	±	±	±
---	---	---	---

(b) For the stopwatch and the electronic balance, write the reading and its uncertainty with the appropriate number of digits.

time =	mass =
--------	--------

– Estimating larger uncertainties in direct measurements



Sometimes the markings on an instrument cannot be confidently read by us and the uncertainty is actually *larger*. For example, imagine you are attempting to measure the bounce height of a ball. The ball moves so quickly that you cannot precisely measure to millimetre accuracy on the adjacent ruler.

In cases like this, you will have to use your judgement – perhaps you are only confident that you can measure to the nearest 5 cm, so your uncertainty is half of this, ± 2.5 cm.

This practice does rely on some subjectivity. If you are unsure, it is always preferable to *overestimate* your uncertainty than to dishonestly claim that it is smaller.

– Uncertainty in an average of trials

Recall that conducting repeated trials and then computing an average helps to reduce random error. While we can assume that the average is more accurate than any of the trials, there is still some uncertainty in the average.

If there are many trials, then you might use a standard deviation as the uncertainty. However, in a school laboratory, we often only conduct very few trials, so a shortcut – using half the range of the trials – is a simpler alternative.

Uncertainty in average (when there are many trials)	± 1 standard deviation
Uncertainty in average (when there are only few trials)	$\pm \frac{\text{highest trial} - \text{lowest trial}}{2}$

Question 5

Determine the average and uncertainty from the following sets of measurement trials.

Trial 1	Trial 2	Trial 3	Average and uncertainty
10.12 m.s ⁻²	8.74 m.s ⁻²	9.37 m.s ⁻²	

Trial 1	Trial 2	Trial 3	Average and uncertainty
77.53 mL	72.81 mL	79.22 mL	

Trial 1	Trial 2	Trial 3	Average and uncertainty
100.31 Ω	100.24 Ω	100.17 Ω	

7 – Using uncertainty to assess values

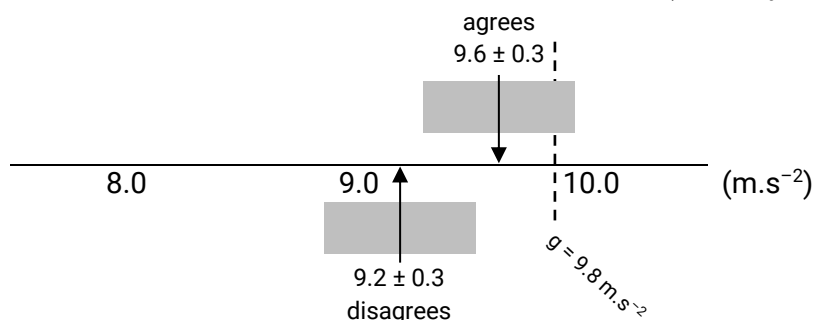
Recall that when we used the concept of *error*, an accurate measurement was one that was close to the accepted value. However, an analysis of errors on its own does not give us a complete view of how good a value is. Because of the randomness of some errors, there is a probability that a highly accurate/low error value is arrived at by pure luck anyway.

We can use uncertainties to better assess the values that we obtain in experiments: small uncertainties give an indication that measurements were made with good precision.

Comparing a measurement with an accepted value

We can say that a measurement *agrees* with the accepted value when the accepted value lies inside the measurement's uncertainty bounds. For example, when comparing the generally accepted value of acceleration due to gravity at Earth's surface ($g = 9.8 \text{ m.s}^{-2}$), a measurement of:

- $9.6 \pm 0.3 \text{ m.s}^{-2}$ agrees, because 9.8 m.s^{-2} lies inside the range of 9.3 and 9.9 m.s^{-2}
- $9.2 \pm 0.3 \text{ m.s}^{-2}$ does not agree, because 9.8 m.s^{-2} lies outside the range of 8.9 and 9.5 m.s^{-2}
- They are both precise because their uncertainties are less than 5% (arbitrary criteria).



The arrows point to values on the number line and the grey bars are the uncertainty ranges. Only one of the measurements includes the accepted value.

Question 1

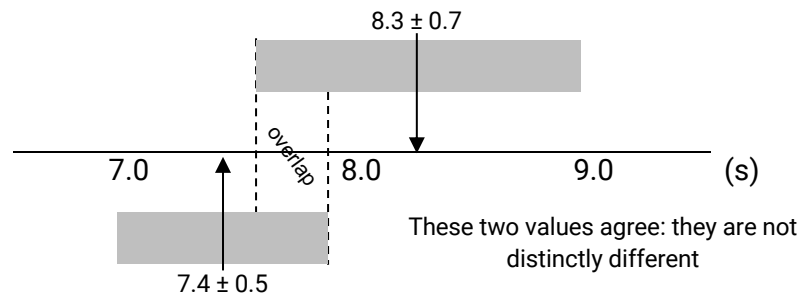
Do these measurements agree with the accepted values? Also, determine if they measurements are precise (using the criteria that <5% uncertainty is high precision).

Accepted value	Measurement	Agrees?	Precise?
9.8 m.s^{-2}	$10.1 \pm 0.4 \text{ m.s}^{-2}$		
$6.626 \times 10^{-34} \text{ J.s}$	$(5.9 \pm 0.5) \times 10^{-34} \text{ J.s}$		
10.0Ω	$9.4 \pm 0.7 \Omega$		
100°C	$102 \pm 2^\circ\text{C}$		
22.4 L	$23.0 \pm 0.4 \text{ L}$		

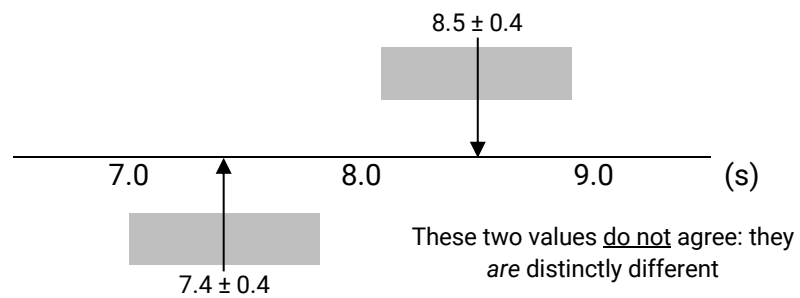
Comparing two values with uncertainties to see if they agree

Two measurements with uncertainties agree if their uncertainty ranges overlap. For example,

- 7.4 ± 0.5 s and 8.3 ± 0.7 s agree with each other because their uncertainty ranges overlap.



- 7.4 ± 0.4 s and 8.5 ± 0.4 s do not agree with each other – their uncertainty ranges do not overlap.



Question 2

Are these pairs of measurements distinctly different to each other?

Value A	Value B	Agree/Do not agree
23.6 ± 0.5 kg	24.0 ± 0.5 kg	
810 ± 10 s	850 ± 20 s	
3490 ± 10 m	3501 ± 5 m	
$(6.2 \pm 0.7) \times 10^{-4}$ T	$(4.8 \pm 0.6) \times 10^{-4}$ T	
7.22 ± 0.04 N	7.15 ± 0.04 N	

Question 3

A team of sports physicists has been hired by an event organiser to verify the length of their marathon course, which is supposed to be 42.195 km long. They use two methods – directly with a car odometer (42.20 ± 0.05 km) and using a map (42.180 ± 0.005 km). Do the measurements agree with each other? Do either of them agree with the required distance? Which one is more precise?

8 – Estimating uncertainty from a graph

Recall that linear regression can be used to establish relationships between variables, and that the gradient of a straight-line graph can be used to determine physical values.

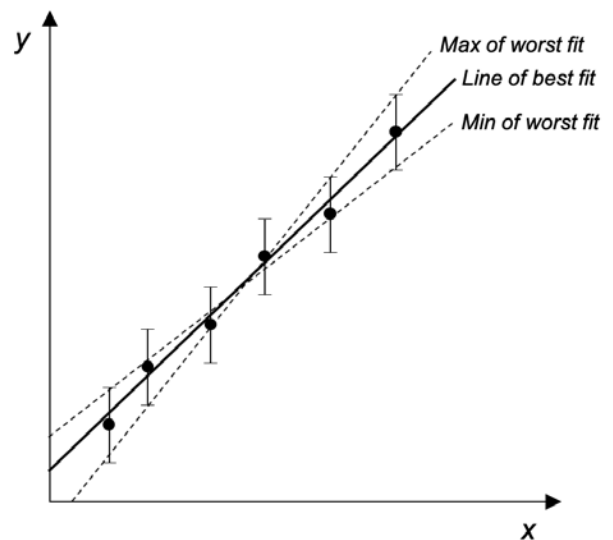
Just like any value that has its ultimate origins in experimental data, there is also uncertainty in the gradient of a straight-line graph.

Determining the gradient & its uncertainty

Plot the data points as normal and draw the line of best fit. Determine the gradient of this line in the usual way.

Next, draw *uncertainty bars* for each data point. These are bars that extend above and below each data point for the same values as their uncertainties, with caps on their ends (see *right*).

Then, draw two *lines of worst fit* – the steepest and shallowest lines that still pass through most or all of the uncertainty bars.



Measure the maximum and minimum gradients for the lines of worst fit, and then calculate the uncertainty:

Uncertainty in line of best fit (LOBF) gradient

$$\pm \frac{\text{max worst fit gradient} - \text{min worst fit gradient}}{2}$$

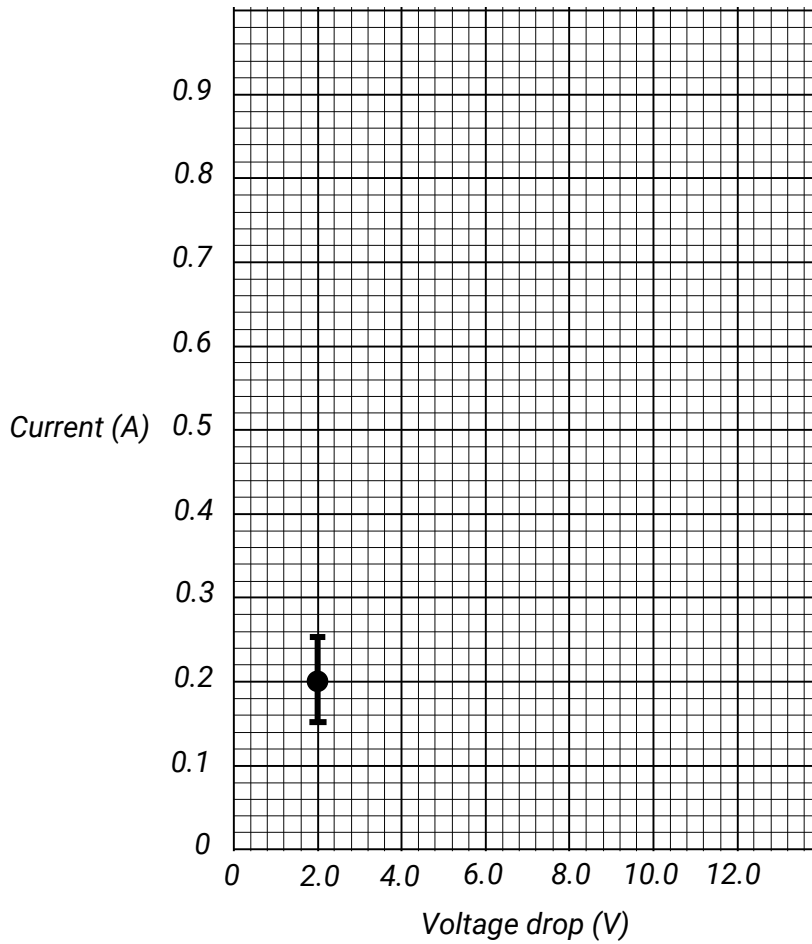
Step 1

Plot the following data, with their respective error bars, on the grid on the next page.

Current when voltage is applied across a resistor

Voltage applied, V (V)	Current, I (A)
2.0	0.20 ± 0.05
4.0	0.31 ± 0.05
6.0	0.46 ± 0.05
8.0	0.64 ± 0.05
10.0	0.79 ± 0.05

Current vs voltage applied across a resistor



Step 2

Draw the line of best fit using the data points as a guide.

Step 3

Draw two lines of worst fit. Compared to the line of best fit, one is shallower and the other is steeper. They should pass through as many error bars as possible.

Step 4

Determine the gradients of the lines of best and worst fit. Write them here.

	Gradients
Line of best fit (LOBF)	
Line of worst fit – maximum (steepest)	
Line of worst fit – minimum (shallowest)	

Step 5

Calculate the uncertainty in the line of best fit (LOBF) gradient using the equation

$$\text{Uncertainty in LOBF gradient} = \frac{\text{max worst fit gradient} - \text{min worst fit gradient}}{2}$$

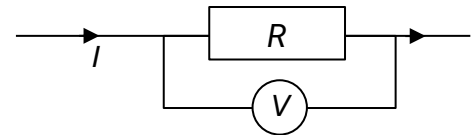
Write the line of best fit gradient with its uncertainty here

±

Using a gradient with uncertainty to derive a physical value

Now that we have a gradient and uncertainty, we can use it to derive a physical value.

The data used for our graph comes from measurements of the current I flowing through a resistor when there is voltage V across it. These variables are related by Ohm's law, $V = IR$, where R is the electrical resistance of the resistor.



When rewritten into the linear form $Y = AX + B$, Ohm's law becomes $I = \frac{1}{R}V$, so the gradient corresponds to $\frac{1}{\text{resistance}}$.

Step 6

Using the line-of-best-fit gradient, estimate the electrical resistance of the resistor.

Step 7

The relative uncertainty in resistance is equal to the relative uncertainty in the gradient.

$$\frac{\Delta R}{R} = \frac{\Delta \text{LOBF gradient}}{\text{LOBF gradient}}$$

Given that you have R from step 6, and the gradient and uncertainty step 5, calculate the uncertainty in resistance, ΔR .

Write the electrical resistance with its uncertainty here

±

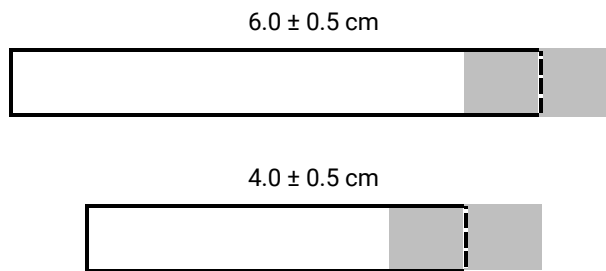


9 – Estimating uncertainty in results

When values with uncertainties are used in a calculation, the result also has an uncertainty.

Uncertainties when adding and subtracting

Consider the bars below. The black lines are the approximate sizes of the bars and the shaded areas are the uncertainties.

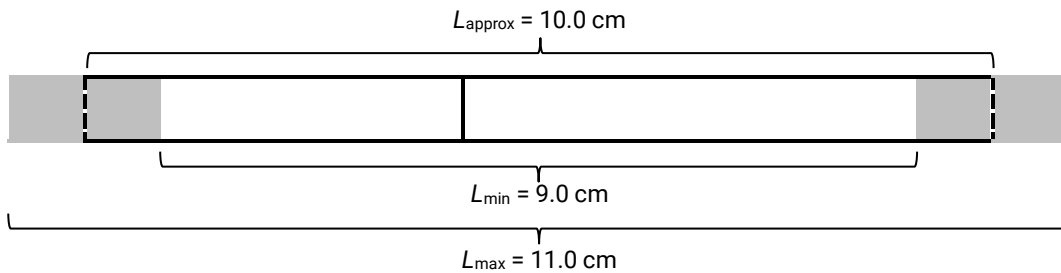


When we combine them into a longer bar, we can determine:

- the approximate length $L_{approx} = 6.0 \text{ cm} + 4.0 \text{ cm} = 10.0 \text{ cm}$

but also because of the uncertainties of each bar, we can infer:

- a minimum possible length, $L_{min} = 5.5 \text{ cm} + 3.5 \text{ cm} = 9.0 \text{ cm}$
- a maximum possible length, $L_{max} = 6.5 \text{ cm} + 4.5 \text{ cm} = 11.0 \text{ cm}$



We can then report the length of the combined bar as $10 \pm 1 \text{ cm}$.

This demonstrates that the uncertainties (0.5 cm and 0.5 cm) have simply been added together.

So when **adding or subtracting** values, their **absolute uncertainties add** together.

When you do these	Do this
$a = b + c + \dots$ $a = b - c - \dots$	$\Delta a = \Delta b + \Delta c + \dots$

Question 1

- (a) In the space below, draw two bars, $L_1 = 7.0 \pm 0.6$ cm and $L_2 = 5.0 \pm 0.8$ cm. Show the uncertainty in their lengths by shading, like on the previous page.
Then, draw the two bars joined together.
Indicate and label the approximate length of the combined bar, and the minimum and maximum possible lengths.

- (b) What is the approximate length of the combined bar?

- (c) What is the maximum possible length of the combined bar, considering the uncertainties of the individual bars?

- (d) What is the minimum possible length of the combined bar?

- (e) Write the length of the bar with its uncertainty in the appropriate format.

Question 2

Perform the following calculations and determine the uncertainties in their results.

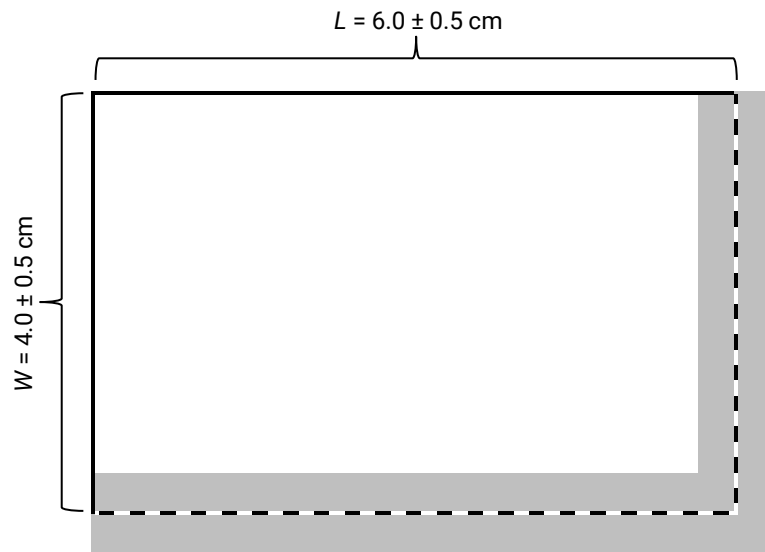
(a) Total mass =
 (450 ± 10) kg + (360 ± 8) kg

(b) Change in temperature =
 (87.3 ± 0.4) °C – (22.6 ± 0.7) °C

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Uncertainties when multiplying or dividing

Consider the rectangle below. The black line is the approximate size of the rectangle and the shaded area is the uncertainty.



We can multiply the approximate lengths of the sides to determine:

- the approximate area of the rectangle, $A_{approx} = 6 \text{ cm} \times 4 \text{ cm} = 24 \text{ cm}^2$

but because of the uncertainties, we can infer:

- a minimum possible area, $A_{min} = 5.5 \text{ cm} \times 3.5 \text{ cm} = 19 \text{ cm}^2$
- a maximum possible area, $A_{max} = 6.5 \text{ cm} \times 4.5 \text{ cm} = 29 \text{ cm}^2$

So we can report the area of the rectangle as $24 \pm 5 \text{ cm}^2$.

It's a bit harder to see here, but when you **multiply or divide** values, **their relative uncertainties add together to give the relative uncertainty in the result.**

When you do these	Do this
$a = b \times c \times \dots$ $a = b \div c \div \dots$	$\frac{\Delta a}{a} = \frac{\Delta b}{b} + \frac{\Delta c}{c} + \dots$

Let's try this for the uncertainty for our rectangle example.

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta W}{W} \rightarrow \Delta A = A \left(\frac{\Delta L}{L} + \frac{\Delta W}{W} \right) = 24 \text{ cm}^2 \left(\frac{0.5}{6.0} + \frac{0.5}{4.0} \right) = 5 \text{ cm}^2$$

which is what we found geometrically.

Question 3

(a) In the space below, draw a rectangle with sides $L = 14.0 \pm 1.0$ cm and $W = 3.0 \pm 0.5$ cm. Label the sides with these values.

Show the uncertainty in the area by shading, like on the previous page.

(b) What is the approximate area of the rectangle?

(c) What is the maximum possible area of the rectangle, considering the uncertainties of each side?

(d) What is the minimum possible area of the rectangle?

(e) Write the area of the rectangle with its uncertainty in the appropriate format.

(f) Use the uncertainty equation to calculate and verify the uncertainty that you have determined geometrically.

Question 4

Perform the following calculations and determine the uncertainties in the results.

(a) Force = $(8.2 \pm 0.4) \text{ kg} \times (3.2 \pm 0.2) \text{ m.s}^{-2}$

(b) Speed = $(52.7 \pm 0.8) \text{ m} \div (6.2 \pm 0.5) \text{ s}$

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10 – Assessing reliability in experiments

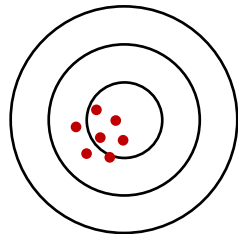
Reliability refers to the consistency in results – repetition returns results that lie within a small margin of error. There are two ways of looking at reliability:

- *Internal reliability* is when repeated trials within an experiment are consistent. This is sometimes also called *precision*.
- *External reliability* is when the results from one experiment are consistent with those from other experiments that are conducted the same way.

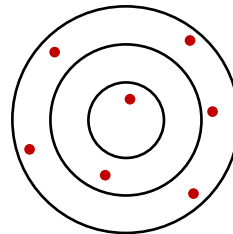
When assessing reliability, sometimes it is enough to make broad subjective judgements (for example, “overall, the results appear to be roughly consistent”) but it is preferable to fall back on some kind of quantitative basis.

Evaluating reliability

One way to assess reliability might be to quantify the spread of trials around an average – large spreads are unreliable, and smaller spreads are reliable.



Data from reliable trials are clustered closely.



Data from unreliable trials are much more spread out.

We could judge a set of trials to be reliable if the *relative uncertainty* of the trials is less than some arbitrary limit, say, *less than 5%*.

For example, the following measurements of the same resistor were collected. The average is shown, and the uncertainty was calculated using:

$$\text{Uncertainty} = \pm \frac{\text{highest trial} - \text{lowest trial}}{2}$$

Trial 1	Trial 2	Trial 3	Average and uncertainty
94.8 Ω	106.3 Ω	100.2 Ω	100 ± 6 Ω

The relative uncertainty of this data set is $\frac{6 \Omega}{100 \Omega}$ or 6%. According to our criteria, these trials are not reliable. Perhaps it might be better to say that they have *low reliability*.

Question 1

A physics class is set the task of measuring acceleration due to gravity, g , using pendulums. They go away in groups to do this, and when they have completed their experiments, they return to report their findings. Below are their results.

Physics class results – acceleration due to gravity, g

Group	g ($\text{m}\cdot\text{s}^{-2}$)
A	8.7
B	9.4
C	10.3
D	9.9
E	10.9

(a) Which concept is being demonstrated here? *Internal* or *external* reliability?

(b) Calculate the average and its uncertainty for this set of data.

(c) Assess the reliability of this set of data.

Question 2

Another physics class is investigating the stopping distance of a bicycle from different speeds. A student rides the bike at a designated speed and applies the brakes once it passes a mark on the ground. The remaining students measure the distance to where the bike has stopped. The students conducted 5 trials for every speed. Below is one set of their trials.

Physics class results – Stopping distances at 10 km/h

Trial	Stopping distance (m)
1	6.2
2	6.1
3	6.3
4	6.2
5	6.4

(a) Which concept is being demonstrated here? *Internal* or *external* reliability?

(b) Calculate the average and its uncertainty for this set of data.

(c) Assess the reliability of this set of data.

Question 3

(a) Can simply repeating a measurement or an experiment on its own improve reliability? Why/why not?

Question 4

(a) It can be said that *variability* is the enemy of reliability. What does this mean?

(b) What is the cause of variability, and how can it be minimised?

11 – Assessing validity in experiments

A *valid* experiment is one that examines what is intended – the relationship between an independent variable and a dependent variable with minimal interference from other factors. These other factors might be the variables that we should control (hold constant), or the level of care with which we conduct the experiment and make measurements.

If we only vary the independent variable and keep all the other variables the same, then we can be confident that the effects that we observe are due only to the changes that we have made. We can say that the experiment is valid.

However, if we do not keep the other variables the same, then we cannot be certain that our observations are only due to the independent variable. This would mean that the experiment would be *invalid*.

If we are careless when we conduct experiments, make inaccurate measurements, or use inappropriate equipment, then this also invalidates the experiment.

Question 1

Some students wish to investigate the effect of a force on accelerating toy cars.

They hypothesise that *increasing the force on the cars will proportionally increase their rates of acceleration because of the equation $a = \frac{F}{m}$* .

They test their hypothesis with a range of toy cars. Below is the data that they collected.

Car	Force (N)	Acceleration (m.s ⁻²)
A	0.2	3.5
B	0.4	11.4
C	0.6	15.3
D	0.8	14.7
E	1.0	20.8

(a) What was the independent variable?

(b) What was the dependent variable?

(c) Does the data show a proportional relationship between force and acceleration, as predicted by the students?

(d) Based on this data alone, what can the students conclude about their hypothesis?

Later, the teacher who was observing the students made some related measurements of her own. Her data is displayed below.

Car	Car mass (g)
A	57
B	35
C	39
D	54
E	48

(e) Can we confidently attribute the acceleration of the cars purely to the force applied? What does this mean for our trust in the students' conclusions in part (d)?

(f) Write a short paragraph that assesses the validity of the students' investigation.

(g) How can the validity of the experiment be improved?

Question 2

What are the conditions for a valid experiment?

12 – Introduction to spreadsheets

Computers are powerful tools that can be used to process and analyse large amounts of data. With applications like spreadsheets, you can perform repetitive and complex calculations quickly. You can follow along and try these examples, which use Google Sheets (Microsoft Excel does the same thing in very similar ways).

Part 1 – Simple calculations

To perform a calculation, just click on a cell to select it and type your equation, beginning with “=”. An equals sign tells the spreadsheet to perform the calculation that we’ve typed into the cell.

Let’s try to add the numbers 123.4 and 876.5 together. Try this:

- Click on cell B2 to select it, and type **=123.4+876.5**
Press *Enter* after you have typed your formula. The answer will be displayed in the cell.

	A	B	C	D	E
1		999.9 x			
2		=123.4+876.5			
3					

Sometimes we want to use numbers that are already on the spreadsheet. Cells can be called upon by their cell locations. Below, the cell B3 has been selected.

	A	B	C	D	E
1					
2		999.9			
3		123.4	876.5		
4					

Try this:

- Type **123.4** into cell B3 and **876.5** into cell C3.
- To sum the contents of cells B3 and C3 and show the result in D3, click on cell D3 and type **=B3+C3** and press *Enter*.

	A	B	C	D	E
1					
2		999.9			
3		123.4	876.5	=B3+C3	
4					

If you decide that one of your values needs to be changed, you can enter a new value into the cell, and the spreadsheet re-does the calculation. Try this:

- Change the value in B3 to **432.1**. When you press *Enter*, the value in D3 also changes.

	A	B	C	D	E
1					
2		999.9			
3		432.1	876.5	1308.6	

You can also perform any other type of mathematical operations. Below is a table of the most common ones.

Operation	Spreadsheet operator	Example syntax
Addition	+	=A+B
Subtraction	-	=A-B
Multiplication	*	=A*B
Division	/	=A/B
Indexation, a^b	^	=A^B
Square root, \sqrt{a}	sqrt()	=sqrt(A)
Scientific notation, $\times 10^a$	E	1.23E+02

Part 2 – More complex calculations

–Order of operations (using brackets)

In more complex calculations, we might need to specify the order of operations. We can use brackets just like we could with pen and paper.

Let's try to calculate $\frac{x+y}{2}$, when $x = 5$ and $y = 3$.

Open a new sheet and try the following steps:

- Type **x** into A2 and **y** into A3 (these are just labels)
- Type **5** into B2, and **3** into B3.

Let's display our answer in B5.

- Select cell B5 and type **=(B2+B3)/2** and press *Enter*.

	A	B	C
1			
2	x	5	
3	y	3	
4			
5		=(B2+B3)/2	
6			

The result should now appear in the cell.

- Averaging

Let's say that we want to find the average of 23, 25, 26, 28, 24 and 27. We can use the "average()" function, and the spreadsheet will compute the average of the numbers inside the brackets.

Open a new sheet, then follow the steps below.

- Select cell B2 and type **=average(23, 25, 26, 28, 24, 27)** and press *Enter*.

	A	B	C
1		25.5 x	
2		=average(23, 25, 26, 28, 24, 27)	
3			

But what if we wanted to calculate the average of numbers that are already on the spreadsheet? Well we can call on them with their cell locations! For this exercise, type the values into their own cells between B4 and B9.

- In cell A10, type "**Average**" (this is just a label to remind us what the value means).
- In cell B10, type **=average(B4:B9)** and press *Enter*.

The colon in "B4:B9" tells the spreadsheet to call on all the cells in a contiguous range between B4 and B9.

	A	B	C
1			
2		25.5	
3			
4		23	
5		25	
6		26	
7		28	
8		24	
9		27	
10	Average	=average(B4:B9)	
11			

- Trigonometry

You can use spreadsheets to perform calculations with trigonometry. Note, however, that spreadsheets use *radians* and *not degrees* for trigonometry. Fortunately, it is easy to convert between degrees and radians in spreadsheets.

To convert from degrees to radians, use "*radians()*" where the value in the brackets is an angle in degrees.

Let's try and convert 45° into radians.

Open a new sheet, and try the following steps:

- In cell B2, type **=radians(45)** and press *Enter*.

	A	B	C
1		0.7853981634 x	
2		=radians(45)	
3			

Now let's try and convert $\frac{\pi}{4}$ radians into degrees. We will use the "*degrees()*" function.

Spreadsheets can do calculations involving π , but you must type it as "**pi()**" with empty closed brackets. So let's try that:

- In cell B4, type **=degrees(pi()/4)** and press *Enter*.

	A	B	C
1			
2		0.7853981634	
3		45 x	
4		=degrees(pi()/4)	
5			

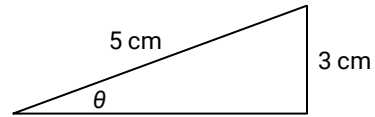
Let's try to find the sine of 45 degrees, i.e. $\sin 45^\circ$. We can use the "*sin()*" function but remember, spreadsheets only accept radians in trigonometry, so we will have to do a conversion.

- In cell B6, type **=sin(radians(45))** and press *Enter*.

	A	B	C
1			
2		0.7853981634	
3			
4		45	
5		0.7071067812 x	
6		=SIN(radians(45))	
7			

We could have done the same for the other trigonometric functions: cosine "*cos()*" and tangent "*tan()*".

What about if we know the sides of a right-angled triangle and we want to find the angle θ ?



Using trigonometry, we would compute $\theta = \sin^{-1}\left(\frac{3}{5}\right)$. In a spreadsheet, we would type `=asin(3/5)`, but remember that this would give us an angle in radians. Let's try to calculate the angle in degrees:

- In cell B8, type `=degrees(asin(3/5))` and press *Enter*.

	A	B	C
1			
2		0.7853981634	
3			
4		45	
5			
6		0.7071067812	
7		36.86989765 x	
8		=degrees(asin(3/5))	

Similar functions exist for arccosine "`acos()`" and arctan "`atan()`".

Part 3 – Repetitive calculations

A very useful feature of spreadsheets is the ability to perform repetitive calculations quickly and easily. You would only need to type a formula once, and then fill multiple cells with the same formulas so that the calculation can be performed with different numbers.

Open a new sheet, let's set up a table.

- Enter the following table headings and data.

	A	B	C	D
1	distance (m)	time (s)	speed (m/s)	
2	36.4	3.2		
3	62.1	8.4		
4	87.2	7.3		
5	18.3	4.9		
6	41.7	1.6		
7				

To calculate speed, we will use the equation $\text{speed} = \frac{\text{distance}}{\text{time}}$. Let's do this for the first row of data.

- In cell C2, type **=A2/B2** and press *Enter*.

	A	B	C	D
1	distance (m)	time (s)	speed (m/s)	
2	36.4	3.2	=A2/B2	
3	62.1	8.4		
4	87.2	7.3		
5	18.3	4.9		
6	41.7	1.6		
7				

It would be inconvenient to type this formula into the spreadsheet again and again for the following rows of data. Instead, try this:

- Click on cell C2 to select it, and then hover your mouse pointer over the square at its bottom right corner – it will turn into a cross. Click and drag to cover the remaining cells in the column (down to C6), and release.

	A	B	C	D
1	distance (m)	time (s)	speed (m/s)	
2	36.4	3.2	11.375	
3	62.1	8.4		
4	87.2	7.3		
5	18.3	4.9		
6	41.7	1.6		
7				

Click and drag!

The cells you have just covered are now filled with copies of the same formula, but now the cell references have been altered for each respective row. The results should now be displayed.

Spreadsheets are handy!

You just need to be careful when using a spreadsheet – it is not a substitute for skill and understanding. If you input garbage, it will output garbage.

13 – Using spreadsheets for graphing

Spreadsheets can be used to plot graphs and to analyse them. You can follow along and try this example, which uses Google Sheets (Microsoft Excel does the same thing in very similar ways).

In this example, we will graph the extension of a spring x as we apply force F to it. These two variables are related by Hooke's law,

$$F = kx$$

We will then attempt to measure its spring constant k .

Step 1

Enter this data into a new spreadsheet. It is important that the independent variable is in the left column and the dependent variable is in the right column, so that the graph will have the correct quantities on each axis.

	A	B	C
1	Force (N)	Spring extn (m)	
2	0.49	0.018	
3	0.98	0.048	
4	1.47	0.066	
5	1.96	0.086	
6	2.45	0.109	
7			

Step 2

Select all the data (including the headings) by clicking and holding in the centre of cell A1, drag to highlight the cells down to and including B6, then release your mouse button.

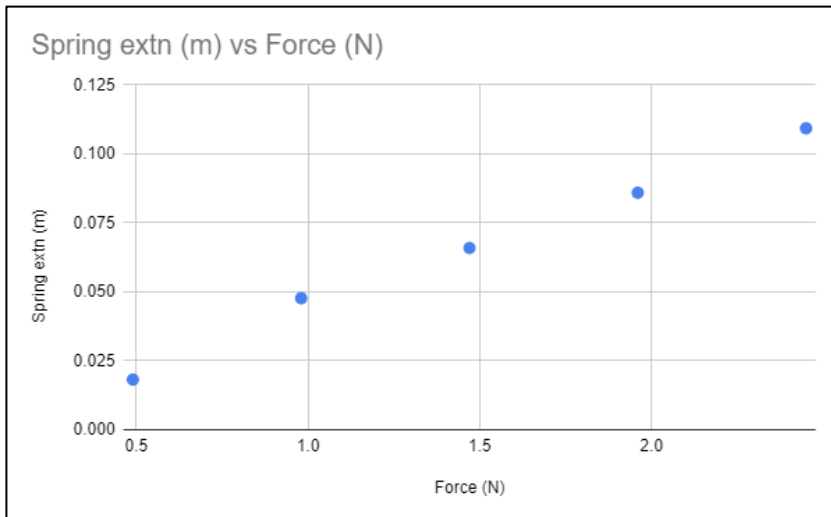
	A	B	C
1	Force (N)	Spring extn (m)	
2	0.49	0.018	
3	0.98	0.048	
4	1.47	0.066	
5	1.96	0.086	
6	2.45	0.109	
7			

Step 3

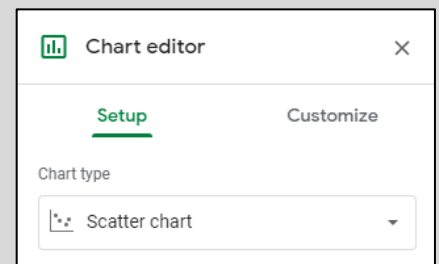
While the cells are still selected, click on the  button in the tool bar.

Alternatively, you can select **Insert > Chart** from the menu bar.

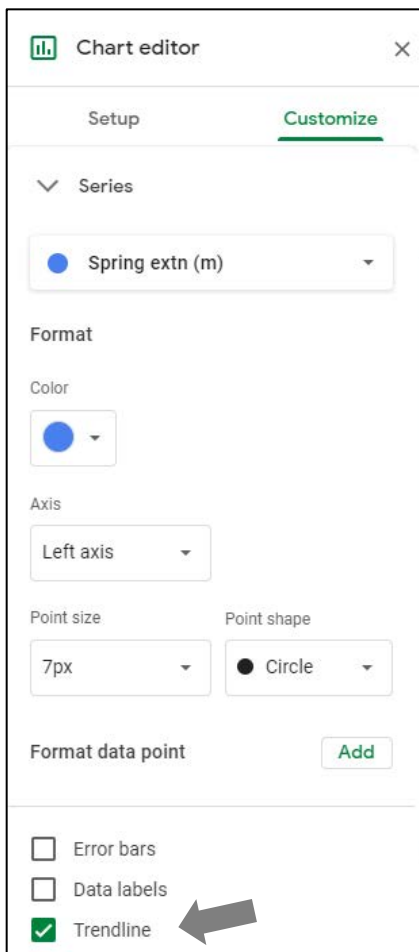
A scatter graph should appear, like the one below.



If a graph other than a scatter graph appears, double-click on the graph to open the **Chart editor** and select **Setup**. In the **Chart type** dropdown menu, select **Scatter Chart**.



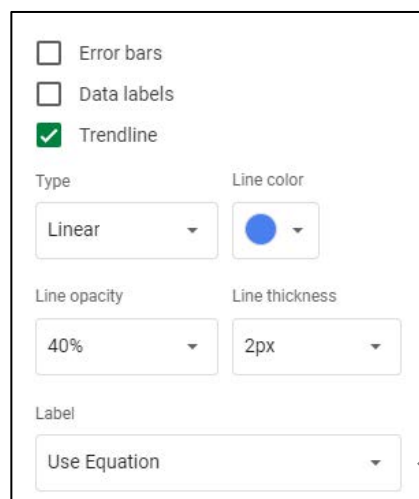
Step 4



← In the **Chart editor**, select the **Customize** tab and expand the **Series** menu.

Place a tick in the **Trendline** box.

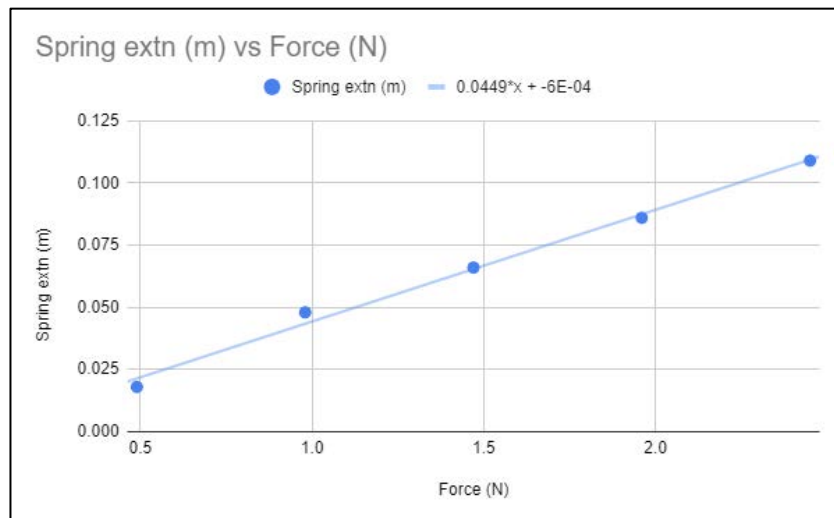
↓ We also want to know the equation of this line. Scroll a bit further down and from the **Label** dropdown menu, select **Use Equation**.



The **Chart editor** has many options and settings that you can set and change. You may wish to explore these when you have finished this activity.

Step 5

You should now have a graph that looks like this one.



Note that the equation for our line is $Y = 0.0449 X - 6 \times 10^{-4}$.

From our understanding of straight line graphs, the gradient of the line is 0.0449 m/N.

Note that:

- **Spring extension, x** is on the Y-axis
- **Force F** is on the X-axis.

Remember that the Hooke's law equation is $F = kx$, so we need to rearrange the equation to the form

$$Y = AX + B$$

$$x = \frac{1}{k}F$$

So therefore

$$\text{gradient} = \frac{1}{k} \rightarrow k = \frac{1}{\text{gradient}} = \frac{1}{0.0449 \text{ m/N}} = 22.2 \text{ N/m}$$

The value for this spring's spring constant is 22.2 N.m^{-1} .

Extension

You can get more information about the line by using the 'LINEST' function. The LINEST function is defined:

$$=linest(Y_values, X_values, true, true)$$

and returns an array of data, which includes the gradient, Y-intercept, and their errors.

In cell A8, type `=linest(B2:B6, A2:A6, true, true)` and press *Enter*.

Input		Output	
	A	B	
1	Force (N)	Spring extn (m)	
2	0.49	0.018	gradient
3	0.98	0.048	
4	1.47	0.066	
5	1.96	0.086	
6	2.45	0.109	
7			
8	<code>=linest(B2:B6, A2:A6, true, true)</code>		

	A	B	
8	0.04489795918	-0.0006	Y-intercept
9	0.002210622695	0.003592584956	
10	0.9927797834	0.003425395354	
11	412.5	3	
12	0.00484	0.0000352	Uncertainty in Y-intercept

An arrow points from the input table to the output table. In the output table, an arrow labeled "gradient" points to cell A8. An arrow labeled "Y-intercept" points to cell B8. An arrow labeled "Uncertainty in gradient" points to cell A12. An arrow labeled "Uncertainty in Y-intercept" points to cell B12.

Notice that the gradient and Y-intercept are given to more significant figures as well as some indication of the uncertainty in the gradient (although this is obtained statistically and not with consideration of the uncertainties in each data point).

14 – Spreadsheets – An exercise

Try analysing this data using a spreadsheet.

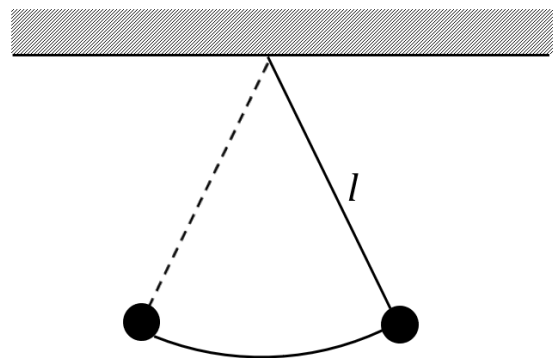
The experiment

The aim of this experiment was to measure the acceleration due to gravity g using a pendulum.

At different pendulum lengths l chosen by the experimenter, the period T of a back and forth swing was timed.

The equation that relates l and T is

$$T = 2\pi \sqrt{\frac{l}{g}}$$



To reduce human-induced timing errors, the experimenters chose to measure the time for 10 periods, then divide the measurement by 10 to obtain a more accurate value for a single period.

Your task

Use a spreadsheet to process the data and determine a value for g using linear regression.

Step 1

Create a table in a spreadsheet like the one below. Enter the data shown.

	A	B	C	D	E	F	G
1	Length (m)	Trial 1 - 10T	Trial 2 - 10T	Trial 3 - 10T	Av - 10T	T	T ²
2	0.2	8.84	8.80	8.76			
3	0.4	12.50	12.56	12.47			
4	0.6	15.31	15.36	15.25			
5	0.8	17.68	17.64	17.64			
6	1.0	19.77	19.83	19.70			

Step 2

In cell E2, type a formula that you could use to calculate the average time for 10 periods ($10T_{av}$), using data from the trials (the data in cells B2, C2 and D2).

Write your formula here

Fill the rest of the cells in column E (down to E6) with similar formulae.

Step 3

In cell F2, type a formula that you could use to calculate the time for one period of the pendulum T , using the average you calculated in step 2.

Write your formula here


Fill the rest of the cells in column F (down to F6) with similar formulae.

Step 4

In cell G2, type a formula that you could use to calculate the square of a single period, T^2 .

Write your formula here

Fill the rest of the cells in column G (down to G6) with similar formulae.

TIP: You can reduce the number of decimal places displayed by selecting the appropriate cells and clicking  in the tool bar. This does not round the number off – the remaining digits are still stored.

Step 5

Once you have a table of processed data, you are ready to plot a graph.

With some algebra, we can rearrange the pendulum equation to the $Y = AX + B$ form

$$T^2 = \frac{4\pi^2}{g} l$$

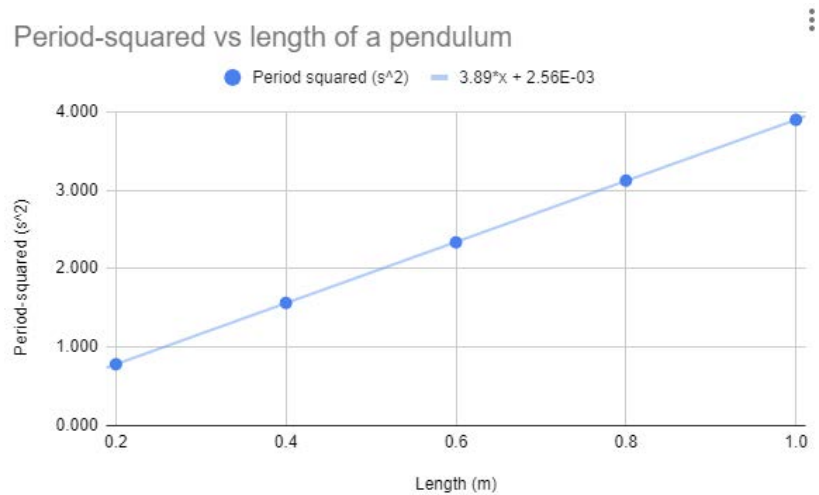
which tells us that we should plot a graph with T^2 on the Y axis and l on the X-axis.

So, plot a graph of *period-squared* vs *length*. You will need to select cells A1:A6 and G1:G6 to do this.

Include a trendline (line of best fit) and its equation.

TIP: To select data that are not next to each other on the spreadsheet, hold down the **Ctrl** key on your keyboard while selecting cells.

Your graph may look like this:



If your graph does not look like this, then you might need to change a few settings in the **Chart editor**:

- If a *scatter graph* was not initially generated automatically, you may need to change it in **Chart editor > Set up > Chart type** and select *Scatter*
- If chart and axis titles are missing, then you can add these in **Chart editor > Customize > Chart & axis and titles** and change each title in turn.

Step 6

Write your gradient here

From the equation in step 5, we can infer this expression involving the gradient of the line and g :

$$\text{gradient} = \frac{4\pi^2}{g}$$

Use this expression to calculate an experimental value for the acceleration due to gravity, g , from the data that was collected.

Extension

Use **LINEST** to determine the gradient to more significant figures, and its uncertainty. Calculate a new value for g .

Write your gradient and its uncertainty here

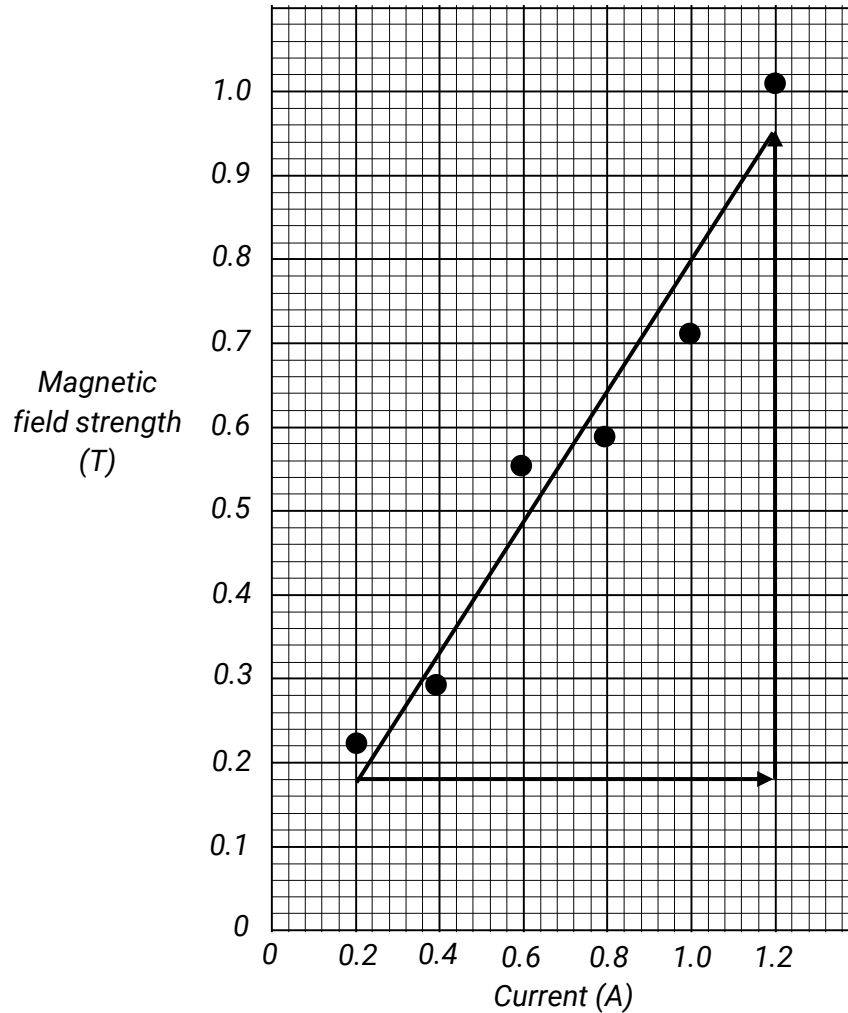
Write a new value for g and its uncertainty here

4A – Linear regression

1. $I = \frac{1}{R}V, F = Eq$

2. (a)

Magnetic field strength vs coil current inside a solenoid



- (b) Y-variable – Magnetic field strength, B
X-variable – Current, I
Gradient – Magnetic permeability constant, μ
Y-intercept – None

(c) $\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{0.76}{1.0} = 0.76 \text{ T} \cdot \text{A}^{-1}$

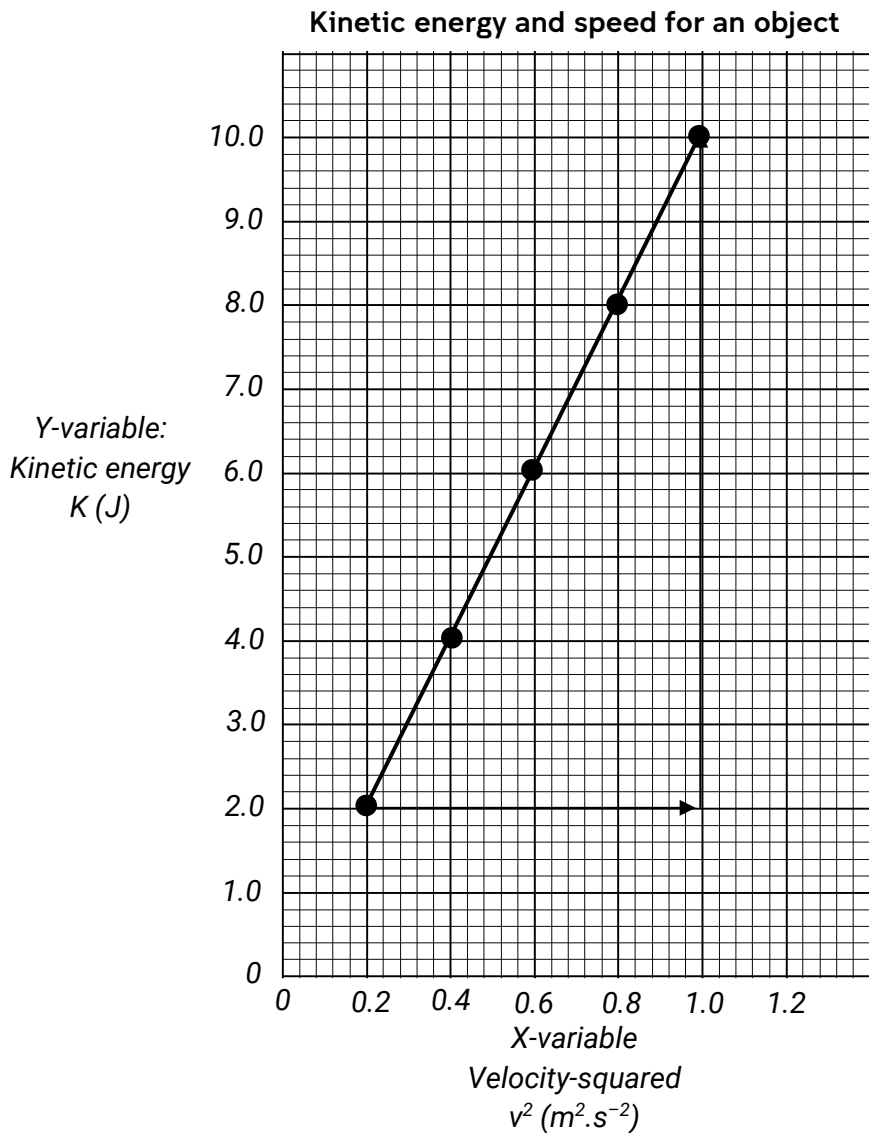
3. (a) $v = at + u$
(b) acceleration, $2.4 \text{ m} \cdot \text{s}^{-2}$
(c) initial speed, $3 \text{ m} \cdot \text{s}^{-1}$

4B – Advanced linear regression

1. (a) $K = \frac{1}{2} m v^2$
- (b) Y-variable – kinetic energy, K
X-variable – velocity-squared, v^2
- (c)

Independent variable	Dependent variable	Processed data
Speed, v ($\text{m}\cdot\text{s}^{-1}$)	Kinetic energy, K (J)	Speed-squared v^2 ($\text{m}^2\cdot\text{s}^{-2}$)
0.45	2.0	0.20
0.63	4.0	0.40
0.77	6.0	0.59
0.89	8.0	0.79
0.99	10.0	0.98

- (d) gradient = $\frac{1}{2}m$
- (e) No, we should not expect a Y-intercept. There was no 'B' term when we put the equation into a straight-line form.
- (f)



(g) gradient = $\frac{1}{2}m \rightarrow m = 2 \times \text{gradient} = 2 \times \frac{8.0}{0.8} = 10 \text{ kg}$

2. (a) $v = f\lambda \rightarrow \lambda = \frac{v}{f} \rightarrow \lambda = v \frac{1}{f}$

(b) Y-variable – wavelength, λ

X-variable – inverse of frequency, $\frac{1}{f}$

(c) Gradient = velocity, v

3. (a)

Equation in linear form $Y = AX + B$	Graph axes	Gradient
$I = I_0 \frac{1}{d^2}$	Y-axis: Intensity I X-axis: inverse of distance squared, $\frac{1}{d^2}$	I_0 Reference intensity

(b)

Equation in linear form $Y = AX + B$	Graph axes	Gradient
$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1$	Y-axis: sine of refracted angle, $\sin \theta_2$ X-axis: sine of incident angle, $\sin \theta_1$	$\frac{n_1}{n_2}$ ratio of refractive indices

(c)

Equation in linear form $Y = AX + B$	Graph axes	Gradient
$\ln \frac{N}{N_0} = -\lambda t$	Y-axis: natural log of the fraction of material remaining, $\ln \frac{N}{N_0}$ X-axis: time, t	$-\lambda$ decay constant

5 – Accuracy and error

1.

True value	Observed value	Absolute error	Percentage error	Accuracy	Accurate?
2.638 m	2.715 m	0.077 m	3%	97%	Yes
15.26 s	13.98 s	1.28 s	8%	92%	No
83 400 kg	84 200 kg	800 kg	1%	99%	Yes
45 N	41 N	4 N	9%	91%	No

2. (a)

Number of coins	"True" values	Observed values
1	15.55 g	14.61 g
2	31.10 g	31.06 g
3	46.65 g	45.68 g
4	62.20 g	61.58 g
5	77.75 g	77.14 g

This is a demonstration of systematic error, since each observation is consistently lower.

(b) The values vary randomly around 100°C – it is a demonstration of random error.

3. Sensitive measuring instruments respond appropriately to variances in quantities being measured, meaning that it is more likely that a measurement will be close to the true value. Using measuring instruments of appropriate resolution means that a measurement is more likely to be made close its true value with a small uncertainty.
4. This technique improves accuracy by increasing the size of the observation relative to the error. Human-induced timing errors stay about the same in absolute terms (~0.5 seconds), but by increasing the length of the observation, the error becomes a smaller fraction.
5. Taring an electronic balance ensures that it reads zero when there is nothing placed on it. If the balance was not tared, then there is a possibility that every reading made will be offset from their actual values.
6. Random errors cause values to vary randomly above and below a true value, in roughly even proportions. A technique like averaging seeks the middle, so an average is likely to approach the true value.

6 – Estimating uncertainty in measurements

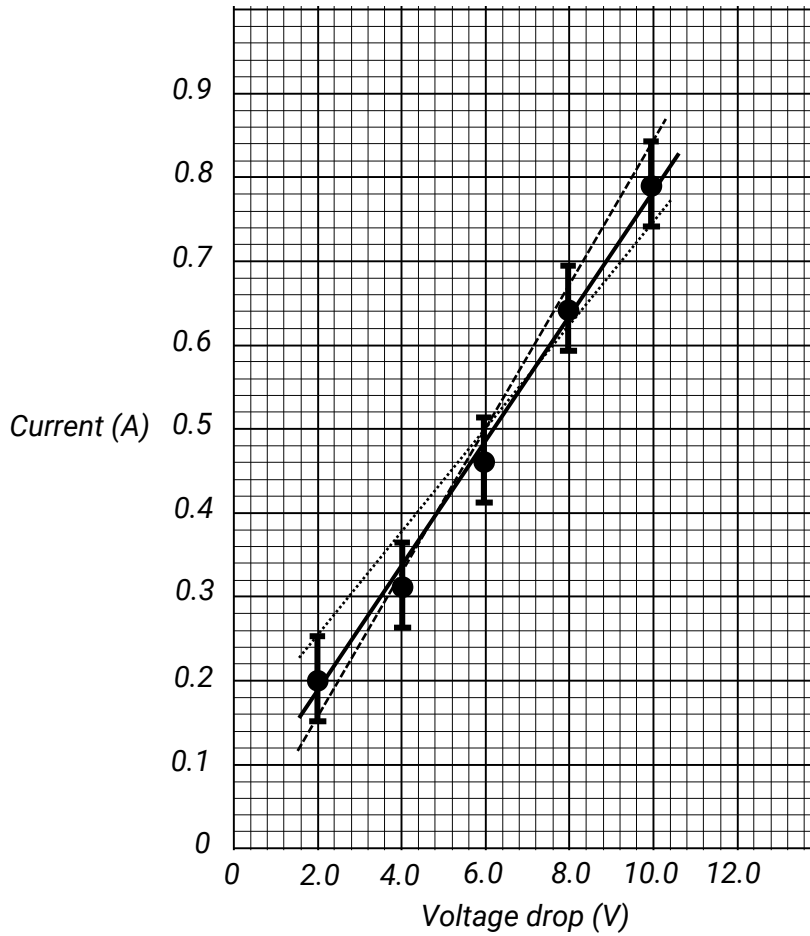
1. 7.0 ± 0.3 cm – We can have more confidence in this measurement.
2. 51.78 ± 0.08 m 0.287 ± 0.005 A
 $841\,100 \pm 500$ kg $(6.3 \pm 0.5) \times 10^4$ V
3. 1% 3%
14% 3%
4. (a) ± 0.005 seconds, ± 0.5 mA, ± 0.05 g, $\pm 0.5^\circ$
(b) 7.060 ± 0.005 seconds, 97.00 ± 0.05 g
5. 9.4 ± 0.7 m.s⁻², 77 ± 3 mL, 100.24 ± 0.07 Ω

7 – Using uncertainties to assess values

1. Agrees (precise), does not agree (imprecise), agrees (imprecise), agrees (imprecise), does not agree (precise).
2. Agree, disagree, agree, disagree, agree.
3. The two measurements agree with each other. Only the measurement made with the car odometer agrees with the required marathon distance. The map method is more precise, because it has a smaller uncertainty.

8 – Estimating uncertainty from a graph

Current vs voltage applied across a resistor



	Gradients
Line of best fit	$0.075 \Omega^{-1}$
Line of worst fit – maximum	$0.086 \Omega^{-1}$
Line of worst fit – minimum	$0.061 \Omega^{-1}$

Uncertainty in gradient = $\pm 0.0125 \Omega^{-1}$

Gradient = $0.08 \pm 0.01 \Omega^{-1}$

$R = 1/0.075 \Omega^{-1} = 12.5 \Omega$

$\Delta R = 12.5 \Omega \times (0.0125/0.075) = 2.1 \Omega$

Thus, $R = 13 \pm 2 \Omega$.

9 – Estimating uncertainty in results

- (b) 12.0 cm, (c) 13.4 cm, (d) 10.6 cm, (e) 12.0 ± 1.4 cm
- (a) 810 ± 20 kg, (b) 65 ± 1 °C
- (b) 42.0 cm^2 , (c) 52.5 cm^2 , (d) 32.5 cm^2 , (e) $42 \pm 10 \text{ cm}^2$, (f) $40 \pm 10 \text{ cm}^2$.
- (a) 26 ± 3 N, (b) $8.5 \pm 0.8 \text{ m.s}^{-1}$.

10 – Assessing reliability in experiments

1. (a) External reliability, (b) $9.8 \pm 1.1 \text{ m.s}^{-2}$, (c) 11% relative uncertainty – low reliability.
2. (a) Internal reliability, (b) $6.2 \pm 0.2 \text{ m}$, (c) 3% relative uncertainty – high reliability.
3. Simple repetition of measurements or experiments does not on its own improve reliability. If nothing has been done to address the causes of variability in the data, then repetitions may just continue to show the variations that are markers of unreliability.
4. (a) Reliability means that measurements or experiments return very similar results every time they are repeated, that is, there is minimal variation between each result. If there is a lot of variation, then the results are not reliable.
(b) Variability is caused by sources of random error. These could be uncontrolled variables, or influences from environmental conditions that cause random fluctuations. They could also come from sloppy work. Variability can be reduced by controlling variables, taking many trials and finding averages, and reducing relative random errors.
(Note that sources of systematic errors do not affect reliability – these errors usually affect measurements in a set of trials the same way, e.g. shifting them all in one direction, rather than causing them to vary randomly – it is possible to have inaccurate yet reliable results because of this)

11 – Assessing validity in experiments

1. (a) Independent variable – Force
(b) Dependent variable – Acceleration
(c) The data does not show a proportional relationship between force and acceleration.
(d) Based on this data, the students might conclude that their hypothesis was false.
(e) The teacher’s data shows that they did not control the masses of the cars. This means that we cannot trust the conclusion in (d).
(f) The student’s investigation was not valid because they did not control all variables.
(g) The validity could be improved by controlling the mass of the toy cars, i.e. keeping them all the same mass.
2. In a valid experiment, only the independent variable should be varied; all other variables should be controlled (held constant). Also, the equipment used when conducting the experiment must be appropriately chosen, and measurements should be made carefully with instruments of appropriate type and resolution.

14 – Spreadsheets – An exercise

1.

	A	B	C	D	E	F	G
1	Length (m)	Trial 1 - 10T	Trial 2 - 10T	Trial 3 - 10T	Av - 10T	T	T ²
2	0.2	8.84	8.80	8.76	8.83	0.883	0.780
3	0.4	12.50	12.56	12.47	12.48	1.248	1.558
4	0.6	15.31	15.36	15.25	15.34	1.534	2.352
5	0.8	17.68	17.64	17.64	17.66	1.766	3.119
6	1.0	19.77	19.83	19.70	19.76	1.976	3.905

2. =average(B2, C2, D2) or =average(B2:D2)

3. =E2/10

4. =F2^2

6. Gradient = $3.91 \text{ s}^2 \cdot \text{m}^{-1}$, $g = 10.1 \text{ m} \cdot \text{s}^{-2}$.

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